NATIONAL TEST IN MATHEMATICS COURSE B
AUTUMN 2005

Directions

Test time — 240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources — **Part I:** ”Formulas for the National Test in Mathematics Course B”
Please note that calculators are not allowed in this part.

Part II: Calculators and ”Formulas for the National Test in Mathematics Course B”.

Test material — The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 17 problems. **Part I** consists of 8 problems and **Part II** consists of 9 problems.

For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 43 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test

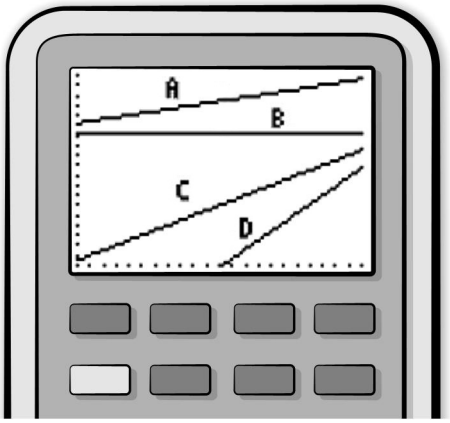
Pass:	12 points
Pass with distinction:	25 points of which at least 7 ”Pass with distinction”-points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”-points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Solve the equation $x^2 - 2x - 24 = 0$ (2/0)

2. The figure shows the window of a graphing calculator where four straight lines *A*, *B*, *C* and *D* have been drawn.



Arrange the lines according to the values of their gradients. Start with the line with the lowest value. *Only answer is required* (1/0)

3. *M* is the centre of the circle below.

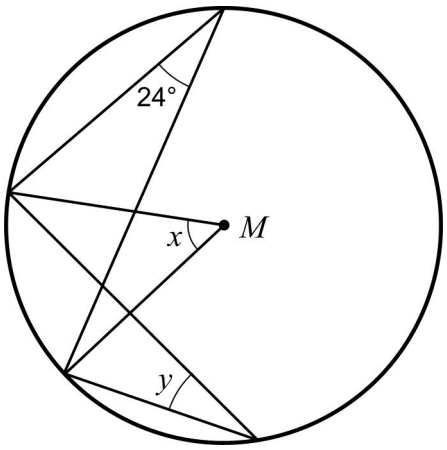


Figure not drawn to scale.

a) How large is the angle *x*? *Only answer is required* (1/0)

b) How large is the angle *y*? *Only answer is required* (1/0)

4. In a shop, the price of apples is x SEK/kg and the price of oranges is y SEK/kg.

You know that the following two relations are true: $\begin{cases} 2x + 3y = 69 \\ x + 2y = 42 \end{cases}$

a) Solve the simultaneous equations above. (2/0)

b) Calculate the value of the expression $3x + 4y$. In words, explain what the expression and the calculated value represent. (1/1)

5. Write down *one* value of x that satisfies the inequality $40 + 10x < -5$. Justify your answer. (1/0)

6. a) In a coordinate system, draw a straight line that passes through the point (2, 3) and has a negative gradient of your choice. (1/0)

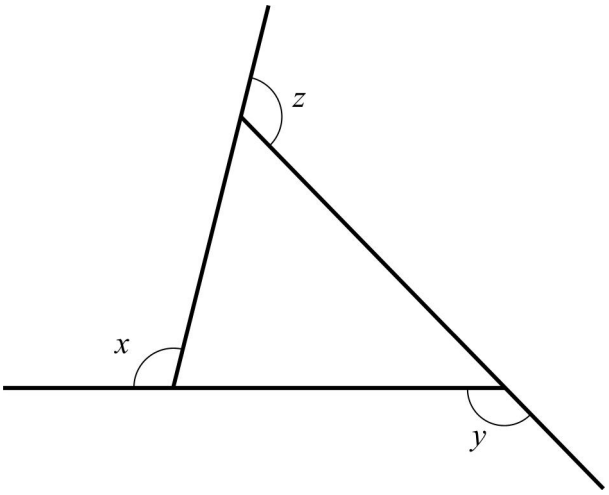
b) Determine the equation of the line. (2/0)

7. In the function $f(x) = x^2 + 4ax - 15$, a is a constant.

Find a such that $f(-3) = 0$ (0/2)

8. x, y and z are exterior angles of the triangle below.

Show that $x + y + z = 360°$ (0/1/¤)



Part II

This part consists of 9 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

9. Simplify as far as possible

a) $(x+4)(x-4)+2(x+8)$ (1/0)

b) $(3a)^2-3a^2$ (1/0)

10. In the triangle ABC, DE is parallel to AC. Calculate the length of DE. (2/0)

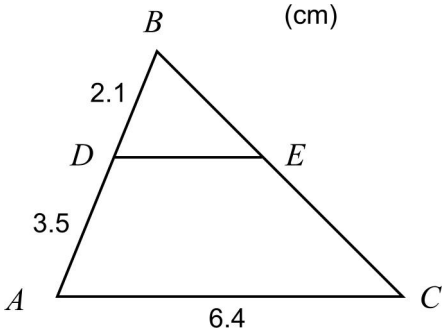


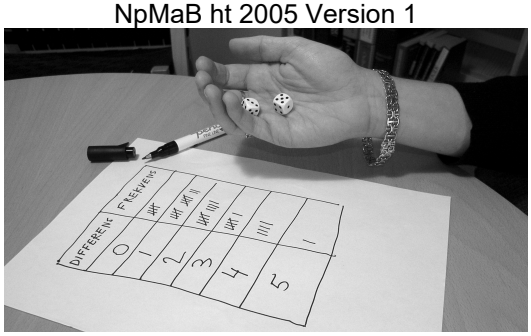
Figure not drawn to scale.

11. Vilma carries out an experiment during a maths lesson. She throws two ordinary six-sided symmetric dice 200 times. After each throw she writes down the difference between the largest and the smallest number of dots that the dice show. Vilma writes down the results in the following table of frequency.

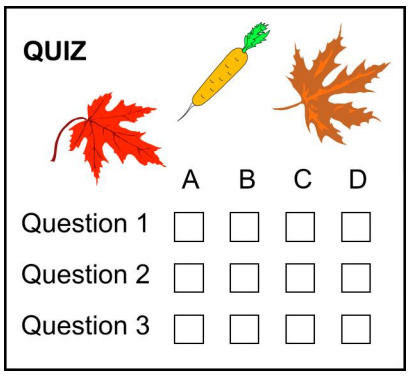
Difference	Frequency
0	22
1	68
2	51
3	37
4	18
5	4

a) What is the experimental probability of getting a difference of 1 according to Vilma's experiment? (1/0)

b) What is the theoretical probability of getting a difference of 1? (1/1)


DIFFERENCE
FREQUENCY
0
1
2
3
4
5

12. Filip follows his parents to the autumn party at the Garden Club. There is a quiz where you can win a nice prize if you answer three questions correctly. For each question there are four alternatives of which only one is correct.



Filip has no idea which answers are the correct ones, so he guesses completely at random.

a) What is the probability that Filip guesses correctly on question 1?
Only answer is required (1/0)

b) What is the probability that Filip guesses correctly on all three questions?
Only answer is required (1/0)

c) What is the probability that Filip guesses correctly on at least one of the questions? (0/2)

13. To be able to exercise at the right intensity, it is useful to know your maximum heart rate. For example it is suitable to have a heart rate which is 70 % of your maximum heart rate when fitness training. The table below shows how the theoretical maximum heart rate varies with age:

Age (years)	Max heart rate (beats/minute))
25	195
30	190
35	185
40	180
45	175
50	170
55	165
60	160
65	155
70	150

a) Use the table above to estimate the maximum heart rate of a twenty-year old.
Only answer is required (1/0)

b) The relationship between maximum heart rate and age seems to be a linear one. Explain how this can be decided with help of the table. (1/0)

c) Find the function that describes how the maximum heart rate varies with age. (0/2)

14. In a magic square the sums of the numbers in the squares are the same for all rows, all columns and all diagonals. In the square below, different expressions have been written in some of the squares.

a) Determine the positive x-value which makes the values of the expressions written in the squares satisfy the conditions for a magic square. (0/2)

b) Calculate the values for each of the nine squares and then draw the whole magic square. (0/1)

$x^2 - 20$	?	?
x^2	$3x + 2$	$x - 2$
$x + 2$	?	?

15. At an upper secondary school it was suggested that the school day should be shifted forward half an hour, that is, the day should begin and end later. Therefore, the student council carried out a survey among the students at the school.

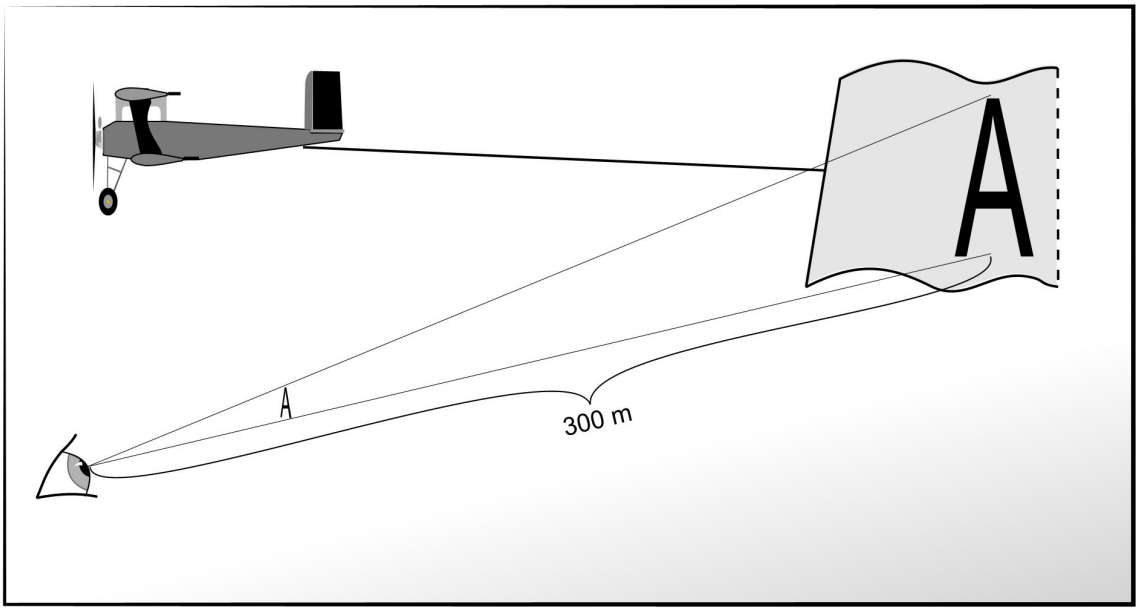
Out of 980 students at the school, 632 answered. Out of these, 70.1 % answered that they wanted to shift the school day.

The student council presents the result of the survey to the school board and claims that the students at the school want a change. The school board says that the survey does not clearly show this, but rather that a majority of the students can actually be against a change.

Can a majority of the students be against the suggestion? (0/2)

16. When Karin is on the Canary Islands she sees a small aeroplane with a banner attached. She starts thinking about the size of the letters, and how large they have to be for the message to be readable from the ground. Karin estimates the plane to be flying at a distance of 300 metres.

Do your own estimations of the measures you need and help Karin calculate an approximate value of how large the letters have to be to be readable from the ground. (0/1/¤)



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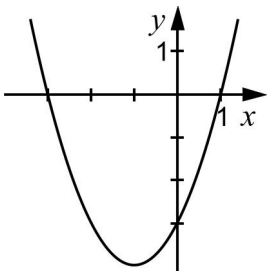
When assessing your work with the following problem, your teacher will take into consideration:

- How well you carry out your calculations
- How well you present and comment on your work
- How well you justify your conclusions
- What mathematical knowledge you show
- How well you use the mathematical language
- How general your solution is

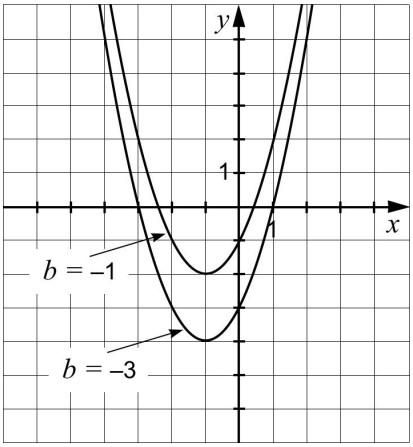
17. The aim of this problem is to investigate how different values of the real constants a and b affect the solutions to the equation $f(x) = 0$, when $f(x) = x^2 + ax + b$

- If $a = 2$ and $b = -3$ then $f(x) = x^2 + 2x - 3$
  The graph of this function can be seen in the figure on the right.



  Solve the equation $f(x) = 0$, when $f(x) = x^2 + 2x - 3$

- In the figure on the right, there are two graphs to $f(x) = x^2 + 2x + b$ with different values of the constant b.



  Investigate and describe, as detailed as you can, how the constant b affects the number of solutions to the equation $f(x) = 0$

- Determine algebraically how the constants a and b affect the number of solutions to the equation $f(x) = 0$ when $f(x) = x^2 + ax + b$ (2/4/¤)