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## NATIONAL TEST IN MATHEMATICS COURSE B

**Autumn 2010**

### Directions

|                                |   |       |            |                        |   |                                |  |
|--------------------------------|---|-------|------------|------------------------|---|--------------------------------|--|
| Test time                      | 240 minutes for Part I and Part II together. <b>We recommend that you spend no more than 60 minutes on Part I.</b>  |       |            |                        |   |                                |  |
| Resources                      | <p><b>Part I:</b> "Formulas for the National Test in Mathematics Course B"<br/> <i>Please note that calculators are not allowed in this part.</i></p> <p><b>Part II:</b> Calculators, also symbolic calculators and "Formulas for the National Test in Mathematics Course B".</p>   |       |            |                        |   |                                |  |
| Test material                  | <p>The test material should be handed in together with your solutions.</p> <p>Write your name, the name of your education programme/adult education on all sheets of paper you hand in.</p> <p><i>Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.</i></p>   |       |            |                        |   |                                |  |
| The test                       | <p>The test consists of a total of 18 problems. <b>Part I</b> consists of 8 problems and <b>Part II</b> consists of 10 problems.</p> <p>For some problems (where it says <i>Only answer is required</i>) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.</p> <p>Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.</p> <p>Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.</p> |       |            |                        |   |                                |  |
| Score and mark levels          | <p>The maximum score is 41 points.</p> <p>The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1). Some problems are marked with <math>\alpha</math>, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for "Pass with Special Distinction".</p> <p>Lower limit for the mark on the test</p> <table style="width: 100%;"> <tr> <td>Pass:</td><td>12 points.</td></tr> <tr> <td>Pass with distinction:</td><td>25 points of which at least 6 "Pass with distinction"-points.</td></tr> <tr> <td>Pass with special distinction:</td><td>25 points of which at least 13 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the <math>\alpha</math>-problems give the opportunity to show.</td></tr> </table>                | Pass: | 12 points. | Pass with distinction: | 25 points of which at least 6 "Pass with distinction"-points. | Pass with special distinction: | 25 points of which at least 13 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the $\alpha$ -problems give the opportunity to show. |
| Pass:                          | 12 points.  |       |            |                        |   |                                |  |
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| Pass with special distinction: | 25 points of which at least 13 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the $\alpha$ -problems give the opportunity to show.  |       |            |                        |   |                                |  |

## Part I

**This part consists of 8 problems that should be solved without the aid of a calculator.** Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Solve the simultaneous equations  $\begin{cases} x + 3y = 10 \\ 2x - y = 6 \end{cases}$  (2/0)

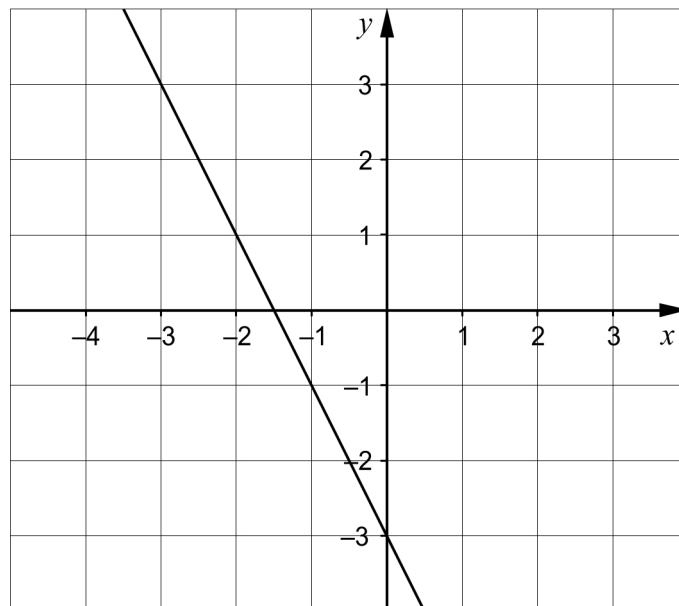
2. Jens is participating in a dressage competition. There are twelve participants and Jens is one of them. The start order is determined the following way: each of the participants takes a note from a riding helmet where there are twelve notes with the numbers 1 to 12 (one number on each note). Jens is the first one to take a note. He would prefer to be the last one to ride.

What is the probability that Jens will be the last one to ride?

*Only answer is required* (1/0)

3. Solve the equation  $x^2 - 2x - 24 = 0$  (2/0)

4. Find the equation of the line in the figure on the form  $y = kx + m$  (2/0)



5. Simplify as far as possible

a)  $(x + 4)(x - 4) + 2(x + 8)$  (1/0)

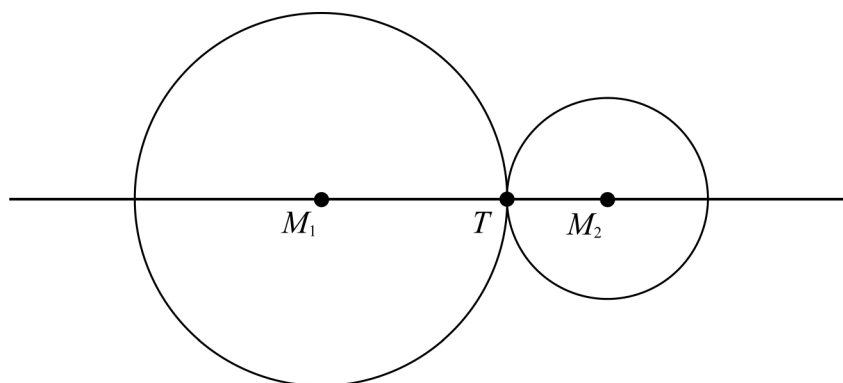
b)  $(3a)^2 - 3a^2$  (1/0)

6. A bowl of sweets contains 4 raspberry sweets and 6 liquorice sweets. Lukas and Emma take one sweet each from the bowl without looking.

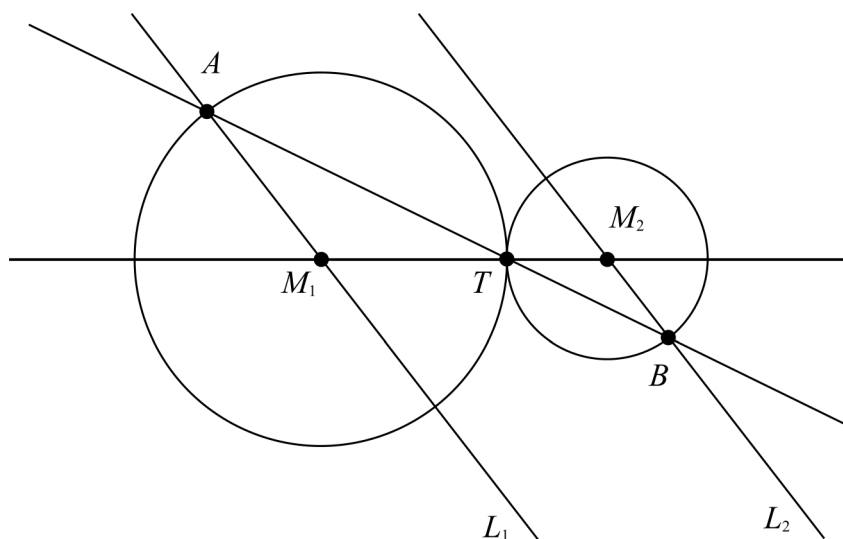
What is the probability that both Lukas and Emma get a raspberry sweet? (1/1)

7. Two straight lines have the equations  $y = 3x - 1$  and  $y - 3x - 1 = 0$ . Investigate whether the lines are parallel. (0/1)

8. Two circles touch each other at a point  $T$ . A line that passes through the centres of the circles  $M_1$  and  $M_2$  also passes through the point of tangency  $T$ .



A straight line is drawn through the point of tangency and intersects the circles at points  $A$  and  $B$ . Another two lines  $L_1$  and  $L_2$  are drawn. Line  $L_1$  passes through  $M_1$  and  $A$ . Line  $L_2$  passes through  $M_2$  and  $B$ .



Show that the lines  $L_1$  and  $L_2$  are parallel. (0/2/∞)

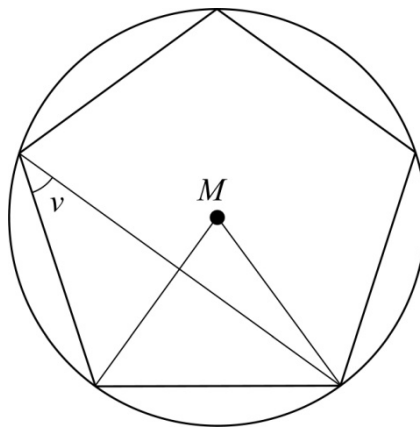
## Part II

**This part consists of 10 problems and you may use a calculator when solving them.**  
Please note that you may begin working on Part II without a calculator.

9. The Alm and Bok families are having dinner at a pizzeria. The Alm family order two Vesuvio pizzas and three Margerita pizzas and pay SEK 345. The Bok family order one Vesuvio pizza and two Margerita pizzas and pay SEK 205.

What is the price of one Vesuvio pizza and one Margerita pizza respectively? (2/0)

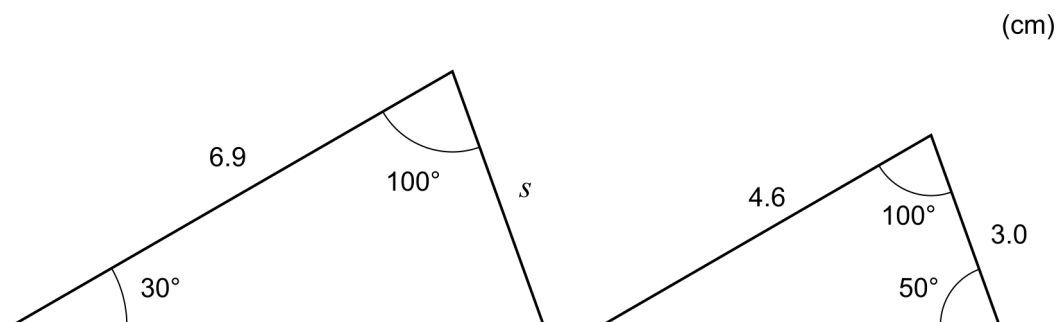
10. A regular five-sided polygon is inscribed in a circle with centre  $M$ .



Determine the angle  $v$ .

(2/0)

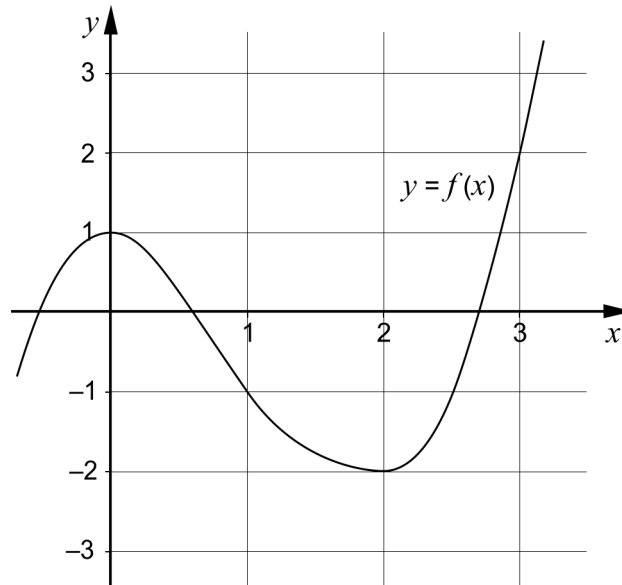
11. The figure shows two triangles.



Calculate the length of side  $s$ .

(1/1)

12. The figure shows the graph of the function  $y = f(x)$



- a) Determine  $f(1)$  *Only answer is required* (1/0)
- b) For what  $x$  is  $f(x) = 2$ ? *Only answer is required* (1/0)

13. A company has 50 employees. During wage negotiations, there was a proposal to raise the hourly wage by SEK 5 for the group with the lowest hourly wage and for the group with the highest hourly wage, see table.

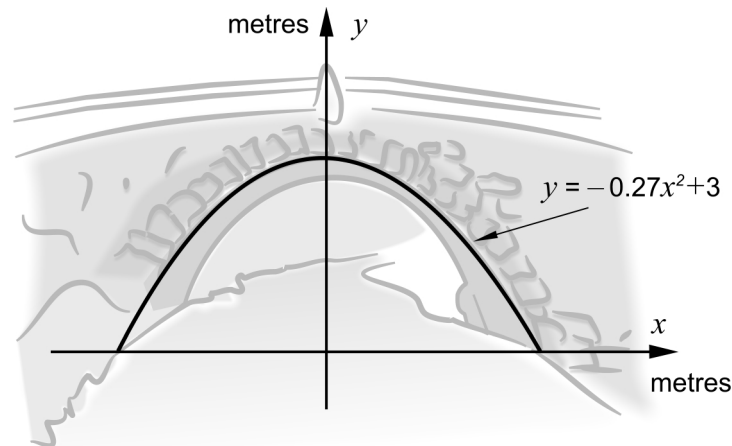
| Number of employees | Hourly wage (SEK) | New hourly wage (SEK) |
|---------------------|-------------------|-----------------------|
| 21                  | 90                | 95                    |
| 20                  | 100               | 100                   |
| 6                   | 110               | 110                   |
| 3                   | 120               | 125                   |

The manager: "Then we will get a higher wage level and an increased wage differential in the company."

The union representative: "I don't agree. I'm of the opinion that the wage level and the wage differential remain unchanged."

- a) Investigate how the medium, the median and the range of distribution change if the proposal is realized. (2/0)
- b) Use your results to find arguments that the union representative and the manager can use to support their statements. (0/1/?)

14. In Sweden they began to build stone bridges with wedge-shaped stones during the 17th century. The picture shows an example of such a bridge over a river.



The arch of the bridge in the picture has a shape that can be described by the quadratic function  $y = -0.27x^2 + 3$  where  $y$  is the height of the arch above the water surface.

- a) Determine the maximum height of the arch above the water surface. *Only answer is required* (1/0)
- b) Approximately how wide is the river under the arch of the bridge? (0/2)

15. Viktor and Emilia calculate the probabilities of penalty throws in basketball. They estimate that the probability of a "hit" is 0.7 and the probability of a "miss" is 0.3 in each throw.

Viktor correctly calculates the probability of two "hit" to 0.49 and of two "miss" to 0.09. Emilia also gets 0.49 and 0.09 but when they take a look at their calculations they see they have carried them out differently.

Viktor's method:  $P(\text{hit}, \text{hit}) = 0.7 \cdot 0.7 = 0.49$   
 $P(\text{miss}, \text{miss}) = 0.3 \cdot 0.3 = 0.09$

Emilia's method:  $P(\text{hit}, \text{hit}) = 0.7 \cdot 0.7 = 0.49$   
 $P(\text{miss}, \text{miss}) = 0.49 - (0.7 - 0.3) = 0.09$

To get the probability of two "miss" Emilia first calculates the probability of two "hit". From this result she then subtracts the difference between the probability of a "hit" and the probability of a "miss".

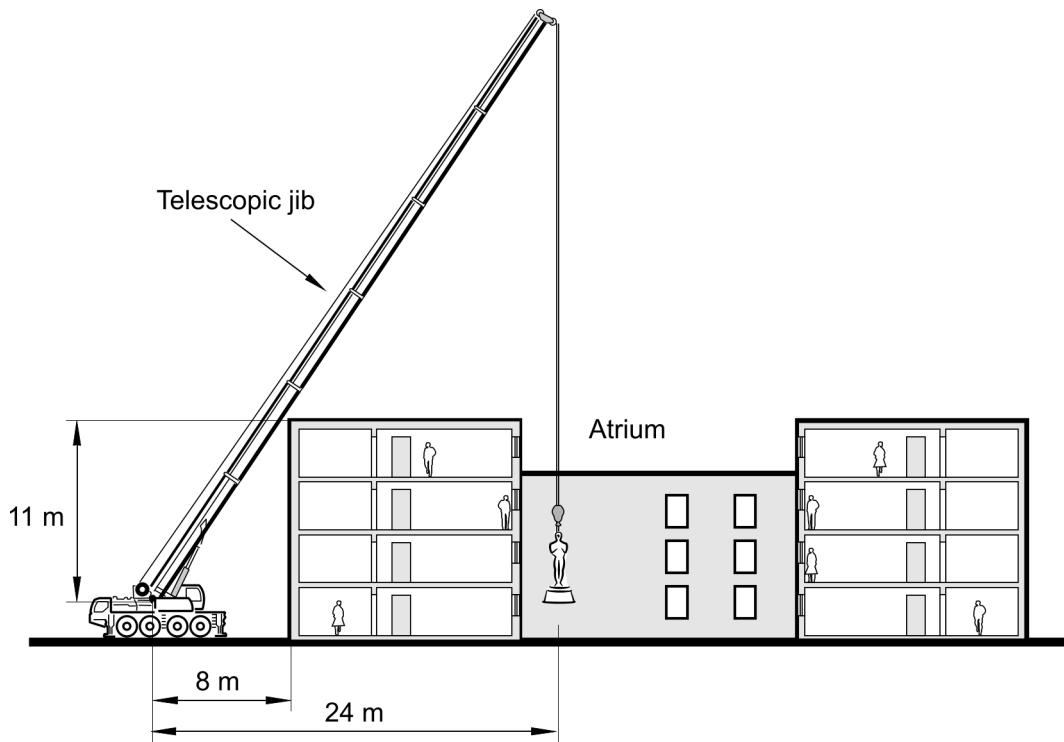
- a) Investigate whether Emilia's method of calculating the probability of two "miss" works if the probability of a "hit" is 0.9 (0/1)
- b) Investigate whether Emilia's method of calculating the probability of two "miss" works *independently* of the probability of a "hit". (0/1/?)

16. You have been commissioned by Kran & Lyft AB to plan for the lifting of a statue. The statue will be lifted over a roof and into an atrium.

The figure below shows the building in cross-section.

The company has access to three crane lorries with different lengths on their telescopic jibs. One of the crane lorries has a telescopic jib that can be extended to 36 metres. The other two have telescopic jibs that can be extended to 48 metres and 54 metres respectively.

For security reasons, the crane lorry must be positioned so that the abutment of the jib is 8 metres from the wall of the house. The distance to the spot where the statue is to be placed is then 24 metres. The vertical distance from the abutment of the jib to the roof is 11 metres for all crane lorries. See figure.

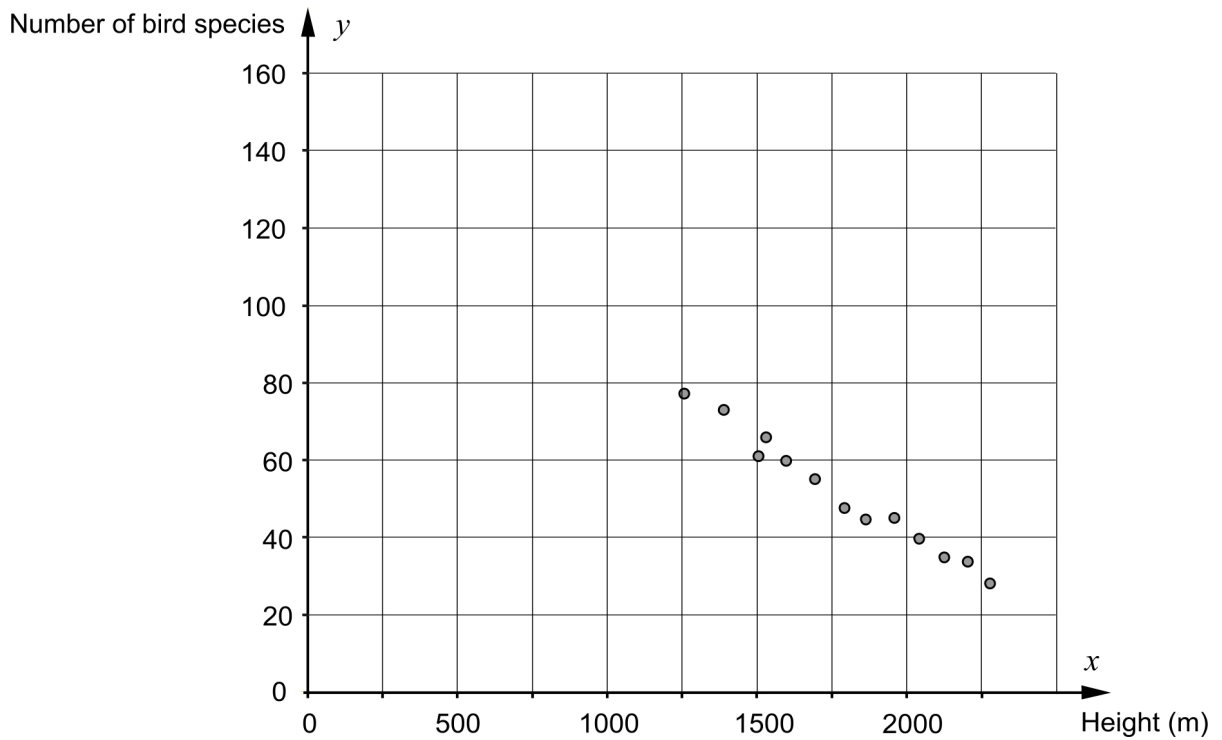


You have been told that the most profitable thing to do is to use the crane lorry with the shortest telescopic jib that can still carry out the lifting.

Which crane lorry should you choose for it to be the most profitable choice?

(0/2)

17. Researchers have studied the number of bird species at Mount Karimui in New Guinea. They have found a linear relationship between the height above sea level and the number of bird species. Their results are put together in the coordinate system below.



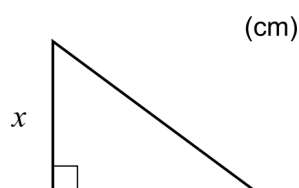
- a) Draw a straight line that adjusts as well as possible to the points above. Determine a relationship of the form  $y = kx + m$  for the straight line you have drawn. (0/2)
- b) The height of Mount Karimui is 2500 metres. How many bird species are there on the top of the mountain according to the relationship you have determined? (0/1)



**When assessing your work with this problem the teacher will take into consideration:**

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

18. In this problem you will investigate a certain type of right-angled triangle. In these triangles, the difference in length between the short leg (i.e. the shortest side) and the hypotenuse is twice the difference in length between the short leg and the long leg.



The figure shows such right-angled triangle. The length of the short leg is  $x$  cm. The length of the long leg is 3 cm longer than the short leg. The length of the hypotenuse is 6 cm longer than the short leg.

- Write down expressions for the lengths of the long leg and the hypotenuse. Then calculate the length of the short leg.

In another triangle of this type the long leg is 2 cm longer than the short leg. The hypotenuse is 4 cm longer than the short leg.

- Calculate the length of the short leg in this triangle.

|                  | <b>The difference in length between the short and long legs</b> | <b>The difference in length between the short leg and the hypotenuse</b> | <b>The length of the short leg</b> |
|------------------|---|--|------------------------------------|
| The 1st triangle | 3 cm  | 6 cm   | ?                                  |
| The 2nd triangle | 2 cm  | 4 cm   | ?                                  |
| ...              | ...   | ...  | ...                                |

With this type of triangle there is a relationship between the length of the short leg and the difference in length between the short and the long legs.

- Investigate this type of triangle. Based on your investigation, formulate a conclusion about the relationship between the length of the short leg and the difference in length between the short and long legs.

(2/4/π)