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NATIONAL TEST IN MATHEMATICS COURSE B

AUTUMN 2011

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.						
Resources	Part I: "Formulas for the National Test in Mathematics Course B" <i>Please note that calculators are not allowed in this part.</i> Part II: Calculators, also symbolic calculators and "Formulas for the National Test in Mathematics Course B".						
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. <i>Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.</i>						
The test	The test consists of a total of 17 problems. Part I consists of 9 problems and Part II consists of 8 problems. For some problems (where it says <i>Only answer is required</i>) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.						
Score and mark levels	The maximum score is 42 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1). Some problems are marked with α, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for "Pass with Special Distinction". Lower limit for the mark on the test <table style="width: 100%;"> <tr> <td>Pass:</td><td>13 points.</td></tr> <tr> <td>Pass with distinction:</td><td>26 points of which at least 6 "Pass with distinction"-points.</td></tr> <tr> <td>Pass with special distinction:</td><td>26 points of which at least 13 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the α-problems give the opportunity to show.</td></tr> </table>	Pass:	13 points.	Pass with distinction:	26 points of which at least 6 "Pass with distinction"-points.	Pass with special distinction:	26 points of which at least 13 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the α -problems give the opportunity to show.
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Part I

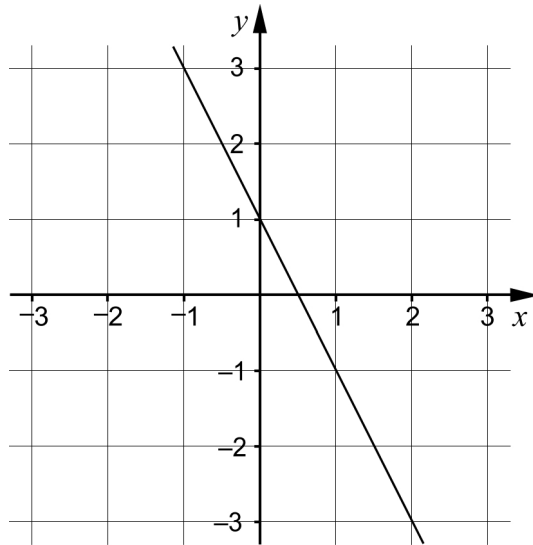
This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Which of the functions A – F corresponds to the graph below?

Only answer is required

(1/0)

- A. $y = 2x + 1$
- B. $y = -2x + 1$
- C. $y = -0.5x + 1$
- D. $y = 0.5x + 1$
- E. $y = 0.5x - 1$
- F. $y = x + 0.5$



2. Solve the equation $x^2 - 8x - 9 = 0$

(2/0)

3. Solve the simultaneous equations $\begin{cases} 2y - 3x = 30 \\ 2y + x = -2 \end{cases}$

(2/0)

4. Simplify the following expressions as far as possible:

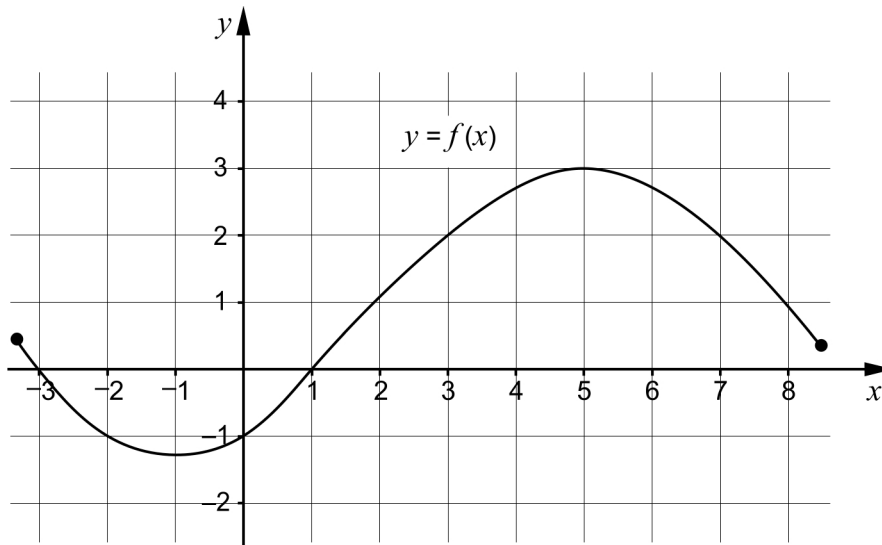
a) $3a \cdot 3a + 3a + 3a$

(1/0)

b) $(x + 4)(x - 4) + 2(x^2 - 8)$

(2/0)

5. The figure shows the graph of the function $y = f(x)$

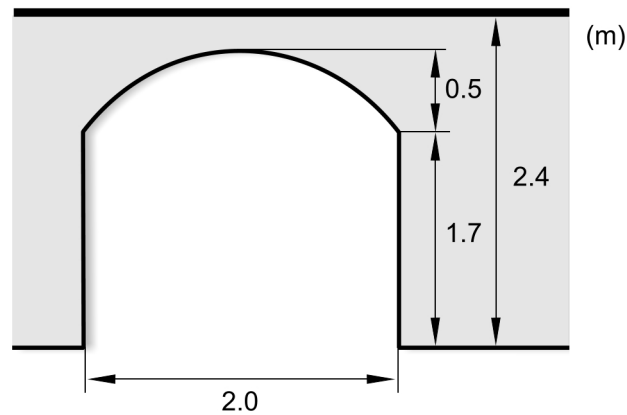


- a) Determine $f(0)$ *Only answer is required* (1/0)
- b) For what value of x is $f(x) = 3$? *Only answer is required* (1/0)
6. At one school, the board of the school council consists of two girls and three boys. By drawing lots, first one person is to be appointed treasurer and then a second person is appointed secretary.
- a) What is the probability that a girl is appointed secretary at the draw? *Only answer is required* (1/0)
- b) What is the probability that the draw results in the treasurer and the secretary being of different gender? (1/1)
7. It holds for the linear function f with gradient k that:
- $k > 0$
 - The graph of f passes through the point $(1, 5)$

Is it possible for the graph of f to also pass through the point $(3, 3)$? Justify your answer.

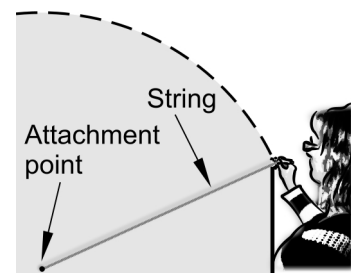
(0/1)

8. Ivan's and Anna's grandfather is going to create an arch between two rooms in his summer cottage. He wants the vault to have the shape of an arc with measurements according to the sketch.



"We can help you draw an arc on the wall. You can then use it when you are going to saw out the vault", said Ivan.

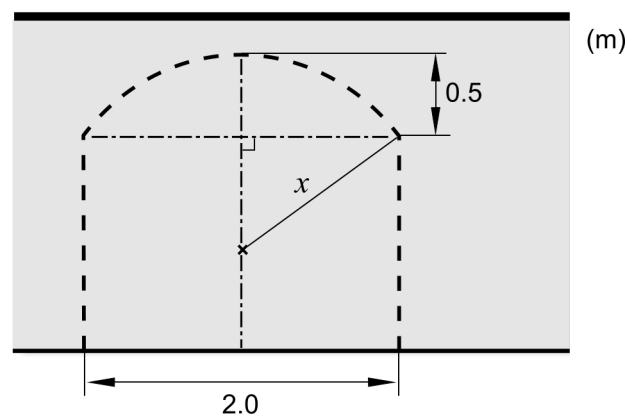
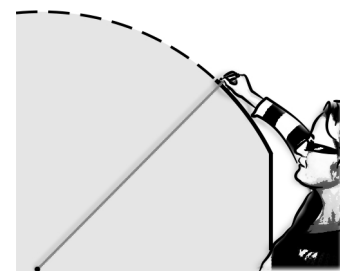
"To get a smooth arc, we can use a piece of string and a pencil like this", said Anna. See pictures.



"But how do we decide the radius of the arc?" Grandfather wondered.

"We can calculate it", said Ivan, and drew a new sketch with measurements.

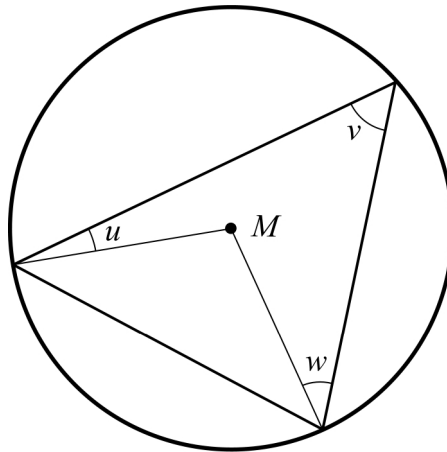
See sketch below.



Use Ivan's sketch and calculate the length of the radius x .

(0/3)

9. A triangle is inscribed in a circle according to the figure below. The centre of the circle M is inside the triangle.



Show that $v = u + w$

(0/2/□)

Part II

This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. Determine an equation to the straight line that passes through the points (2, 5) and (6, 11). (2/0)
11. Lisa is going downhill skiing in Tärnaby during her Christmas holidays. She reads about the ski pass prices on the Internet. For youths, an eight-day ski pass is SEK 995. If more days are wanted, it costs SEK 85 per extra day. It is also possible to buy a season ski pass at SEK 2 395.

Ski pass	Adult	Youth
8 days*	SEK 1 240	SEK 995
Season pass	SEK 3 100	SEK 2 395
*Extra day	SEK 105	SEK 85

Lisa thinks about when it pays to buy a season pass. To get an answer to her thoughts, she writes down the following inequality.

$$995 + 85 \cdot x < 2395$$

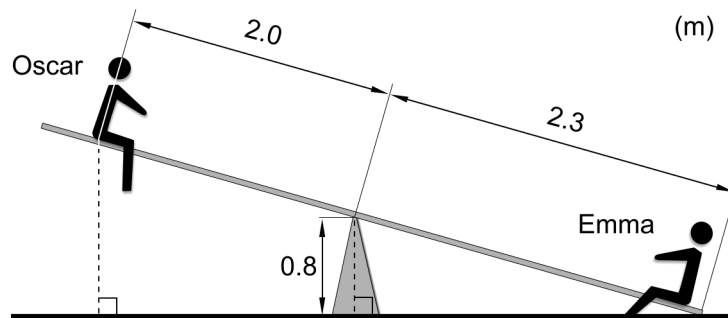
- a) Solve Lisa's inequality. (1/0)
- b) Help Lisa get an answer to her thoughts by interpreting your solution to the inequality. (0/1)

12. Oscar and Emma are going to ride on a seesaw. The plank they sit on is attached in the middle onto a 0.8-metre high support.

Oscar sits 2.0 metres from the middle of the plank. Emma sits on the opposite side so the seesaw tilts over and the end of the plank rests on the ground on Emma's side, see figure.

How high above the ground does Oscar sit?

(2/0)



13. At shot put, the trajectory of the shot can be described by a simple mathematical model. On one occasion, the trajectory of the shot was measured and could be described by the function

$$y = -0.07x^2 + 0.8x + 2$$

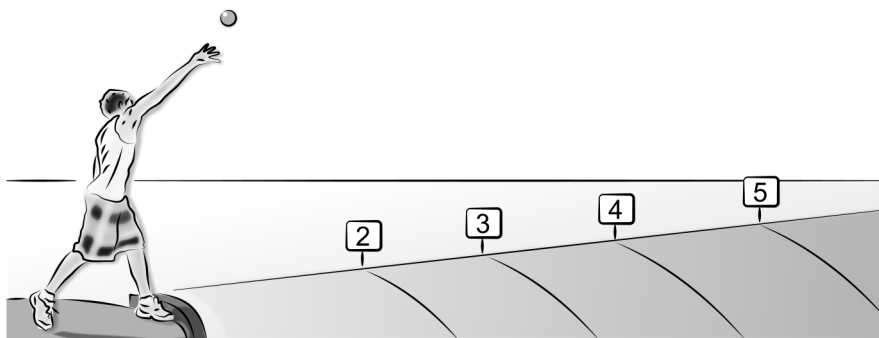
y is the height of the shot above ground.

x is the distance in metres along the ground from the edge of the circle.

There are markings along the ground for different distances. The shot was close to its maximum height when it passed the marking for 6 metres.

- a) What was the height of the shot when it passed the marking for 6 metres? (1/0)

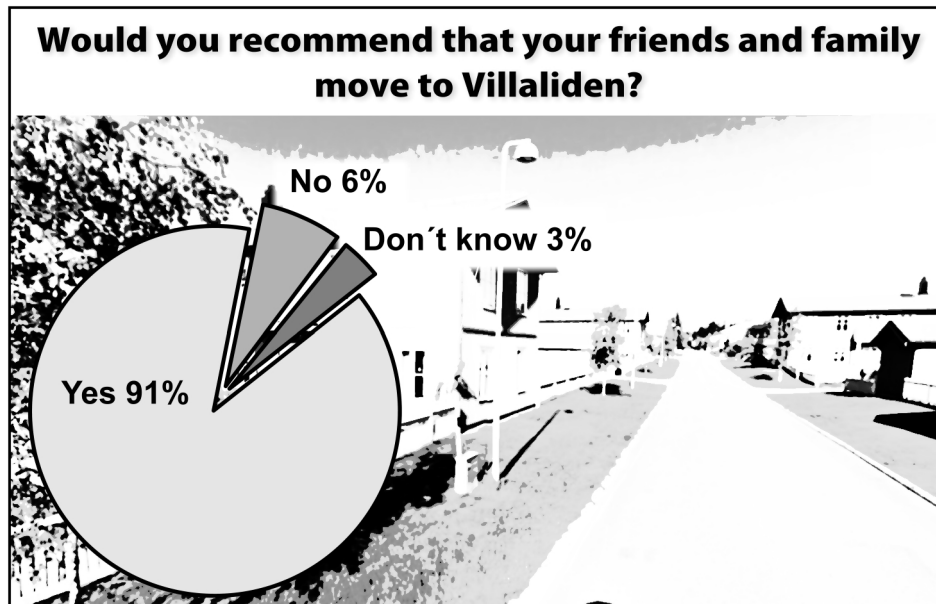
- b) What was the length of the shot, that is, how far from the edge of the circle did the shot hit the ground? (0/2)



14. To investigate the residents' comfort in the area Villaliden, the municipality sent out a survey to each of the 1 337 households in the area. The survey was answered by 54 % of the households.

The survey contained, among other questions, the following one: "Would you recommend that your friends and family move to Villaliden?" Out of those who answered the survey, 91 % answered "Yes" to this question.

The results of the survey were compiled in a brochure. The picture shows the front page of this brochure.



- a) In what way is the information on the front page misleading? (1/0)
- b) Investigate between what percentages the part answering "Yes" could be if every household had answered the survey. (0/2)

15. In a mathematics textbook it states that:

"If the difference between two numbers is 1 then the difference between the square of the greater number and the square of the lesser number is always as great as the sum of the numbers."

Show that this holds true for all such numbers.

(0/2/□)

16. At a staff meeting at a company, the manager informed her employees that the salary level at the company was to be increased with SEK 2 000 per month.

Later, during the coffee break, the following discussion between two employees was heard:

"Didn't the manager say that the salaries would rise so that the mean salary would rise by SEK 2 000?"

"No, I think she said that the salaries would rise so that the median salary would rise by SEK 2 000."

The company has 50 employees and their salaries before the rise can be seen in the table.

Number of employees	Monthly salary (SEK)
21	25 000
20	27 000
6	29 000
3	32 000

A rise of the mean salary or the median salary can lead to different increases in salary costs for the company.

- a) Investigate how large the increase of salary costs can be at most and at least, if the *mean salary* rises by SEK 2 000. You can assume that no salaries are cut.

(1/0)

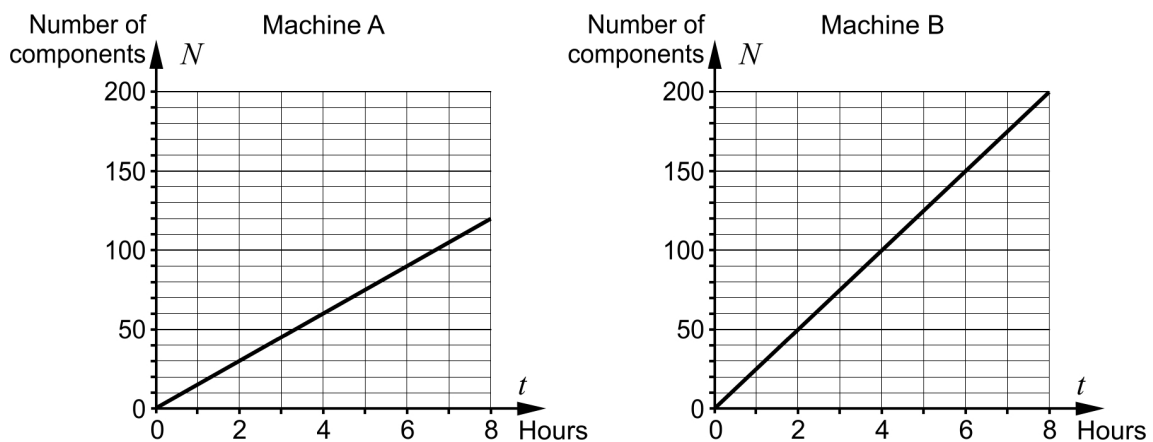
- b) Investigate how large the increase of salary costs can be at most and at least, if the *median salary* rises by SEK 2 000. You can assume that no salaries are cut.

(0/2/□)

When assessing your work with this problem the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

17. Barbro and Conny run a small business. They have two machines that both produce the same component for car engines. The newer machine is more environmentally-friendly but works slower, and the older machine is less environmentally-friendly but works faster. Each machine must be run by one person. How fast the machines work can be seen from the graphs.



- How many components can the two machines produce together at most during an 8-hour working day?
- Write down the relationship between the number of produced components N and the time t in hours for Machine A. Write down the corresponding relationship for Machine B.

Machine A is used as much as possible since it is more environmentally-friendly. Each working day is eight hours long, at most.

- During one working day, 270 components are to be produced. Both Conny and Barbro can run the machines during the day. How long should each machine be run in order to be as environmentally-friendly as possible?
- Another working day, only Barbro can work with the machines and she can therefore only run one machine at the time. She is going to produce 172 components and starts with Machine A. After how long time should she switch to Machine B?

Some days, one of them has to be away on customer visits. Barbro and Conny discuss how they then should plan their work with the machines and come to the following conclusion:

- For the sake of the environment, Machine A should be used as much as possible.
- For safety's reasons, one person should only run one machine at the time .

- Help Barbro and Conny to sort out how long each machine should be run during a working day when only one of them can run the machines, depending on how many components that is to be produced during the day.

(3/3/□)