NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2002 (Syllabus 2000)

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” Please note calculators are not allowed in this part. **Part II:** Calculators, and “Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 15 problems. **Part I** consists of 6 problems and **Part II** consists of 9 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 42 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 “Pass”-points and 1 “Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 12 points Pass with distinction: 24 points of which at least 6 “Pass with distinction points”. Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: ______________________________ School: ________________________

Education programme/adult education: ____

Part I

This part consists of 6 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate the following.

a) $y = 2x^3 - 5$ *Only answer is required* (1/0)

b) $y = e^{4x}$ *Only answer is required* (1/0)

2. The function $y = x^2 - 4x + 8$ has a minimum point.

By using the derivative, find the x-coordinate for this point. (2/0)

3. In January 2001, Karin deposited 3000 crowns into a savings account. The interest on the account is 4 %. Karin continues to deposit 3000 crowns into the account in January each year.

Which of the following describes how much money will be available in the account directly after her deposit in year 2010 if no withdrawals are made?

A) $\dfrac{3000(1.04^9 - 1)}{1.04 - 1}$ B) $3000 \cdot 1.04^9$ C) $\dfrac{3000(1.04^{11} - 1)}{1.04 - 1}$

D) $3000 \cdot 1.04^{10}$ E) $3000 \cdot 1.04^{11}$ F) $\dfrac{3000(1.04^{10} - 1)}{1.04 - 1}$

Only answer is required (1/0)

4. Which of the following values is the closest approximation to lg80?

A) 0.8 B) 0.9 C) 1.9 D) 2.9 E) 8.0 F) 800

Only answer is required (1/0)

5. Find the minimum value for the function $f(x)=\frac{x^4}{4}+x^3$ (0/3)

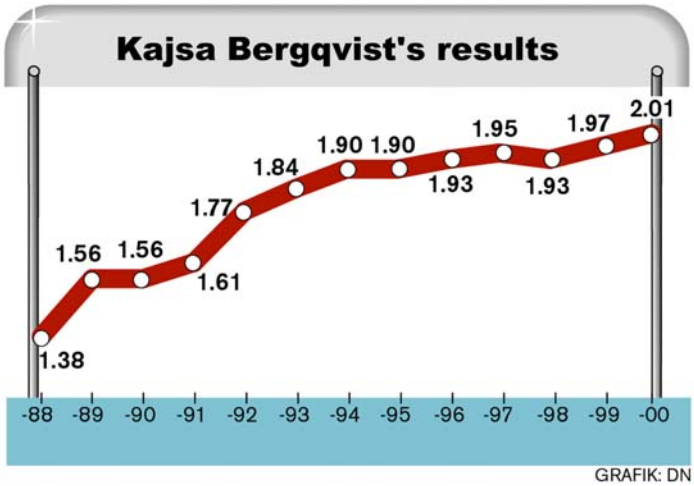
6. a) Explain with the help of a graph, why the derivative of a constant function is zero. (0/1)

b) Explain with the help of the definition of a derivative, why the derivative of a constant function is zero. (0/2/¤)

Part II

This part consists of 9 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

7. The figure shows Kajsa Bergqvist's outdoor high-jump results from 1988 to 2000.



What is the average rate of change for her results from 1988 to 2000? (1/0)

8. The following equation is given
$10\ 000 \cdot x^7 = 16\ 000$

a) Formulate a question that pertains to a realistic situation and can be answered by solving this equation. (1/0)

b) Solve the equation and give the answer to the question that you formulated. (2/0)

9. Develop and simplify the following expression as far as possible
$(x+1)^3+(x-2)^2$ (2/0)

10. Anders, Bodil and Carina were asked to simplify the expression $\dfrac{(4+h)^2-4^2}{h}$

Anders did it this way:

$$\frac{(4+h)^2-4^2}{h}=\frac{16+h^2-16}{h}=\frac{h^2}{h}=h$$

Bodil did it this way:

$$\frac{(4+h)^2-4^2}{h}=\frac{16+8h+h^2-16}{h}=\frac{8h+h^2}{h}=8+h$$

Carina did it this way:

$$\frac{(4+h)^2-4^2}{h}=\frac{16+8h+h^2-16}{h}=\frac{8h+h^2}{h}=8h+h=9h$$

Not everyone has done it correctly.

What errors exist? Motivate your answer. (2/1)

11. A patient with heart trouble has received artificial cardiac valves via an operation. When the cardiac valves are closing, the pressure in the carotid artery can be described by the following model

$P = 95 \cdot e^{-0.65 \cdot t}$

where P is the pressure in units mm Hg and t is the time in seconds from when the cardiac valves begin to close.

a) Calculate the pressure after 0.2 seconds. *Only answer is required* (1/0)

b) Find $P'(0.1)$ (1/0)

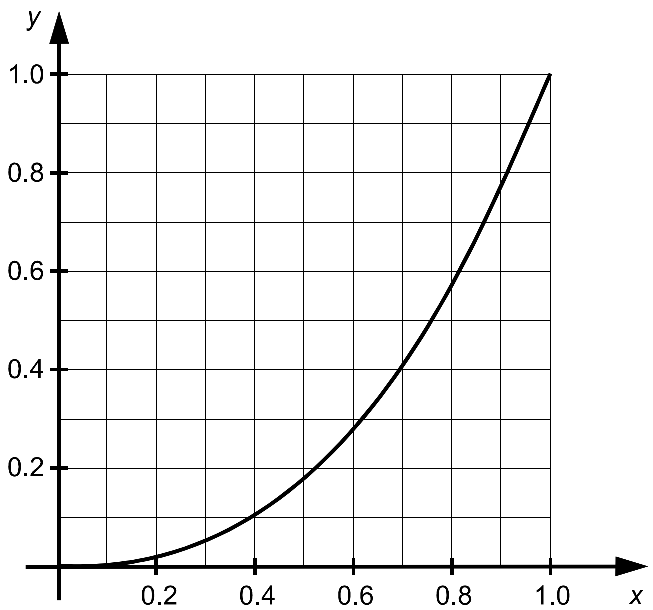
c) What does $P'(0.1)$ tell you about the pressure in the carotid artery? (0/1)

The manufacturer has said that it should take at most 0.5 seconds for the artificial valves to close. When the valves have closed the pressure has dropped to 70 mm Hg.

d) How long does it take the valves to close for this patient? (2/0)

12. In the following coordinate system the graph for the function $f(x) = x^{2.5}$ has been drawn $f(x) = x^{2.5}$ has been drawn.

Find $f'(0.6)$ in two different ways. (2/1)



13.

In 1960 there were approximately 20 000 grey seals in the Baltic Sea. Due to the high levels of environmental pollutants, the number of seals then decreased dramatically. The decrease was exponential and in 1980 there were only 2 000 grey seals left.

a) What was the average yearly percent decrease of the number of grey seals between 1960 and 1980? (0/2)

The seal population has partially recovered since 1980. Today there are approximately 12 000 grey seals in the Baltic Sea. According to a prognosis from the Environmental Protection Agency, the number of grey seals will increase exponentially at a rate of 6.5 % per year for the next few years.

b) In what year will the number of grey seals again reach 20 000 if the prognosis holds true? (0/2)

14. The function f fulfills the following two conditions

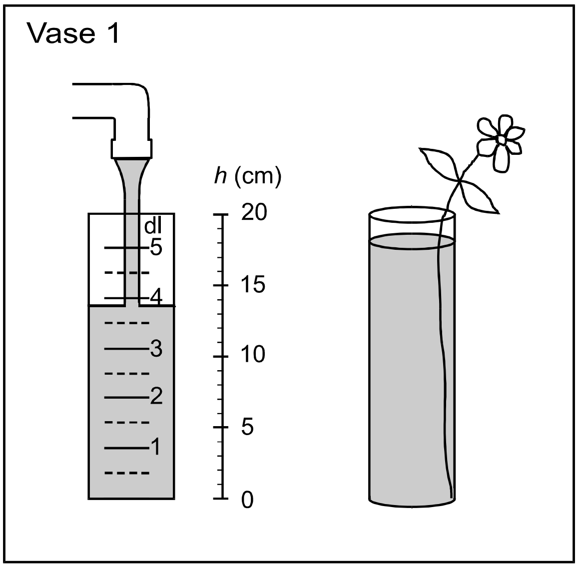
$f(2) = 5$

$-1 \leq f'(x) \leq 2$

Which values can $f(10)$ take? (0/2)

When assessing your work with problem 15 the teacher will consider the following:
- How well you argue your conclusions
- How well you use mathematical vocabulary and symbols
- How well you carry out your calculations
- How well you draw figures as well as how well you account for and annotate your work

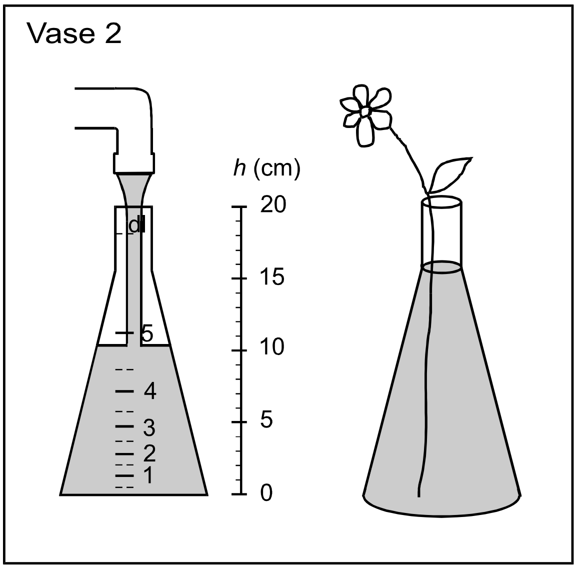
15. The following question involves five different glass vases. All of the vases are 20 cm tall and hold 5.6 dl.
One cylindrical glass vase is filled with water similar to the figure below. The height of the water surface *h* cm over the vase's bottom is a function of the volume of water *x* dl that has run down into the vase.

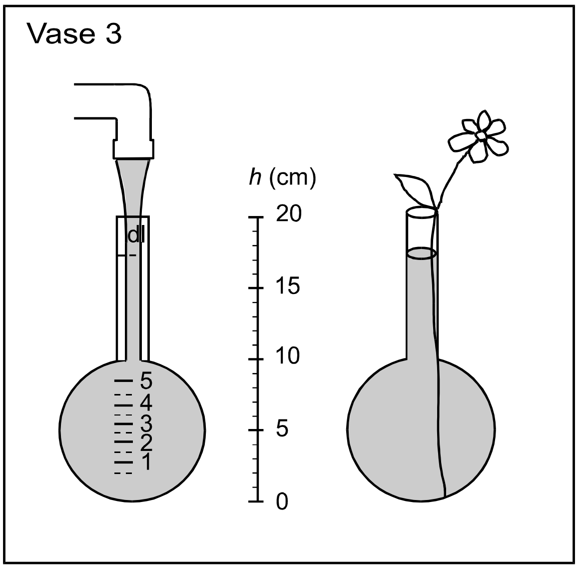


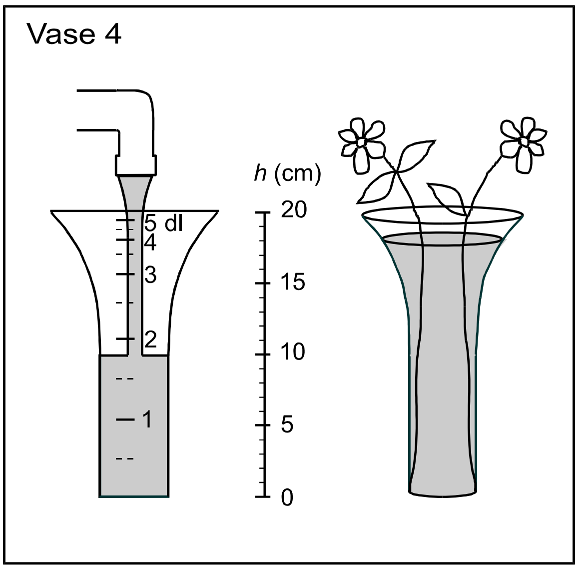
Choose two values for volume *x* and read from the figure the corresponding values for the height of the water surface *h*.

- Calculate the rate of change quotient $\frac{\Delta h}{\Delta x}$ for the read values.
- Explain with words what this rate of change quotient means.

In the figures below you can see how water is filled into three other glass vases. The height of the water surface *h* cm is a function of the volume of water *x* dl which has run down into a vase.



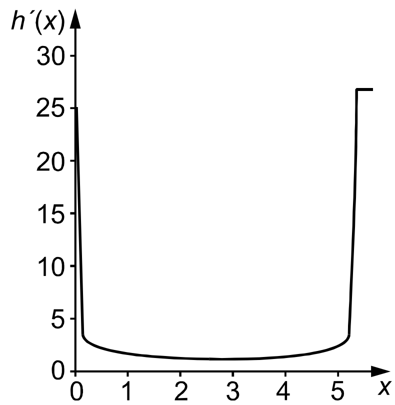




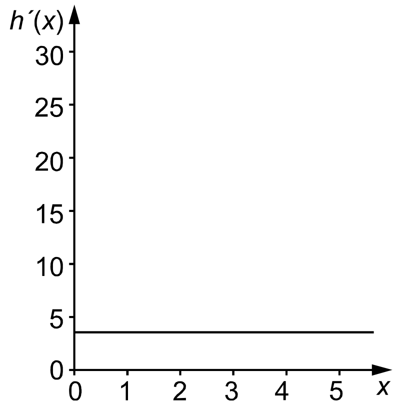
Here are four graphs that have been drawn. They show the graphs to the derivative $h'(x)$ for each one of the glass vases from the two previous pages.

- Pair together the graphs A, B, C and D with corresponding vases 1, 2, 3 and 4.
  Motivate for each pair why the vase belongs together with the graph.

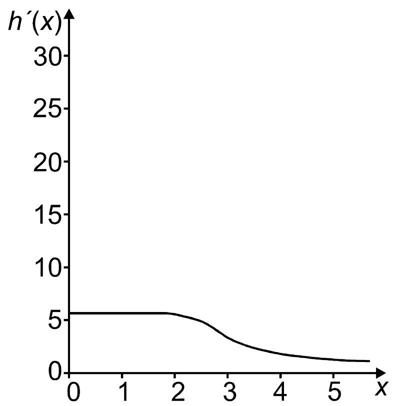
Graph A



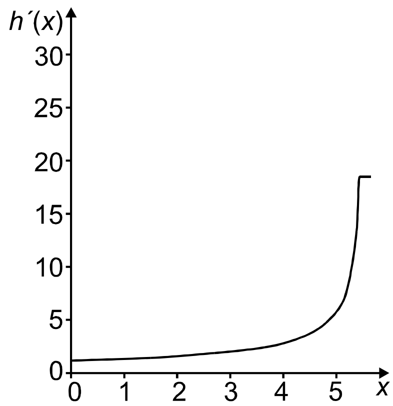
Graph B



Graph C

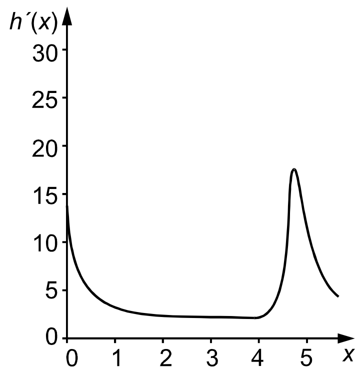


Graph D



In the figure below, the graph for the derivative $h'(x)$ is shown for a fifth glass vase.

- Draw a sketch of what this vase could look like. Motivate why the vase can look like that.



(3/4/¤)