NATIONAL TEST IN MATHEMATICS COURSE B
SPRING 2006

Directions

Test time — 240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources — **Part I:** ”Formulas for the National Test in Mathematics Course B”
Please note that calculators are not allowed in this part.

Part II: Calculators and ”Formulas for the National Test in Mathematics Course B”.

Test material — The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 18 problems. **Part I** consists of 8 problems and **Part II** consists of 10 problems.

For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 43 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	25 points of which at least 7 ”Pass with distinction”-points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”-points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. A straight line passes through the points (0, 2) and (4, 0).

a) Draw the line in a coordinate system. *Only answer is required* (1/0)

b) Determine the equation of the line. *Only answer is required* (1/0)

2. Solve the equation $x^2 + 6x - 7 = 0$ (2/0)

3. Solve the simultaneous equations $\begin{cases} 3x + 4y = -2 \\ x + 10y = 8 \end{cases}$ (2/0)

4. Solve the equation $x^2 = x$ *Only answer is required* (1/0)

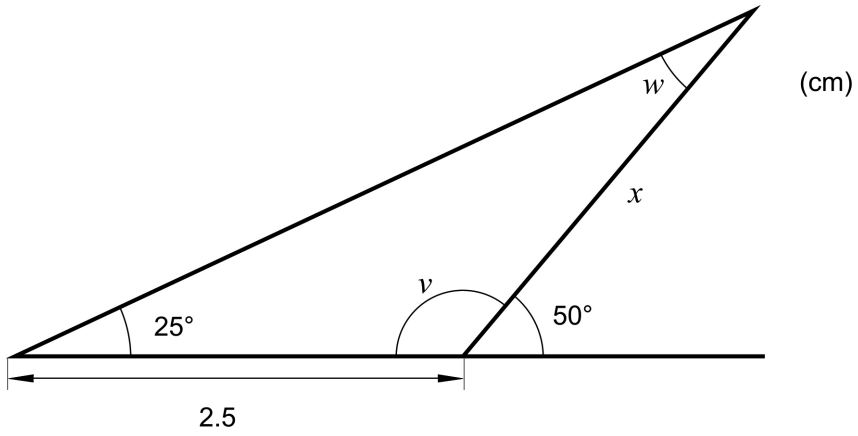
5.

In an art lottery with three prizes 15 people participate with one lottery ticket each. The winning tickets are drawn in value order. If you have won, you cannot receive further prizes. Emelie does not win the first or second prize.

What is the probability at this point that Emily wins third prize?

Only answer is required (1/0)

6.



a) Determine the angles v and w. (1/0)

b) Determine x. Justify your answer. (1/0)

7. Free kicks and throw-ins are examples of set pieces in football. In both of these situations, the game has temporarily stopped and the ball will soon be in-play again. In a set piece a player set the ball going. The ball then followed a path that can be described by the formula

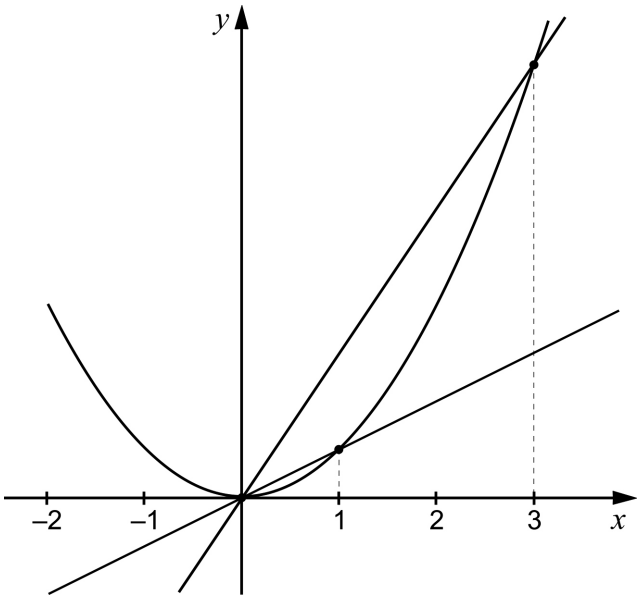
$$y = 2.0 + 0.62x - 0.043x^2$$

where y is the height in metres above the ground and x the distance in metres along the ground from the spot where the player was.

Did the player take a free kick or a throw-in? Justify your answer. (0/1)

Throw-in Free kick

8.



Two straight lines $y = 1.5x$ and $y = 0.5x$ each intersects the graph of the function $y = ax^2 + bx$ at the origin and also at yet another point, see the figure above. The function $y = ax^2 + bx$ has its minimum point at the origin.

Determine *a* and *b*. (0/3/¤)

Del II

This part consists of 10 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

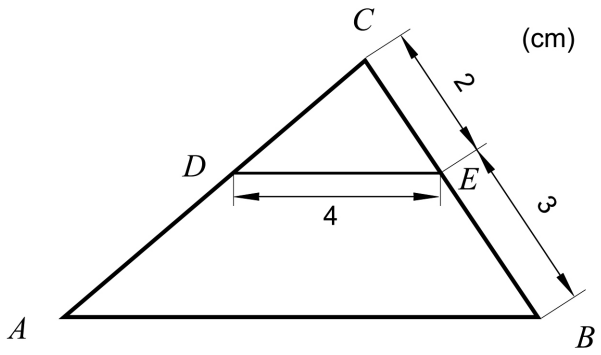
9. Simplify the following expressions as far as possible

a) $(x-3)(x+3)+10$ *Only answer is required* (1/0)

b) $x(x+6)-2(4+3x)$ *Only answer is required* (1/0)

10. In the triangle ABC, the segment DE is parallel to the side AB.

Determine AB. (2/0)



The figure is not drawn to scale

11. The table below shows the turnover of the six largest companies dealing with fast food in Sweden in 2004.

	Mc-Donald's	Burger King	Max	Frasses	Pizza Hut	Sandy's
Turnover (million SEK)	3 846	526	482	176	90	60

(Source: DN grafik)

Choose the measure of location (mean or median) that you think best describes the companies' turnover. Justify your choice of measure of location and calculate it. (2/0)

12. High-altitude sickness is a generic term of a number of unpleasant symptoms that you may be stricken with in high altitudes. It may for example be a headache, loss of appetite or nausea.

Sara and Martina are interested in mountain climbing. They are thinking of climbing the highest peak of Mt Kilimanjaro which is 5 900 metres above sea level, but they are worried about the high altitude. They have read that you can be stricken with high-altitude sickness if the air pressure is less than 700 hectoPascal (hPa).

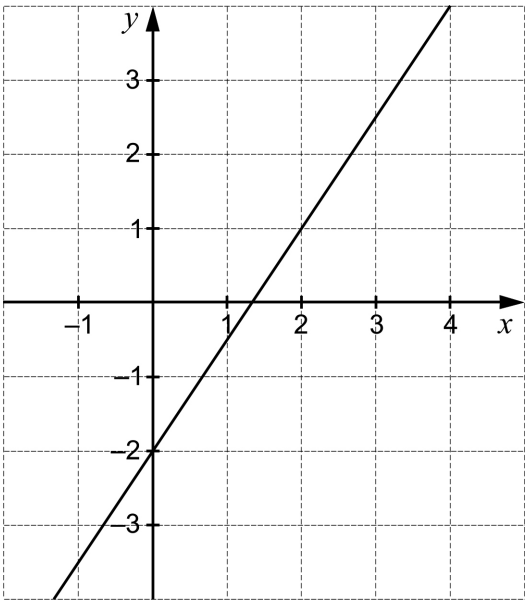
The air pressure can be described by the function $f(x) = 1013 \cdot 0.887^x$, where $f(x)$ is the air pressure in hectoPascal and x is the height above sea level in kilometres.

a) Calculate $f(0)$ and describe in words what you have calculated. (2/0)

b) If Sara and Martina do attempt to reach the peak of Mt Kilimanjaro, is there any risk that they will be stricken with high-altitude sickness? (0/1)

13. Erika is going to write down simultaneous equations with the solution $x = 2$ and $y = 1$. She decides that one of the equations is going to be $3x - 2y = 4$.
In the figure you can see the graph of this equation.

Give Erika a suggestion for the other equation.



(0/2)

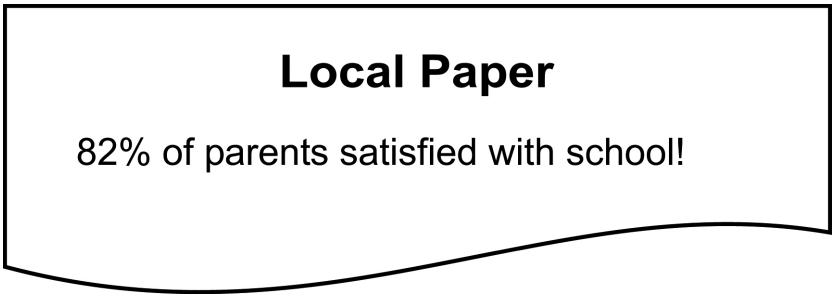
14. The Andersson family have an agreement with their electricity company that every month they should pay a set fee and a fee for every kilowatt-hour (kWh) consumed.

One month, the cost was SEK 295. That month the family had consumed 300kWh. The next month the cost was SEK 415. The consumption was then 460kWh.

a) Determine the monthly cost SEK y as a function of x, where x is the number of kWh consumed. (0/2)

b) The family estimates that they consume 5000 kWh per year. Calculate the total annual cost at this level of consumption. (0/1)

15.



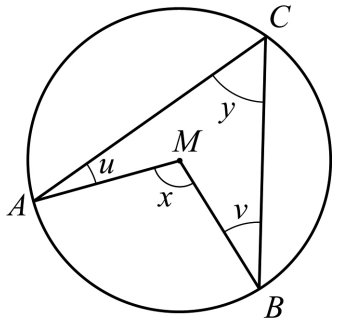
In a municipality a questionnaire was sent out to find out if parents were satisfied with the school. The survey was a full survey and 3330 questionnaires were sent out to all parents. 82% of the 1631 who answered were satisfied. The falling off group was not investigated further.

The local newspaper used the headline “82% of parents satisfied with school!”

Investigate how the proportion of parents who are satisfied can vary depending on the possible answers of those who did not respond to the questionnaire and comment on the headline of the local paper. (0/3)

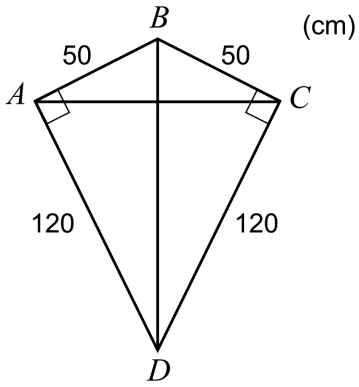
16. A, B and C are three points on a circle with centre M. M lies between the segments AC and BC. In the figure, four angles x, y, u and v are indicated. There is a relationship between these four angles.

Show that this relationship can be written as $x = u + v + y$ (0/2/¤)



17. A kite with measurements according to the figure below is going to be built. The segments AC and BD form a right angle. The angles BAD and BCD are right.

What are the lengths of the segments AC and BD? (1/2/¤)



When assessing your work with the following problem, your teacher will take into consideration:

- How well you carry out your calculations
- How well you present and comment on your work
- How well you justify your conclusions
- What mathematical knowledge you show
- How well you use mathematical language
- How general your solution is

18. Simon and Oskar have invented a simple game with two six-sided dice. They use two dice of different colour. First, they make an opening throw. Then they take turns throwing the two dice. For example, if the opening throw is a blue 5 and a red 3, the winner is the one who first gets the exact same result as in the opening throw, which is a blue 5 and a red 3.

Throw with two dice

Each round of the game starts with a new opening throw. Simon and Oskar play several rounds and they think that many throws are needed before someone wins. Once they play with two similar white dice. They then find that they do not need as many throws in each round.

Your task is to investigate how the length of the game is affected if the two dice of different colour are replaced with two identical white dice.

The following points may be of help in your investigation:

- Once, when they play with two dice of different colour, Simon gets a blue 5 and a red 3 in the opening throw. Then it is Oskar's turn to throw the dice.

  * What is the probability that Oskar does **not** win at once with this throw?

  * Investigate the probability that no one has won after two, three or many throws.

- Once, when they play with two identical white dice, Oskar throws a 2 and a 4 in the opening throw. Then it is Simon's turn to start throwing. Investigate the probability that no one has won after one, two or many throws.

- Does it matter what the opening throw is when they use two dice of different colour? Does it matter what the opening throw is when they use two identical white dice?

- Compare the probabilities that no one has won after many throws. Describe how the length of the game is affected if it is played with two dice of different colour or two identical white dice. (3/4/¤)