

Concerning test material in general, the Swedish Board of Education refers to the Official Secrets Act, the regulation about secrecy, 4th chapter 3rd paragraph. For this material, the secrecy is valid until the expiration of June 2014.

## NATIONAL TEST IN MATHEMATICS COURSE B SPRING 2008

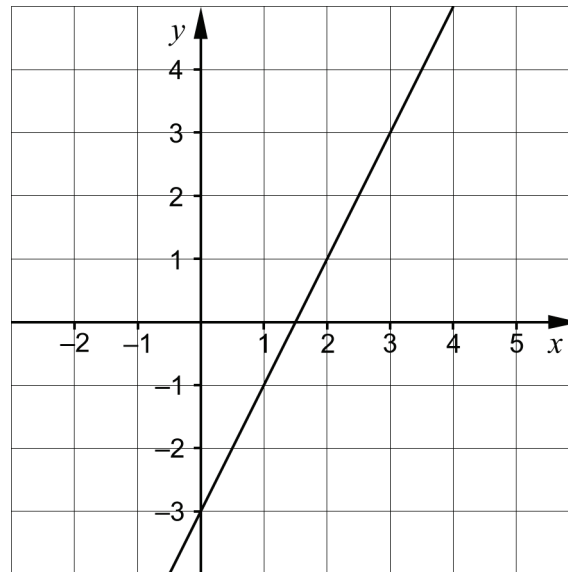
### Directions

- |                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
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| Test time                      | 240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| Resources                      | <p><b>Part I:</b> "Formulas for the National Test in Mathematics Course B"<br/> <i>Please note that calculators are not allowed in this part.</i></p> <p><b>Part II:</b> Calculators, also symbolic calculators and "Formulas for the National Test in Mathematics Course B".</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| Test material                  | <p>The test material should be handed in together with your solutions.</p> <p>Write your name, the name of your education programme/adult education on all sheets of paper you hand in.</p> <p><i>Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.</i></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| The test                       | <p>The test consists of a total of 17 problems. <b>Part I</b> consists of 10 problems and <b>Part II</b> consists of 7 problems.</p> <p>For some problems (where it says <i>Only answer is required</i>) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.</p> <p>Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.</p> <p>Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.</p>            |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| Score and mark levels          | <p>The maximum score is 40 points.</p> <p>The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1). Some problems are marked with <math>\boxplus</math>, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for "Pass with Special Distinction".</p> <p>Lower limit for the mark on the test</p> <table border="0" style="margin-left: 20px;"> <tr> <td>Pass:</td> <td>12 points</td> </tr> <tr> <td>Pass with distinction:</td> <td>23 points of which at least 6 "Pass with distinction"-points.</td> </tr> <tr> <td>Pass with special distinction:</td> <td>23 points of which at least 12 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the <math>\boxplus</math>-problems give the opportunity to show.</td> </tr> </table> | Pass: | 12 points | Pass with distinction: | 23 points of which at least 6 "Pass with distinction"-points. | Pass with special distinction: | 23 points of which at least 12 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the $\boxplus$ -problems give the opportunity to show. |
| Pass:                          | 12 points                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| Pass with distinction:         | 23 points of which at least 6 "Pass with distinction"-points.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |
| Pass with special distinction: | 23 points of which at least 12 "Pass with distinction"-points. You also have to show most of the "Pass with special distinction" qualities that the $\boxplus$ -problems give the opportunity to show.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |           |                        |                                                               |                                |                                                                                                                                                                                                        |

## Part I

**This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.**

1. Find an equation to the line in the figure below.



*Only answer is required* (1/0)

2. Solve the equation  $x^2 - 4x + 3 = 0$  (2/0)

3. Simplify the expression  $(x + 3)(x - 3) + (x + 2)^2$  as far as possible. (2/0)

4. A bowl of sweets contains only 4 raspberry sweets and 6 liquorice sweets. Lucas and Emma take one sweet each from the bowl without looking.

What is the probability that both Lukas and Emma get a raspberry sweet? (1/1)

5. Solve the simultaneous equations 
$$\begin{cases} y + 2x = 1 \\ y - 5x = 29 \end{cases}$$

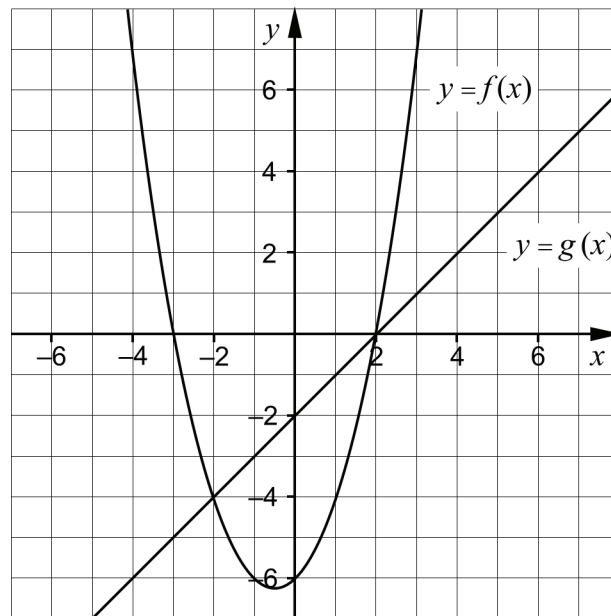
(2/0)

6.

a) Solve the inequality  $3x - 3 > x + 5$  (1/0)

b) Give the smallest integer that satisfies the inequality.  
Only answer is required (1/0)

7. The figure below shows the graphs of the functions  $y = f(x)$  and  $y = g(x)$

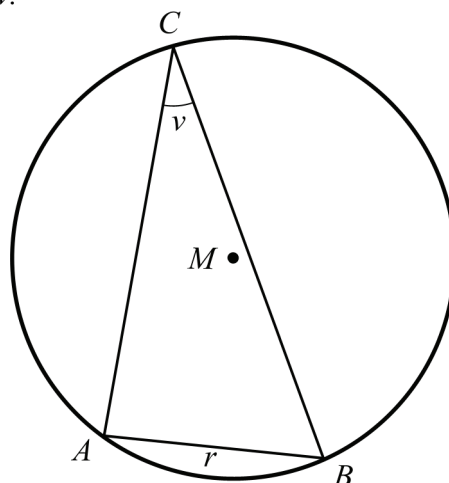


a) Determine  $f(3) - g(3)$  Only answer is required (1/0)

b) For which values of  $x$  is  $f(x) < g(x)$  Only answer is required (0/1)

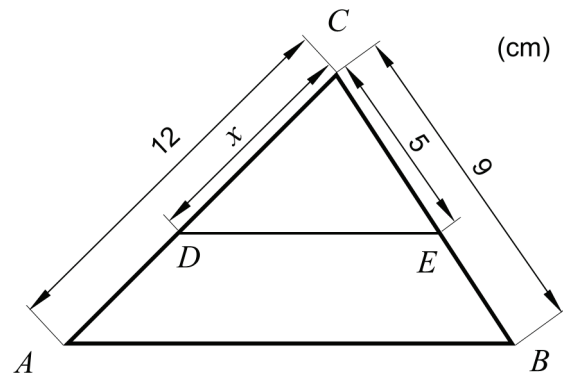
8. The triangle  $ABC$  has its corners on a circle with centre  $M$  and radius  $r$  cm.  
Side  $AB$  is  $r$  cm.

Calculate the angle  $v$ .



(0/2)

9. Side  $AC$  is 12 cm and side  $BC$  9 cm in the triangle  $ABC$ . A transversal intersects side  $AC$  at point  $D$  and side  $BC$  at point  $E$ , so that  $EC$  is 5 cm. The segment  $DC$  is  $x$  cm.



Which two equations 1) to 6) can be used to calculate a correct value of  $x$ ?  
*Only answer is required*

1)  $\frac{x}{5} = \frac{12}{9}$

2)  $\frac{x}{5} = \frac{9}{12}$

3)  $\frac{x}{12} = \frac{4}{9}$

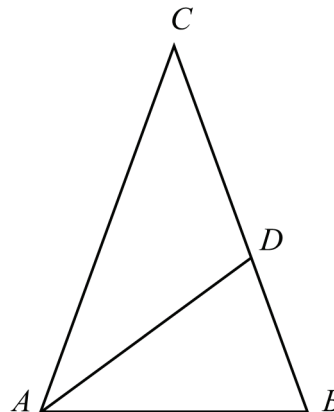
4)  $\frac{x}{12} = \frac{9}{4}$

5)  $\frac{12-x}{x} = \frac{5}{9}$

6)  $\frac{12-x}{x} = \frac{4}{5}$

(1/1)

10. Side  $AC$  has the same length as side  $BC$  in the triangle  $ABC$ . The segment  $AD$  divides the angle  $CAB$  in two equal parts. Show that angle  $ADC$  is always three times angle  $CAD$ .

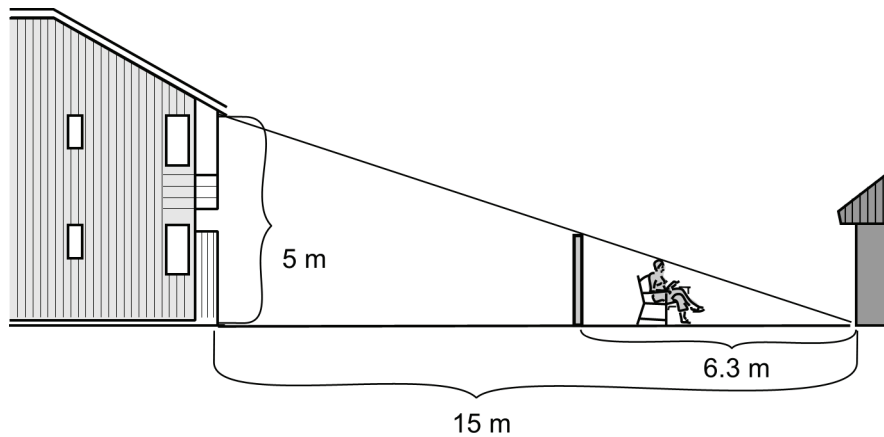


(0/2/∞)

## Part II

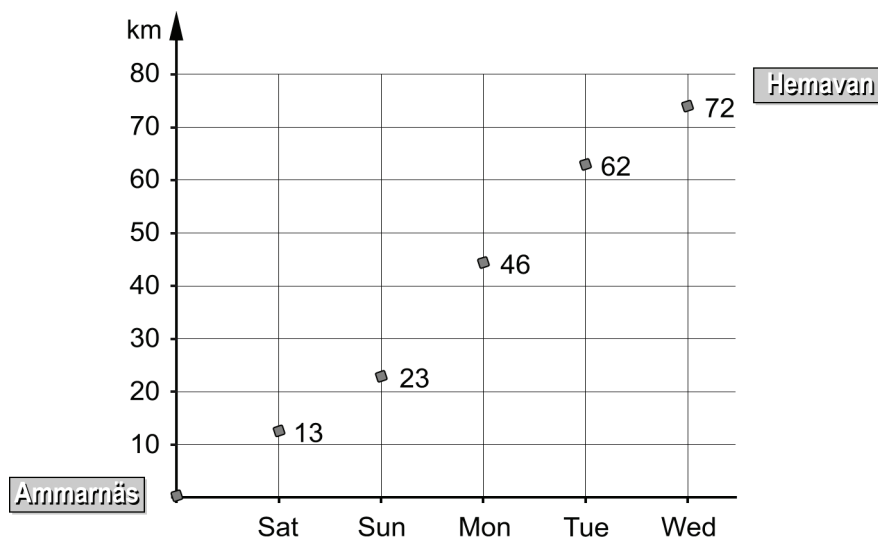
**This part consists of 7 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.**

11. The Svensson family has decided to build a fence so that people can't look onto their patio. What must the height of the fence be to shut off the patio from the neighbours' view?



(2/0)

12. Mia and Pia start their mountain tour in Ammarnäs. Their destination is Hemavan, 72 km away. Every night they write down how far from Ammarnäs they are. The diagram below illustrates their walking-tour.



- a) What was the length of a daily stage on average? (1/0)
- b) What was the range of distribution in the length of the daily stages? (2/0)

13. In a community there are two upper secondary schools, East and West. East has 1350 students and West 520 students. There has been a discussion within the community about banning the sale of sweets in school cafeterias. To find out the students' views the students' councils have together conducted a survey. Some SP-classes at each school were chosen and asked the question:

"Do you think one should be allowed to buy sweets in school?"

The answers can be seen in the table below:

| School | Number<br>not replying | Number<br>of Yes | Number<br>of No |
|--------|------------------------|------------------|-----------------|
| East   | 17                     | 27               | 58              |
| West   | 30                     | 49               | 16              |

The students' councils summed up the survey the following way:

$$\text{Share "Yes": } \frac{27 + 49}{27 + 49 + 58 + 16} \approx 51 \%$$

*Conclusion: A majority of the upper secondary students thinks it should be allowed to buy sweets in school.*

Some students have critical opinions on the survey and the students' councils' conclusion.

Give two critical opinions.

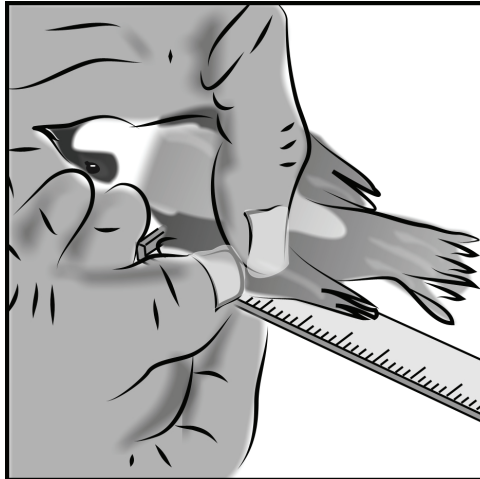
(2/0)

14. Anna and Pär use two six-sided dice. They each roll the dice once and calculate the difference between the number of spots on the two dice. Anna wins if the difference is 1 or 5 and Pär wins if the difference is 3 or 4. Investigate if they both have the same chance of winning.



(0/2)

15. When ringing birds, their weight and wing measurements are often measured.



A biologist at Lake Tåkern in Östergötland rings Penduline Tits. Her data leads to the following model

$$y = -0.0060x^2 + 0.36x + 6.0$$

where the weight,  $y$  grams, depends on the wing measurement,  $x$  mm.

She notices that a nestling with wing measurement 10 mm has the same weight as an adult bird.

What should the wing measurement be of an adult Penduline Tit according to the model?

(0/3)

16. A line  $L$  passes through the origin in a coordinate system.  $L$  intersects the line  $y = 2x - 3$  at a point whose  $x$ -coordinate is greater than 50.

What are the possible equations of line  $L$ ?

(0/2/∞)

**When assessing your work with the following problem the teacher will take into consideration**

- How well you justify your conclusions
- How well you carry out your calculations
- How well you present your work
- How well you use mathematical language and expressions

17. Hjalmar and Agnes have been given the task of baking buns and biscuits that they can sell to make money for a school trip. They expect to sell everything they bake.

They write down the two recipes on a piece of paper and decide that the profit should be SEK 4 per bun and SEK 2 per biscuit.

|                                                                                                                                                                                                                              |                                                                                                                                                                                                                        |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Buns: (100 pieces)</b></p> <p>2400 grams flour</p> <p>500 grams butter</p><br><p>425 grams sugar</p> <p>2.5 packages of yeast</p> <p>1.5 litres of milk</p> <p>1 teaspoon salt</p> <p><b>Profit: SEK 4 per bun</b></p> | <p><b>Biscuits (100 pieces)</b></p> <p>600 grams flour</p> <p>400 grams butter</p><br><p>170 grams sugar</p> <p>4 teaspoons baking-powder</p> <p>6 teaspoons vanilla sugar</p> <p><b>Profit: SEK 2 per biscuit</b></p> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

To find out how much they can bake, Agnes and Hjalmar check how much of the ingredients they have at home.

Agnes has 2000 grams of butter and writes down the equation  $5x + 4y = 2000$  and Hjalmar has 6000 grams of flour and writes down the equation  $4x + y = 1000$  where  $x$  is the number of buns and  $y$  the number of biscuits.

The lines for Agnes' and Hjalmar's equations are drawn in figure 1 (see next page).

- Explain which line corresponds to Agnes' equation and which line corresponds to Hjalmar's equation.
- Describe what is applicable to the supply of flour and butter in the areas marked with  $A$  and  $D$  in figure 1.



**Hjalmar:** The profit will be SEK 400 if we bake 100 buns and no biscuits or if we bake 200 biscuits and no buns.

**Agnes:** Yes, that is true, but shouldn't we bake both buns and biscuits? There must be many combinations of the number of buns and biscuits that give the profit SEK 400.

**Hjalmar:** We should also be able to get a larger profit than SEK 400. Why not a profit of SEK 800 or SEK 1400?

- Copy figure 1. Decide on some points ( $x$  buns and  $y$  biscuits) where the profit is SEK 800 and SEK 1400 and draw the relations in the figure.

**Agnes:** If we use my 2000 grams of butter and your 6000 grams of flour and buy the rest of the ingredients, can we then get a profit of SEK 1400?

- Explain to Agnes how she can answer the question by using the figure.

**Hjalmar:** What is the largest profit we can get if we use my flour and your butter?

- Draw the line that corresponds to maximum profit and justify the choice of this line. Then answer Hjalmar's question.

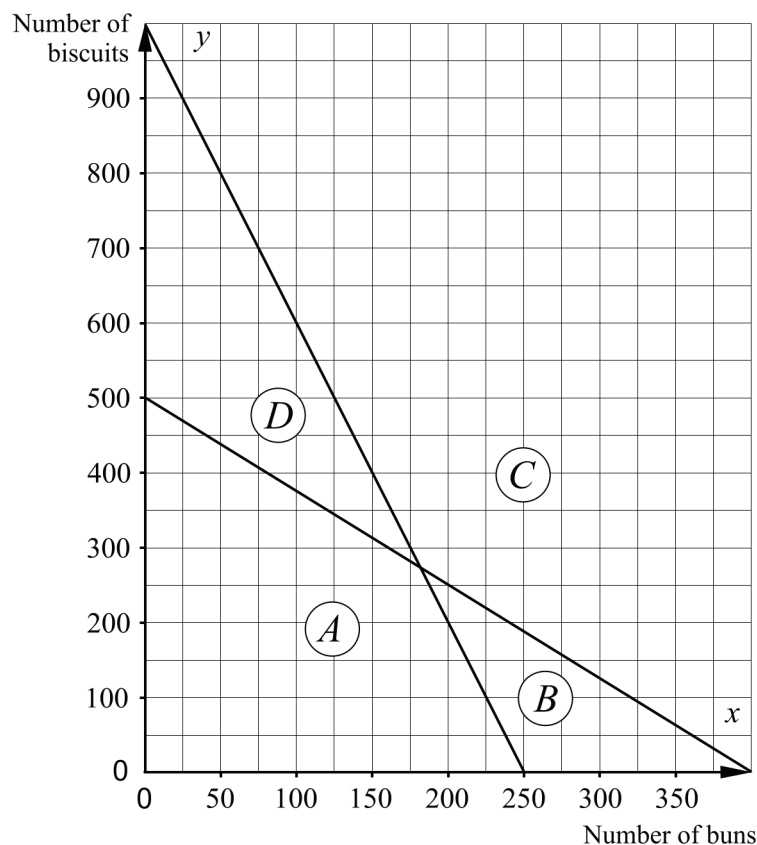


Figure 1