NATIONAL TEST IN MATHEMATICS COURSE B
SPRING 2010

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 60 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course B” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course B”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 9 problems and **Part II** consists of 9 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 43 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 25 points of which at least 6 ”Pass with distinction”-points. Pass with special distinction: 25 points of which at least 13 ”Pass with distinction”-points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

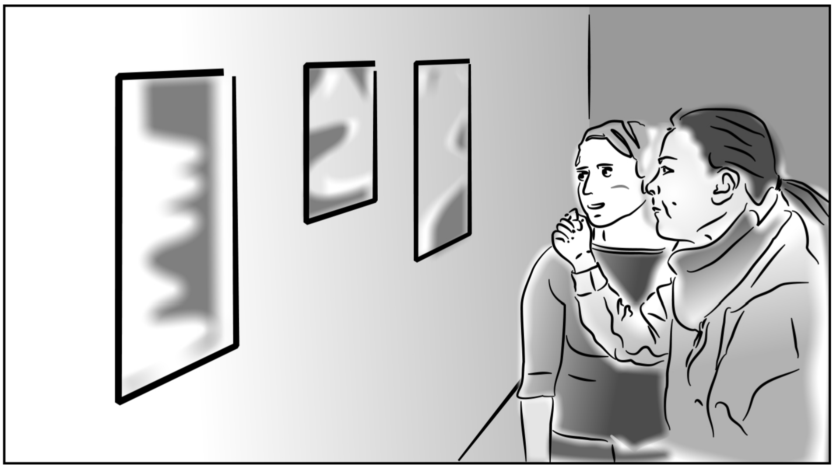
1. Solve the equation $x^2 - 8x - 33 = 0$ (2/0)

2. A straight line passes through the point (0, 2) with a gradient $k = -3$

a) Draw the line in a coordinate system. *Only answer is required* (1/0)

b) Find the equation of the line. *Only answer is required* (1/0)

3. 15 people participate in an art lottery with three prizes. They each have one lottery ticket. The winning lottery tickets are drawn in turn. Those who have won are not allowed to continue taking part in the lottery.



Emelie does not win first or second prize.
What is the probability that Emily now wins the third prize?
Only answer is required (1/0)

4. At a shop the price of apples is x SEK/kg and the price of oranges is y SEK/kg.

The prices of the fruits are part of the following simultaneous equations: $\begin{cases} 2x + 3y = 69 \\ x + 2y = 42 \end{cases}$

a) Solve the simultaneous equations above. (2/0)

b) In words, explain how the equation $x + 2y = 42$ should be interpreted in this context. (0/1)

5. Simplify $(x+3)^2+3(4+x)$ as far as possible. (2/0)

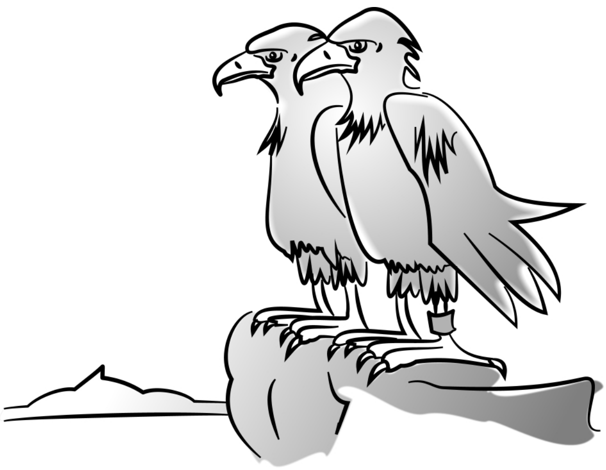
6. Let $f(x)=x^2+5x$

Calculate $f(4)-f(2)$ (1/0)

7. The expression $(x-5)(x+7)$ has the value 28 when $x=7$. The value of the expression is also 28 for another value of *x*.

Determine this *x*-value. (0/2)

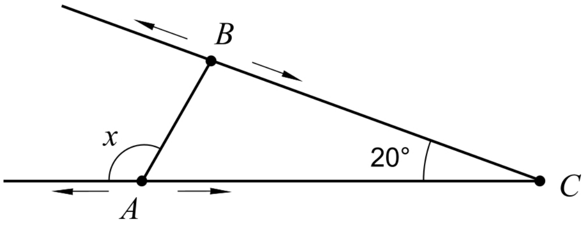
8. The largest predatory bird in Sweden is the sea eagle. Approximately 70 % of the Swedish sea eagles are ringed. Sea eagles who live in pairs remain throughout their lifetime with the same partner.



Calculate the probability that a couple of sea eagles consists of one bird which is ringed and one bird which is not ringed. (1/1)

9. In the figure below the angle at point *C* is $20°$. *A* and *B* are two freely moving points on each side of an angle. The points *A* and *B* can be moved independently, but without coinciding with *C*. When *A* and *B* are moved, the angle *x* is changed. See figure.

What are the possible values of the angle *x*? (0/1/¤)



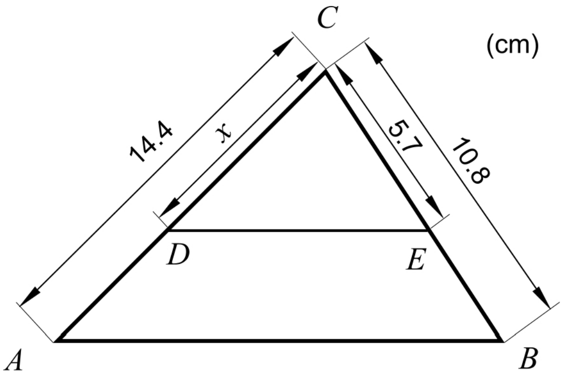
Part II

This part consists of 9 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. Find an equation for the straight line that passes through the points (6, 11) and (10, 16). (2/0)

11. In the triangle *ABC*, side *DE* is parallel to side *AB*.

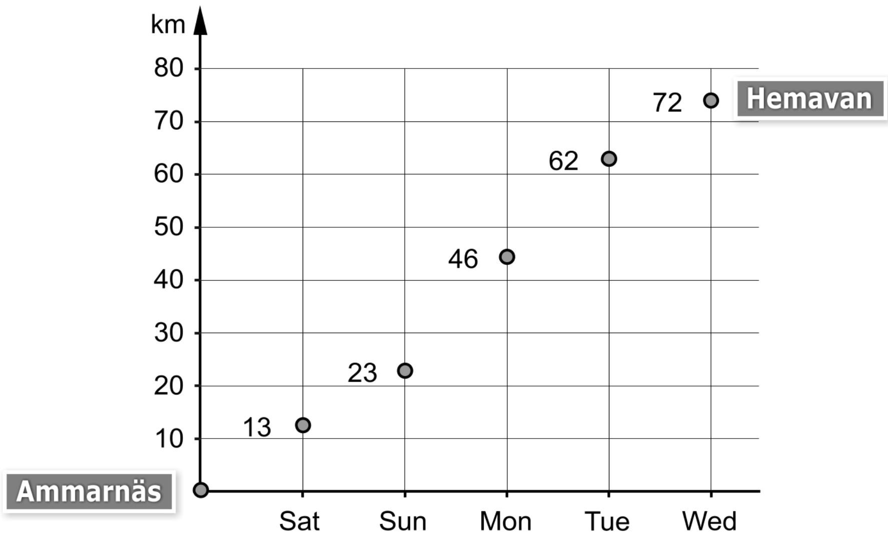
Calculate x.



The figure is not drawn to scale

(2/0)

12. Mia and Pia start their mountain hike in Ammarnäs. Their goal is Hemavan, 72 km away.
Every night they write down how far from Ammarnäs they have come. The diagram below shows how far they have walked each day.



a) What was the average length of their daily stages? (1/0)

b) What was the range of distribution in the length of the daily stages? (1/0)

13. To be able to exercise at the right intensity it is useful to know your maximum pulse. It is for example suitable to have a pulse of around 70 % of the maximum pulse during cardio exercise. The table shows how the theoretical maximum pulse varies with age.

Age (years)	Maximum pulse (beat/minute)
25	195
30	190
35	185
40	180
45	175
50	170
55	165
60	160
65	155
70	150

a) Use the table and estimate the maximum pulse for a 20-year old.
Only answer is required (1/0)

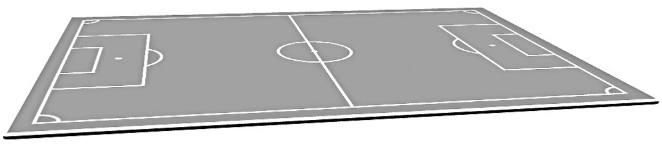
b) There seems to be a linear relation between maximum pulse and age. Explain how this can be determined by using the table. (1/0)

c) Determine the function that describes the relationship between the maximum pulse y beat/minute and age x years. (0/1)

14. A football club is going to set up a new football field. The club carried out a vote among their members to decide whether the football field would have artificial grass or normal grass.
All 1500 members were asked and 1140 answered. Out of these responses 63.8 % said that they wanted artificial grass.

With the decision of the board approaching, one member asked for a further investigation of the non-responders. She said that it is not certain that a majority of the members wants artificial grass.

Could it be true that there is a majority for normal grass among the members? (0/2)



15. To some courses and study programmes in higher education there are sometimes more applicants with maximum points of merit than there are student places. Admission to these courses and study programmes is then sometimes decided by drawing lots.

On one occasion 20 men and 80 women with maximum points of merit applied.

a) What is the probability that the first person admitted through lottery is a man?

Only answer is required (1/0)

Weighted drawing of lots is sometimes used to get a more even distribution between the sexes among the admitted. In this case, with 20 men and 80 women, it means that every male applicant receives several lottery tickets as there are fewer men than women applying to this study programme.

b) Every woman gets one lottery ticket. If every man gets two lottery tickets, what is the probability that the first student admitted through lottery is a man? (0/1)

c) Every woman gets one lottery ticket. If the probability that the first student admitted through lottery is a man should be the same as if it is a woman, how many lottery tickets should each man have? (0/1)

To a certain study programme, the probability that the first student admitted through lottery is a man should be the same as if it is a woman, regardless of how many men and women with maximum points of merit apply.

d) Assume that *m* men and *k* women with maximum points of merit apply to this programme. Suggest how many lottery tickets each man and woman should have. It is possible that both men and women get more than one ticket each. (0/1/¤)

16. The ozone layer in the stratosphere protects us from UV radiation. The thickness of the ozone layer is measured in Dobson Units (DU). What meteorologists mean when they talk about ozone holes over for example the Antarctic is areas where the thickness of the ozone layer is less than 220 DU. Thus, there is not really a hole, but a thinner ozone layer.

Since the 1980s SMHI has measured the thickness of the ozone layer over Norrköping among other places in Sweden. A function based on these measurements from June 1 until December 31 in 2008 is given by:

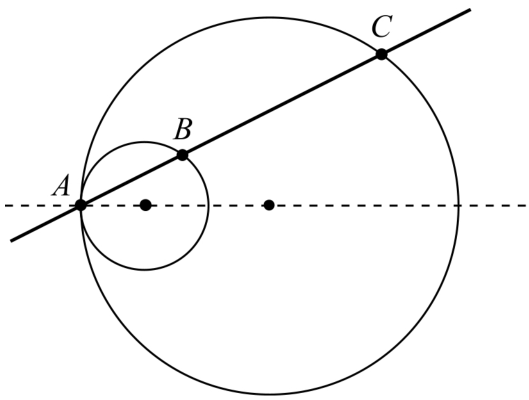
$$f(x) = 0.0052x^2 - 1.4x + 378, \quad 0 \le x \le 210$$

where $f(x)$ is the thickness of the ozone layer in DU and x is the number of days after June 1.

a) Determine $f(0)$ and describe how $f(0)$ should be interpreted in this context. (1/1)

b) Does the function above assume values less than 220 DU? Justify your answer. (0/2)

17. The centres of the two circles in the figure are located on the dotted line. The radii of the circles are 12 cm and 36 cm.



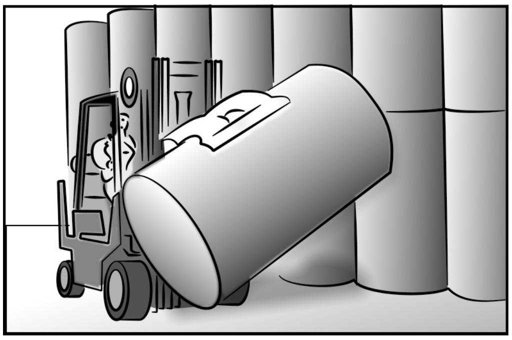
A is a point on the dotted line. Both circles pass through point *A*. A straight line drawn through point *A* intersects the circles in two points, *B* and *C*.

Show that $\dfrac{AB}{AC} = \dfrac{1}{3}$ (0/2/¤)

When assessing your work with this problem the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

18.



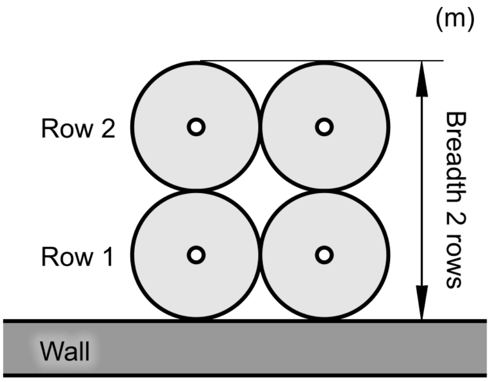
Ware house with paper rolls

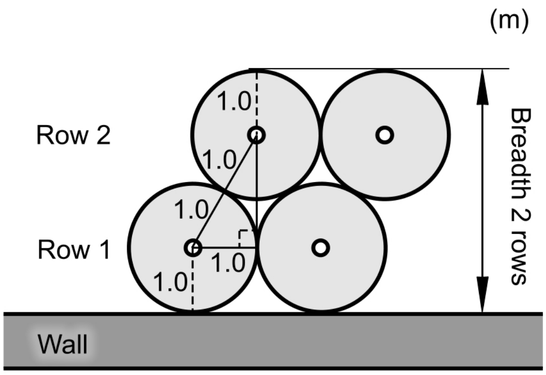
A paper mill has several ware houses where paper rolls are stored.
The paper rolls are stored standing in rows according to Model 1 below.
The radius of the rolls is 1.0 metre.

The management want to try a new storage model for their paper rolls, Model 2, which also can be seen below.

Your task is to compare the two models and investigate how much the total breadth of the stored paper rolls decreases with the new model, Model 2.

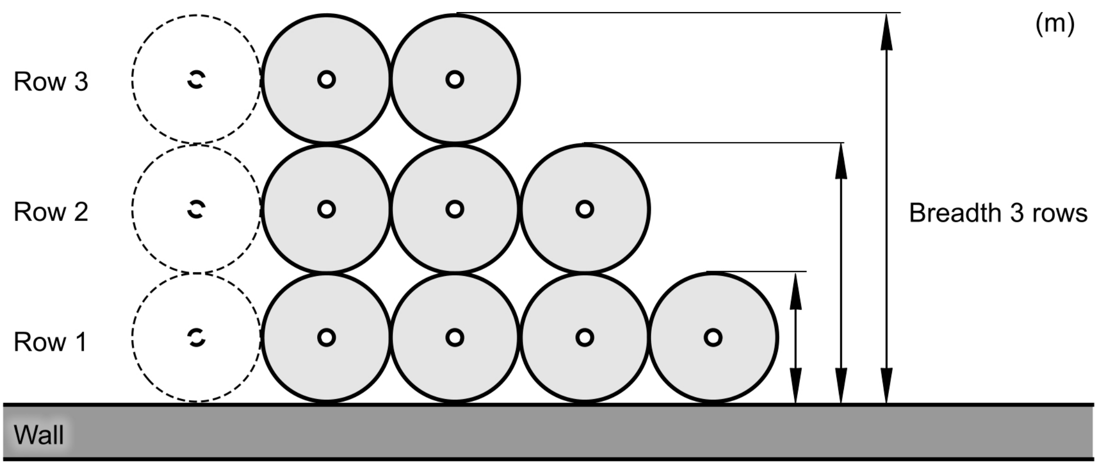
Model 1 (Ware house seen from above) **Model 2** (Ware house seen from above)



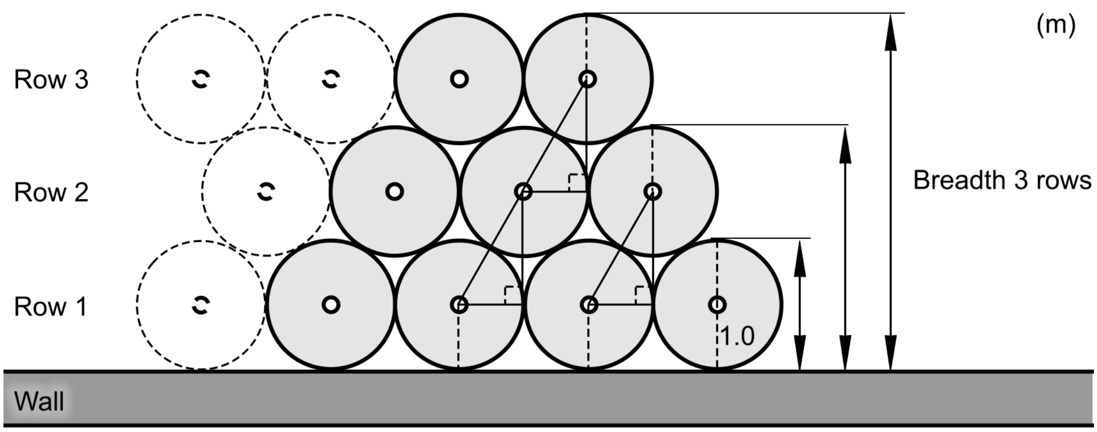


- Determine the total breadth for two rows of Model 1 and Model 2.
- How much less, in percentage terms, is the total breadth with Model 2 compared to Model 1 if two rows are stored?

Model 1 (Ware house seen from above)



Model 2 (Ware house seen from above)



- How much less, in percentage terms, is the total breadth with Model 2 compared to Model 1 if three rows are stored?
- Examine as thoroughly and in as much detail as possible what will happen with this percentage when more and more rows of paper rolls are stored. (3/3/¤)