NATIONAL TEST IN MATHEMATICS COURSE B
SPRING 2011

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 60 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course B” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course B”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 7 problems and **Part II** consists of 9 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test

Pass:	13 points.
Pass with distinction:	25 points of which at least 6 ”Pass with distinction”-points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”-points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Solve the equations

a) $x^2 - 6x - 16 = 0$ (2/0)

b) $x^2 + 4x = 0$ (2/0)

2. A straight line passes through the points (0, 2) and (4, 0)

a) Draw the line in a coordinate system. (1/0)

b) Write down the equation of the line. *Only answer is required* (1/0)

3. Solve the simultaneous equations $\begin{cases} 2x + 3y = 14 \\ x + 2y = 0 \end{cases}$ (2/0)

4. At a ski race with 20 participants, the start is individual. The participants should have bib numbers numbered from 1 to 20, where the number indicates the start order.

Marcus and Erik are participating in the ski race and both of them want to be the last one to start, that is, they both want number 20. Just before the start, the participants randomly pick an envelope which contains a bib number. Marcus is the first one to pick an envelope and Erik gets to pick his envelope immediately after him.

Before they open their envelopes, Marcus says to Erik:”There is a greater chance that number 20 is in my envelope than that is in yours, because I got to pick first.”

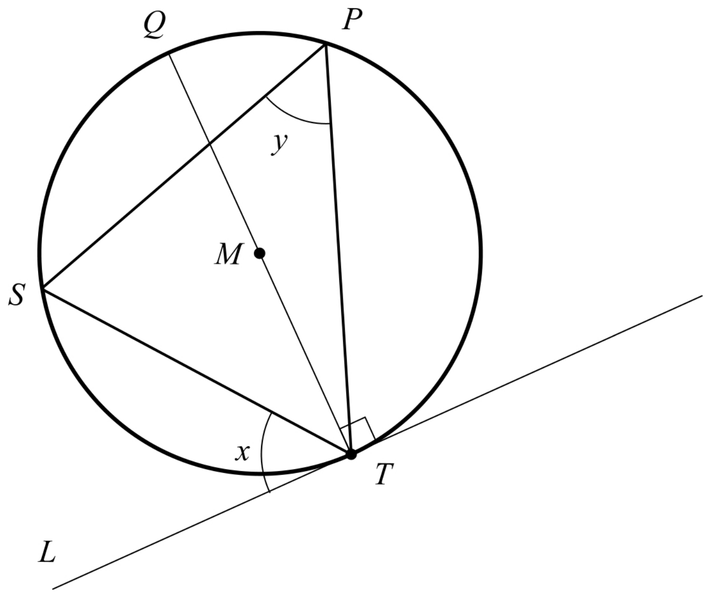
Decide whether Marcus is right by determining the probability that Marcus gets number 20 and also by determining the probability that Erik gets number 20. (1/1)

5. Simplify the expression $(x-2)^2+(3-x)(x-4)$ as far as possible. (2/0)

6. Two lines $y=2x+5$ and $y=kx+m$ intersect at one single point. That point is on the *y*-axis.

What are the possible values of the gradient k ? Justify your answer. (0/1/¤)

7. A line *L* touches a circle at point *T*. *M* is the centre of the circle. The angle between the diameter of the circle *QT* and the line *L* is $90°$. A triangle *PST* is inscribed in the circle with all its corners on the circumference of the circle. See figure.



a) What is the size of angle *y* when angle *x* is $56°$? (0/2)

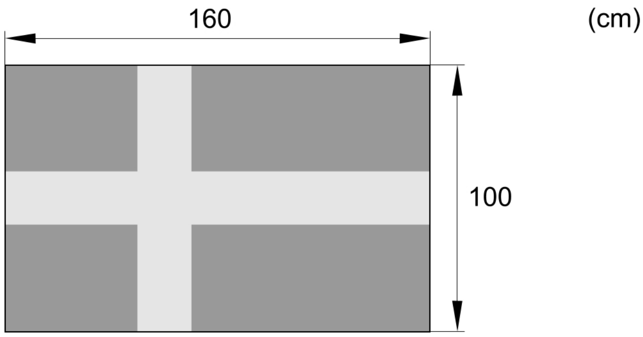
If the points *P* and *S* are moved along the circumference of the circle, the angles *x* and *y* will vary. It holds that angle *x* is $0° < x < 90°$

b) Determine the relationship between the angles *x* and *y*. (0/1/¤)

Part II

This part consists of 9 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

8. A Swedish flag with a long side of 160 cm and a short side of 100 cm satisfies the flag law in force. Anna wants to make a small table flag with a short side of 8 cm.



What length should Anna's flag have if it is to be similar with the large flag? (2/0)

9. The company Round Plastic Inc manufactures, among other things, floorball balls. Each month 50 000 balls are produced.

After complaints from customers, the management of Round Plastic Inc decided to introduce quality control. During one month, the quality of every 200th ball produced was checked. 11 balls of poor quality were found.

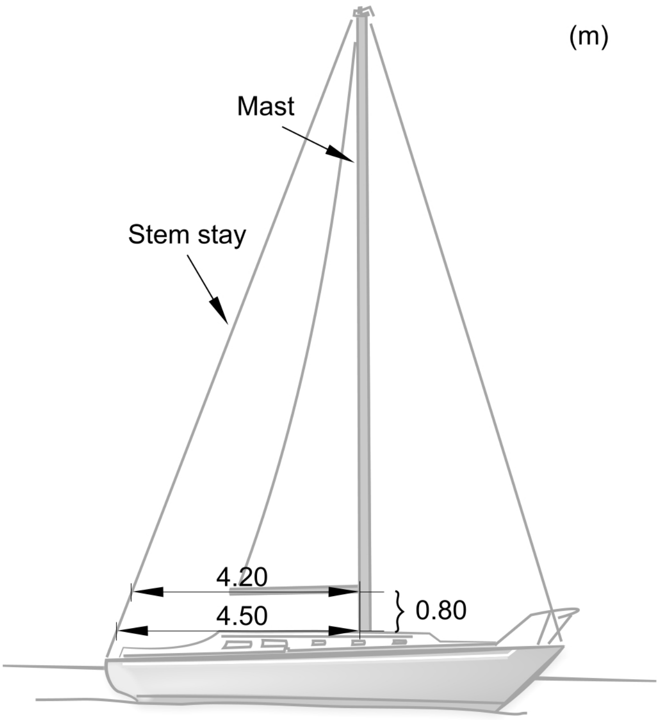
a) A sample survey is described above. What was the size of the sample? (1/0)

b) How many of the balls manufactured during one month can be assumed to be of poor quality? (2/0)

10. A straight line has a gradient $k = 1.2$ and intersects the y-axis at the point (0, 3)

Decide whether the point (175, 207) is on the line. (2/0)

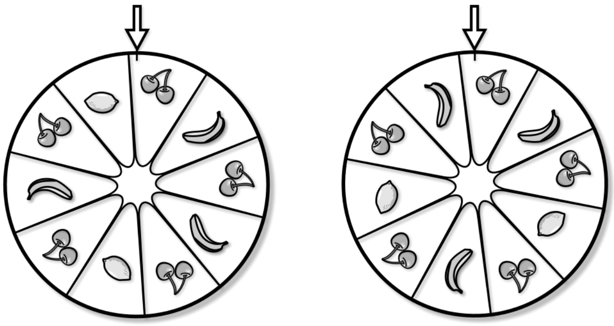
11. Lina and Sara are out sailing in a boat they have borrowed. They sail towards a bridge and start wondering if the mast is too tall for the boat to pass under the bridge. To be able to determine the height of the mast, they take some measurements.



Lina and Sara measure the distance from the foot of the mast and straight out to the stern stay and find that it is 4.50 m. Then they measure the distance from the mast to the stern stay at a point 0.80 m higher up, parallel to the first measurement. That distance is 4.20 m. See figure.

Use the measurements Lina and Sara have taken and calculate the height of the mast. (2/0)

12. Elin, Petter and Ali are at a fun-fair. There is a game with two similar spinning wheels of fortune with pictures of bananas, lemons and cherries. The wheels spin independently of each other. The game yields a prize if the pointers at the two wheels point on the same type of fruit when the wheels stop. See figure.



a) Elin bets that both wheels will stop on cherries.
What is the probability that she will win? (0/1)

b) At the same time that Elin bets on cherries, Petter bets on bananas and Ali on lemons. What is the probability that none of the three wins? (0/2)

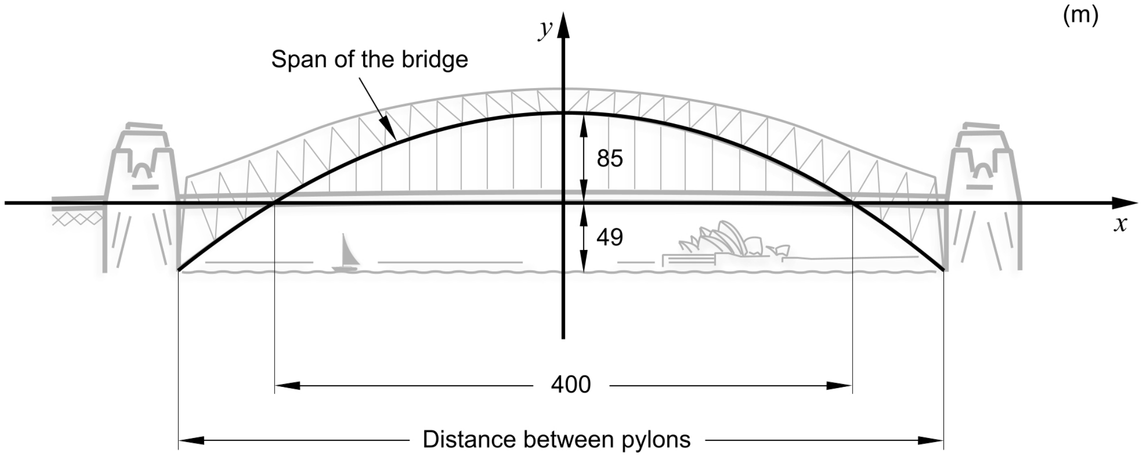
13. Fia runs on a treadmill that can be set at different speeds. On a display she can read how much energy she uses during a workout on the treadmill. The energy is given in the unit kcal.

Fia usually first sets the treadmill on 8 km/h ("low" speed) and then increases the speed to 12 km/h ("high" speed).
The table below shows examples of Fia's workout on the treadmill.

	Time		**Energy consumption**
	”low” speed	”high” speed	
Workout 1	20 min	10 min	300 kcal
Workout 2	10 min	15 min	280 kcal

How much energy per minute (kcal/min) does Fia consume when she runs at the "low" speed and the "high" speed, respectively. (0/3)

14. One of the tourist attractions in Sydney is the large steel arch bridge, Sydney Harbour Bridge. Between the pylons runs the span of the bridge which has the shape of a quadratic curve.
The summit is situated 85 metres above the roadway, which in turn is 49 metres above sea level. The span of the bridge is situated above the roadway along a 400-metre stretch of road. See figure.

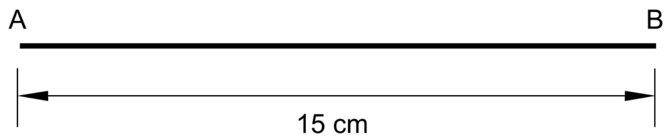


The equation of the quadratic curve that describes the span of the bridge can be written as $y = ax^2 + b$, where a and b are constants.

a) What is the value of constant b in the quadratic curve that describes the span of the bridge? *Only answer is required* (1/0)

b) What is the distance between the pylons? (0/3/¤)

15. A distance AB has a length of 15 cm. The distance can be divided into five parts in different ways. The length of each part must be greater than zero.



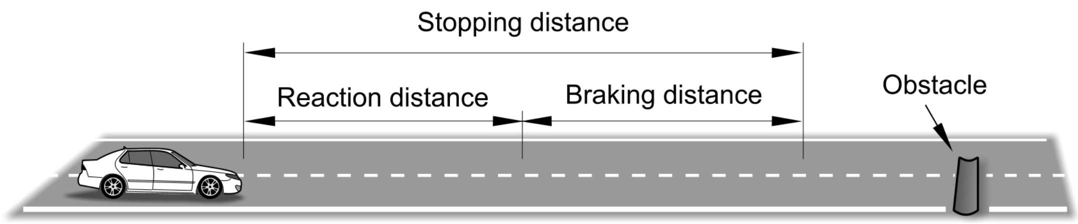
a) Divide the distance AB so that the range for the lengths of the parts is 12.5 cm. (1/1)

b) The range may vary depending on how the distance AB is divided into the five parts. Investigate what possible values there are for the range when the lengths of the five parts are altered. (0/1/¤)

When assessing your work with this problem the teacher will take into consideration:
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

16. When talking about car driving, *stopping distance* is often mentioned in situations when the driver sees an obstacle, brakes and stops.

The stopping distance s can be divided into two parts. The first part, *the reaction distance,* is the distance which the car moves from the moment the driver sees an obstacle until the driver hits the brake. The second part, *the braking distance*, is the distance which the car moves from the moment when the driver starts braking until the car stops, see figure.



The stopping distance s at a certain state of the road can be calculated with the following formula:

$$s = \underbrace{0.27v}_{\text{Reaction distance}} + \underbrace{0.005v^2}_{\text{Braking distance}}$$

where the stopping distance s is given in metres and the velocity v is given in km/h.

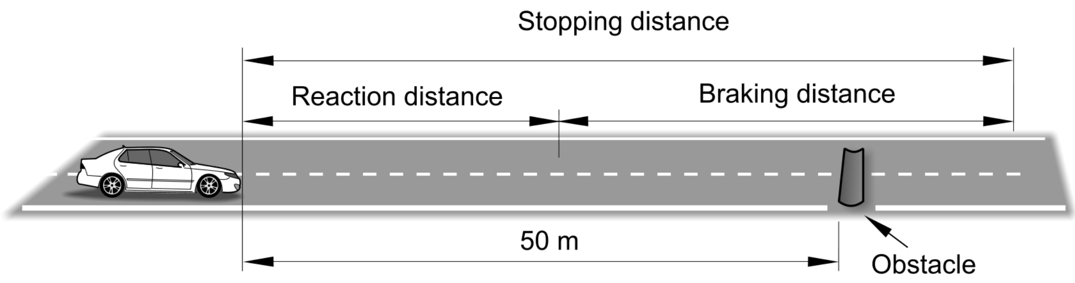
- Calculate reaction distance, braking distance and stopping distance for the velocities 70 km/h, 90 km/h and 110 km/h. Copy the table and fill in your values.

Velocity (km/h)	Reaction distance (m)	Braking distance (m)	Stopping distance (m)
70			
90			
110			

When road driving in the dark, the dipped headlights light up the road approximately 50 metres ahead of the car. That is the soonest the driver can discover an obstacle.

- Investigate for which velocities it is possible to stop in 50 metres.

According to the formula for the stopping distance $s = 0.27v + 0.005v^2$ the driver will not be able to stop for an obstacle which is discovered at a distance of 50 metres if the driver drives at a velocity of 110 km/h. Imagine that the car is able to pass the obstacle and that the driver continues to break. See figure.



- How far after the obstacle will the car stop if the velocity is 110 km/h when the driver discovers the obstacle?

- What is the velocity of the car when it passes the obstacle if the velocity is 110 km/h when the driver discovers the obstacle?

The velocity of the car at the obstacle depends on the original velocity when the driver discovers the obstacle and the distance to the obstacle.
Now imagine that the original velocity of the car is *v* km/h when the driver discovers the obstacle at a distance of 50 metres and that the velocity of the car at the obstacle is *u* km/h.

- Investigate and describe the relation between *u* and *v*. (3/4/¤)