NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2001

Directions

Test time — 240 minutes without a break for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources — **Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.”
Please note calculators are not allowed in this part.

Part II: Calculators, and “Formulas for the National Test in Mathematics Courses C, D and E”.

Test material — The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 15 problems. **Part I** consists of 7 problems and **Part II** consists of 8 problems.

To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 45 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 “Pass”-points and 1 “Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000.

Lower limit for the mark on the test
Pass: 12 points
Pass with distinction: 25 points of which at least 7 “Pass with distinction points”.
Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: _____ School: _____

Education programme/adult education: _____

Part I

This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. $f(x) = x^5 + 7x^2 - 5x + 3$

a) Differentiate $f(x)$ *Only answer is required* (1/0)

b) Give an example of another function that has the same derivative as the given function $f(x)$

Only answer is required (1/0)

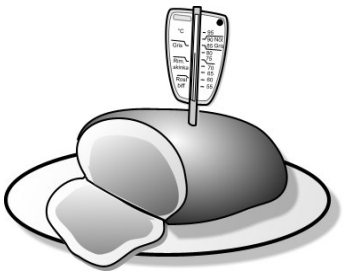
2. Find

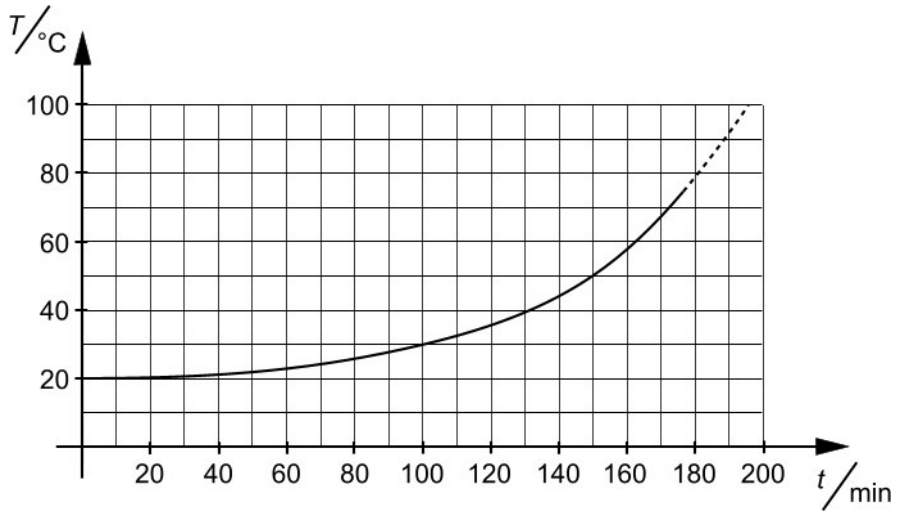
a) $\lg 10\,000$ *Only answer is required* (1/0)

b) $\ln e^2$ *Only answer is required* (1/0)

3.

It takes about 3 hours to roast a Christmas ham. When it is done, the meat thermometer says 77°C. When Tove roasted her Christmas ham the temperature of the ham increased according to the diagram below.





What was the average change of temperature per minute between 100 min and 150 min? (2/0)

4. Which of the alternatives below is a solution to the equation $4^x = 9$?

A) $x = \lg 9$ B) $x = \dfrac{\lg 4}{\lg 9}$ C) $x = \lg 2{,}25$

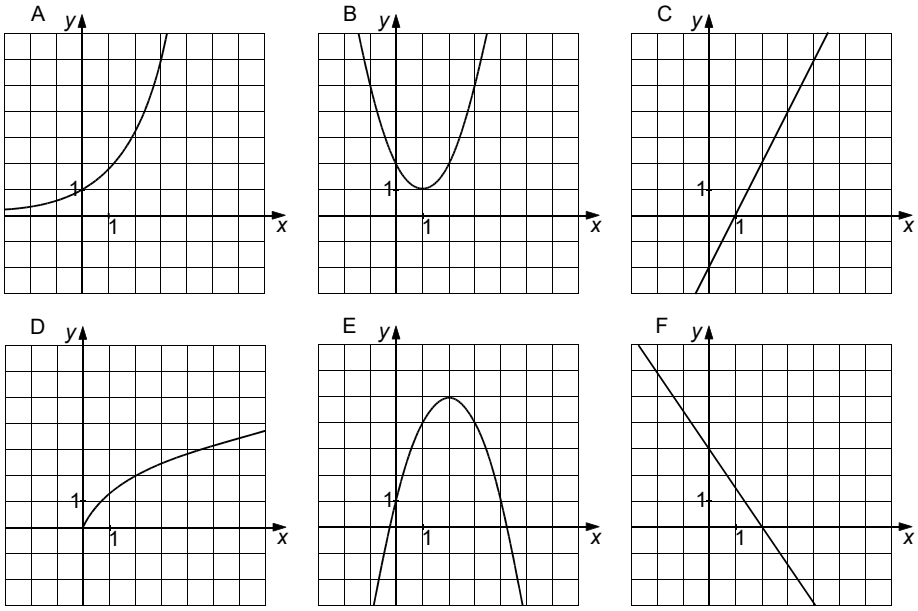
D) $x = \dfrac{\lg 9}{\lg 4}$ E) $x = 2{,}25$ F) $x = 5$

Only answer is required (1/0)

5. Below you can see the graphs to six functions $y = f(x)$

a) In which of the alternatives A-F below is there a graph to a function $y = f(x)$ where $f'(2) = 0$ *Only answer is required* (1/0)

b) In which of the alternatives A-F below is there a graph to a function $y = f(x)$ where $f'(1) < 0$ *Only answer is required* (0/1)



6. The graph of the function $y = x^3 - 45x^2 - 3000x + 1000$ has a local maximum point.
Use the derivative to find the co-ordinates of this point (2/1)

7. The graph to a quadratic function has a local minimum in (−1, 4).

a) Draw a sketch of the graph to the derivative of the function. (0/2)

b) Explain why the graph to the derivative has this appearance. (0/2)

Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

8. Solve the equations

a) $x^{3,5} = 1589$ *Only answer is required* (1/0)

b) $5 \cdot 2^x = 34$ (2/0)

9. Simplify the following expression as far as possible.

a) $\dfrac{x^2 + x}{x}$ (1/0)

b) $\dfrac{x^2 - 1}{x + 1}$ (1/0)

10. Solve the equation $x^4 - 2x^2 = 0$. Answer exactly. (2/0)

11. On New Year's night Gunnar made the following New Year resolution:

"This year I will save 1 SEK the first week, 1.50 SEK the second week, 2.25 SEK the third week and every week 50 % more than the previous week."

How much will he have saved when 52 weeks have passed, if he manages to keep his New Year resolution? (0/3)

12.

The fishing club "The Blue Knot" has released fish in a lake. The number of fish is given by the function $f(t) = 35400 \cdot 0.996^t$ where t is the time in days after the release.

a) In words, explain what $f(140)$ means. (1/0)

b) By how many per cent does the number of fish decrease each day?
Only answer is required (1/0)

c) Rewrite the function using the base e (0/1)

d) Calculate $f'(140)$ and explain in words what you have calculated. (0/2)

13. Experiments have shown that approximately one acre[1] of farmland per person is needed to provide food for the world's population. In 1950 the population was approximately 2.5 billion people and in 1980 it was 4.6 billion people. If we assume that the population grows exponentially, the need for farmland will also grow exponentially. There are about 9 billion acres of farmland on earth.

According to this model, from what year will there not be enough land to provide food for the world's population?

(0/4)

[1] One acre is 4936 m^2 or close to half a hectare.

14. A 30-cm long cord is cut into two pieces. The first part is formed into a circle, and the second part is formed into a square.
Show that the sum of the area of the circle and the square always exceeds 30 cm^2, no matter where the cord is cut. (0/4/¤)

15. This problem concerns the derivative of a quadratic function. We shall examine the graph of the derivative. A general quadratic function can be written as $y = ax^2 + bx + c$ where a, b and c are constants.
One can carry out the general investigation (point three below) or if one prefer, to solve the problem step by step as indicated.

- If $a = 1$, $b = 0$ and $c = 0$ then we have the function $y = x^2$. Draw the graph of y'.

- Choose your own values of a and draw the graph of the derivative of your new functions.
  How does your choice of a affect the graph of the derivative?

- Investigate in as detailed and complete manner as is possible how a, b and c affect the graph of the derivative. (2/4/¤)

When evaluating problem 15 your teacher will look at:
- How well you argument for your conclusions
- How close your solution is to a general solution
- The clarity of your explanations, justifications and conclusions
- How well you use the mathematical language