NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2002

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note calculators are not allowed in this part.* **Part II:** Calculators, and “Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 15 problems. **Part I** consists of 6 problems and **Part II** consists of 9 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 “Pass”-points and 1 “Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 “Pass with distinction points”. Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: ______________________________ School: _________________________

Education programme/adult education: ____

Part I

This part consists of 6 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Find $f'(x)$ when

a) $f(x) = 3x^3 + 4x$ *Only answer is required* (1/0)

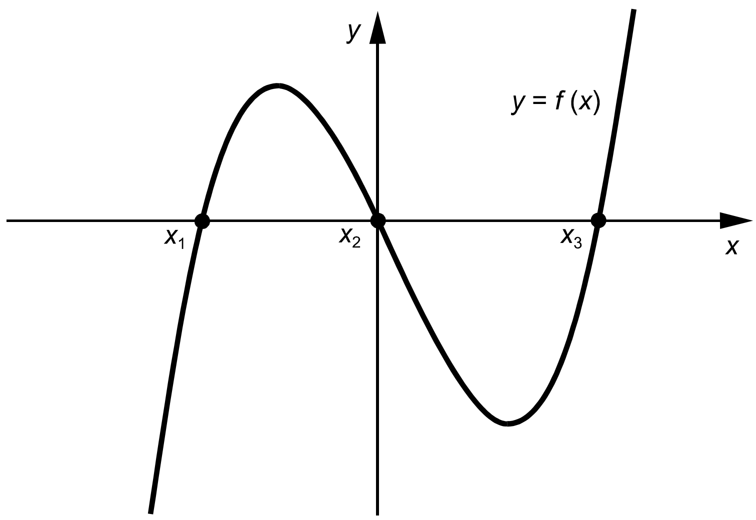
b) $f(x) = 5\mathrm{e}^{4x}$ *Only answer is required* (1/0)

c) $f(x) = 10$ *Only answer is required* (1/0)

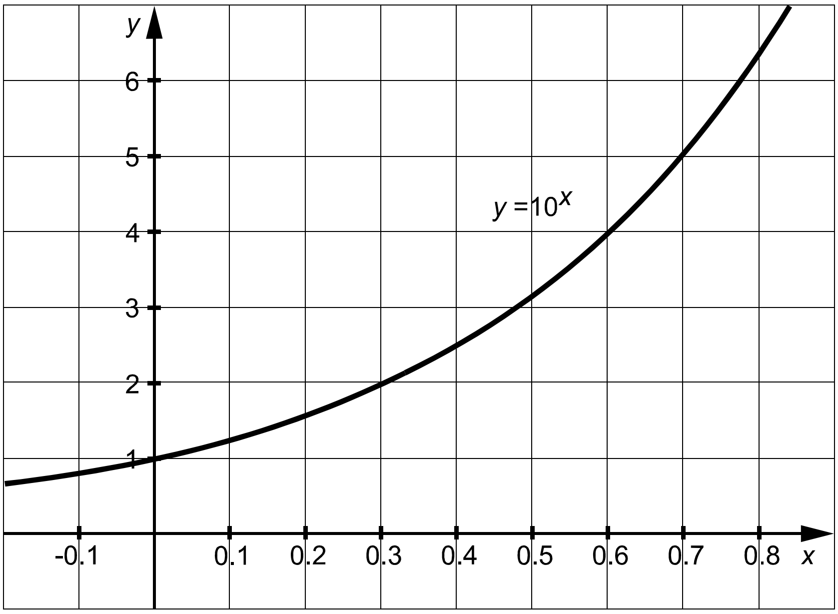
2. Calculate $\lg 100 + \lg 10000$ *Only answer is required* (1/0)

3. The figure below shows the graph to the function $y = x^3 - 2x^2 - 63x$ and its zeroes x_1, x_2 and x_3

Calculate x_1, x_2 and x_3 (3/0)



4.



The figure above shows a part of the graph to the function $y = 10^x$

Use the figure to solve the following problems.

a) Write the number 3 as a power with base 10 *Only answer is required* (1/0)

b) Write down an approximate value to $\lg 5$ *Only answer is required* (1/0)

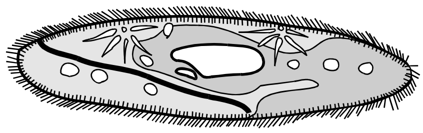
c) Find an approximate value to $10^{1.3}$
Explain how you reached your answer. (0/2)

5.

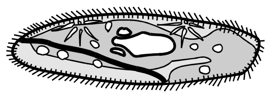
The picture shows a slipper animalcule

In 1934 the Russian biologist Georgyi Frantsevitch Gause carried out an experiment with two different kinds of slipper animalcules: *Paramecium caudatum* and *Paramecium aurelia*.
He cultured the two different kinds separately and found the following relations:

$$f(t) = \frac{105}{1 + 34e^{-1.1244t}}$$ (*Paramecium caudatum*)



$$g(t) = \frac{64}{1 + 15e^{-0.794t}}$$ (*Paramecium aurelia)*



where $f(t)$ and $g(t)$ are the number of slipper animalcules in each culture and t is the time in days from the start of the slipper animalcule cultures.

a) How many *Paramecium caudatum* were there in the culture at the beginning? *Only answer is required* (1/0)

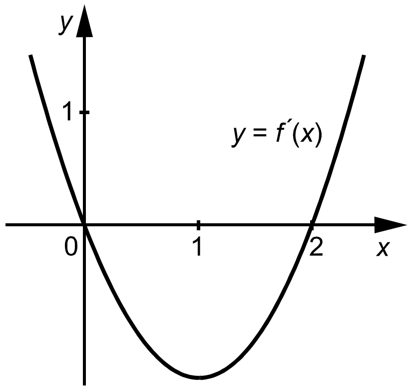
b) Write down a question that can be answered by solving the equation $f(t) = g(t)$ (0/1)

c) Write down a question that can be answered by solving the equation $f'(t) = g'(t)$ (0/1)

d) As time goes, what value would the number of *Paramecium caudatum* have approached according to the relation above? Explain your answer. (0/1)

6. The figure shows the graph to the **derivative** $y = f'(x)$. The derivative is a polynomial of degree 2.

Sketch some of the graphs to the functions that have this derivative in the same coordinate plane. Explain why your graphs have these appearances.



(0/2/¤)

Part II

This part consists of 9 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

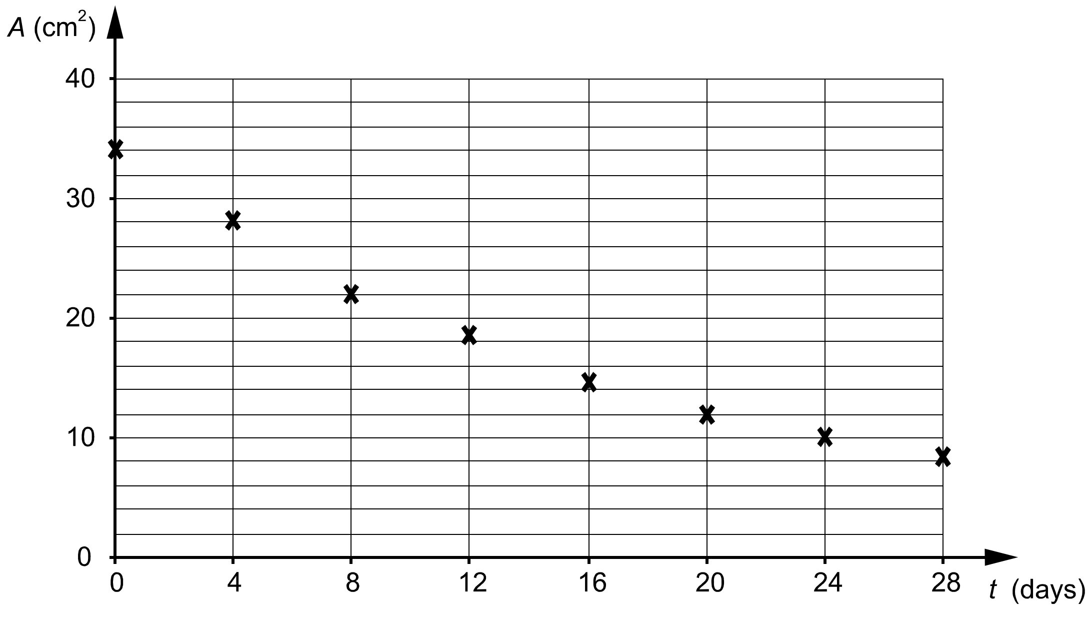
7. Calculate the sum of the first 60 terms in a geometric sum where the first term is 10 and the second is 12. (2/0)

8. A scientist studied the relation between the absorption of oxygen S litres/minute and the step length l centimetres for a number of runners running at a certain speed.
The scientist found the following relation:

$S = 0.0009775 \cdot l^2 - 0.287385 \cdot l + 25.0653$
where $133 \leq l \leq 170$

Use the derivative to show that the absorption of oxygen is at a minimum at a step length of 147 cm. (3/0)

9. To decide how fast a wound would heal its area was measured at 12.00 every fourth day. The result can be seen in the diagram below.



a) Calculate the average daily reduction of the area of the wound from 12.00 day 4 to 12.00 day 24. (1/0)

If the measurements are approximated with an exponential function the relation $A = 34 \cdot 0.95^t$ is obtained where A is the area in square centimetres and t is the time in days.

b) How long does it take before the area of the wound is 3 cm^2 according to this relation? (2/0)

c) How long does it take before the area of the wound decreases by 1 cm^2/day? (0/3)

10.

190 years ago, there lived approximately 32 000 people in Norrbotten county. Today, there live approximately 254 000 people. Assume that the increase has been exponential during the whole period of time.

Calculate the average yearly percentage increase in the number of people in Norrbotten county during this period. (0/2)

11. Find the function $f(x) = kx^4$ (k is a constant) for which it is true that $f'(4) = 1$ (1/1)

12. Anna has solved the equation $\frac{3}{x-3} - \frac{x}{3} = \frac{x}{x-3}$ the following way:

$$\frac{3}{x-3} - \frac{x}{3} = \frac{x}{x-3}$$

$$3 \cdot 3 - x(x-3) = 3x$$

$$9 - x^2 + 3x = 3x$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

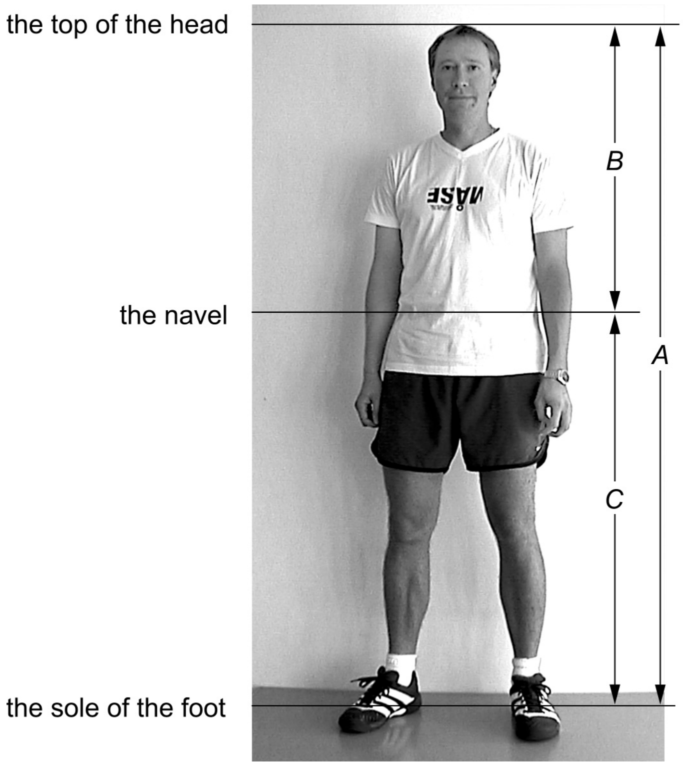
ANSWER: $x_1 = 3$ and $x_2 = -3$

Anna's sister Kajsa says that: 'The solution to the equation is correct, but the answer is wrong!
Explain in what way the answer is wrong. (0/1)

13. The Golden ratio is a division of a line segment into two parts that is considered aesthetically pleasing to the eye.

It is said that the navel divides the length of some human bodies according to the Golden ratio. If a human body fulfills the principle of the Golden ratio, the following relation must be true: $\frac{B}{C} = \frac{C}{A}$

where A is the length of the body, B is the distance between the top of the head and the navel and C is the distance between the navel and the sole of the foot, see figure.



The picture above shows Magnus who is 180 cm. Calculate the distance in centimetres between his navel and the sole of his foot, provided that he is built according to the principle of the Golden ratio. (0/2)

14. If $f(x) = Ax^2$ (A is a constant) then $f'(x) = 2Ax$
Use the definition of the derivative to show this. (0/2/¤)

When assessing your work with problem 15 the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How systematic you are when investigating
- How well you explain your conclusions
- How well you use mathematical words and symbols
- How well you present your work

15.

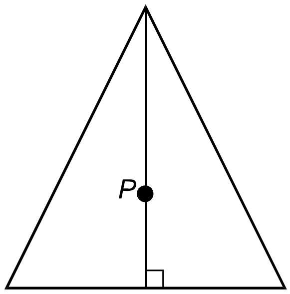


Figure 1

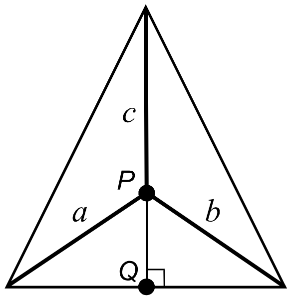


Figure 2

Figure 1 shows an isosceles triangle. The point P lies on one of the triangle's heights. The distances between the point P and the corners of the triangle are of lengths a, b and c, see figure 2. The point Q lies on the middle of the base.

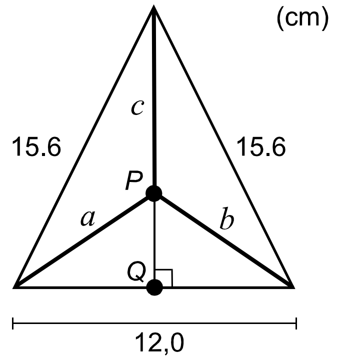


Figure 3

In the following problem you are going to study figure 3.

- Let the distance PQ be 8.0 cm. Calculate the length a.
- Let the distance PQ be 8.0 cm. Calculate the value of the sum of the squares of the lengths a, b and c, that is $a^2+b^2+c^2$.
- Examine and explain, as completely and in as much detail as possible, how the value of the sum $a^2+b^2+c^2$ depends on the distance PQ. Then write down some *different* conclusions regarding how the value of the sum depends on the distance PQ. (3/4/¤)