NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2004

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, and ”Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 15 problems. **Part I** consists of 7 problems and **Part II** consists of 8 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 46 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: In addition to the requirements for ”Pass with distinction” you have to show ”*Pass with special distinction*” *qualities in at least two* of the ¤-problems. You must also have at least 14 ”Pass with distinction”-points.

Name: ______________________________ School: ________________________________

Education programme/adult education: ____

Part I

This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = 2x^3 + 5$ *Only answer is required* (1/0)

b) $f(x) = x(x+1)$ *Only answer is required* (1/0)

c) $f(x) = \mathrm{e}$ *Only answer is required* (1/0)

2. Solve the equations. Give exact answers.

a) $\mathrm{e}^x = 10$ *Only answer is required* (1/0)

b) $2x^7 = 10$ *Only answer is required* (1/0)

c) $(x-1)^3 = 8$ *Only answer is required* (1/0)

3. The function $f(x) = x^3 - 3x$ has a local maximum point.

Determine the x-coordinate of this point. (3/0)

4. For which value of x is the expression $\dfrac{11-x}{x-3}$ not defined?

Only answer is required (1/0)

5. There are several functions $y = f(x)$ that satisfy the conditions:
$f(2) = 0$, $f'(4) = 0$ and $f'(2) = 2$

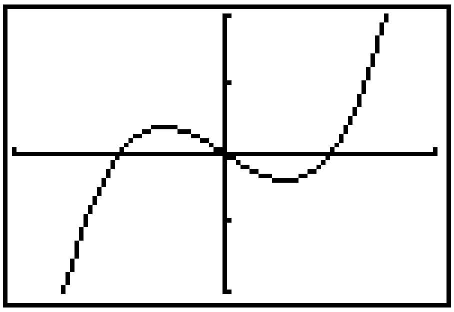
Sketch the graph of one function that satisfies these conditions. (0/2)

6. Kalle solves the equation $x^3 - x = 0$ like this:

$$x^3 - x = 0$$
$$x^3 = x$$
$$\frac{x^3}{x} = \frac{x}{x}$$
$$x^2 = 1$$
$$x = \pm 1$$

ANSWER : $x_1 = 1$ and $x_2 = -1$

He then draws the graph of the function $y = x^3 - x$ on his graphic calculator to check if he has solved the equation correctly. The graph to $y = x^3 - x$ looks like this:



Kalle sees the graph and realises that he must have made a mistake when he solved the equation.

a) How many solutions are there to the equation $x^3 - x = 0$? Justify your answer by the look of the graph. (1/0)

b) Show how Kalle should have solved the problem and explain *why* Kalle's solution is wrong. (0/1/¤)

7. For the function $f(x) = e^{2x}$ it holds that $f(1.1) \approx 9$

a) Determine an approximate value to $f'(1.1)$ (1/1)

b) Show that $f'(3.3) \approx 2 \cdot 9^3$ (0/1/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

8. In a geometric progression the first term is 5000 and the ratio 1.02.

Calculate the sum of the first 36 terms. (2/0)

9. Before the parachute has opened, the parachutist falls in a so-called free fall.

Due to air resistance, the velocity does not increase during the whole free fall. After 10 seconds, the parachutist reaches his/her maximum velocity.

A simple model that describes the distance the parachutist has fallen during the first 10 seconds is:

$$s(t) = -\frac{t^3}{6} + 5t^2$$

where s is the distance fallen in metres and t is the time in seconds after leaving the aircraft.

a) Use the model to calculate how long a distance the parachutist has fallen during the first 10 seconds. (1/0)

b) Use the model to calculate the maximum velocity of the parachutist. (2/0)

10. Sometimes a doctor needs to know the body surface area of a patient to be able to decide how much medicine the patient should be given.

In 1916 two scientists, DuBois and DuBois, presented a formula for calculating the body surface area of a human being:

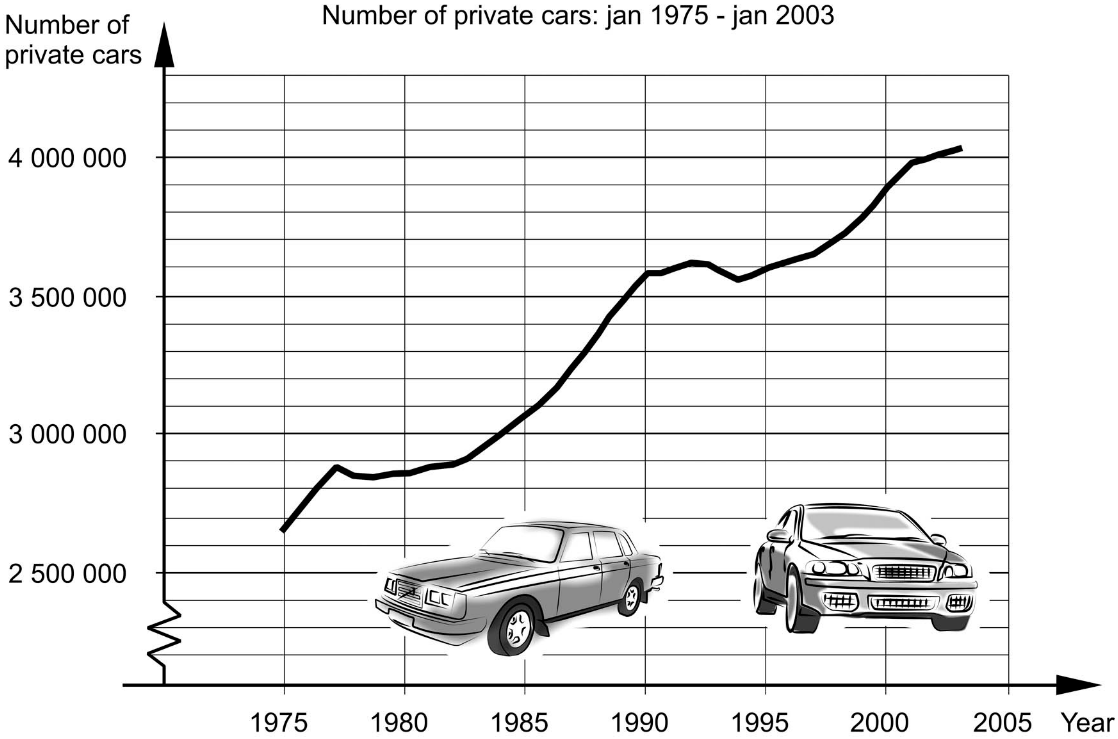
$$\lg A = -0.69364 + 0.725 \lg L + 0.425 \lg M$$

where A is the body surface area in square metres, L is the length in metres and M is the body weight in kilograms.

A doctor is going to write out a prescription for the medicine Methotrexat to a patient that suffers from rheumatism. According to the information about the medicine, the patient should be given 7 mg of medicine per square metre of body surface area. The patient weighs 65 kg and is 1.60 m tall.

What amount of medicine should the patient receive? (2/0)

11. The diagram below shows how the number of private cars in Sweden has changed from January 1975 to January 2003.



a) Calculate the average yearly change in the number of private cars during the years 1980-2000. (2/0)

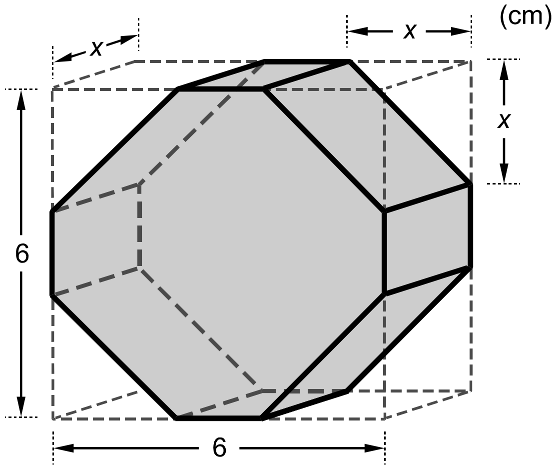
b) Calculate the yearly percentage change in the number of private cars during the years 1980-2000, assuming that the percentage change has been the same each year. (0/2)

c) In parts a) and b) of the problem you have used two different mathematical models to describe how the number of private cars has changed.

Which of these models would you use to estimate the number of private cars in the year 2020? Justify why you would choose this model. (0/1)

12. If $f(x) = Ax^2$, where A is a constant, it holds that $f'(x) = 2Ax$
Use the definition of the derivative to show this. (0/2/¤)

13. A straight eight-sided prism can be created by removing four identical parts from a cuboid. See the figure below.



Which value of x gives the prism of maximum volume? (0/4)

14. In order to follow the progress of small children during their first years, their weight is checked often.
The picture below shows Joel who, four weeks after his birth, weighs 4300 g.

During the fifth week his weight increases by 225 g. If Joel follows the average weight curve provided by the child welfare centre during his first year, his increase in weight will decrease by 3 % each week.

What will Joel's weight be when he is one year old? (0/3)

When assessing your work with this problem your teacher will take into consideration:
- How general your solution is
- How well you justify your conclusion
- How well you carry out your calculations
- How well you present your work
- How well you use the mathematical language

15. In this problem you are going to investigate the area of a rectangle that lies above the x-axis and has two corners on a quadratic curve and two corners on the x-axis.

Figure 1 shows a rectangle with corners on the quadratic curve $y = 27 - 3x^2$ and the x-axis.

- Calculate the area of the rectangle.

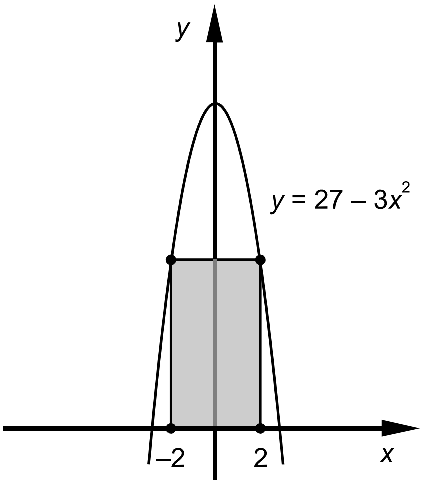


Figure 1

Figure 2 shows the rectangle PQRS with corners on the quadratic curve $y = 27 - kx^2$ and the x-axis, where k is a positive constant.

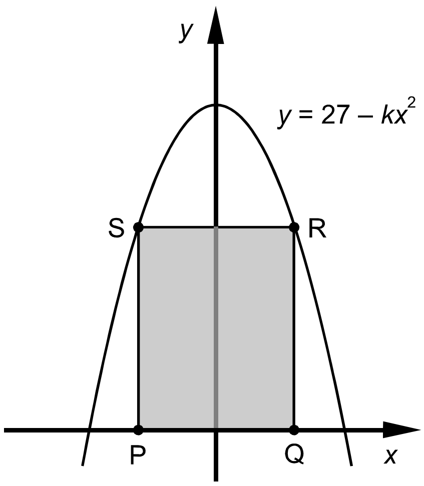


Figure 2

By moving the corners of the rectangle along the curve we can make the area of the rectangle vary.
The area of the rectangle also depends on the value of the constant k.

- Investigate, as thoroughly as possible, how the area of the rectangle depends on the placing of the corners of the rectangle and the value of the constant k. (3/5/¤)