NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2005

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C and D.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators and ”Formulas for the National Test in Mathematics Courses C and D”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 19 problems. **Part I** consists of 8 problems and **Part II** consists of 11 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 19 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: 26 points of which at least 13 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

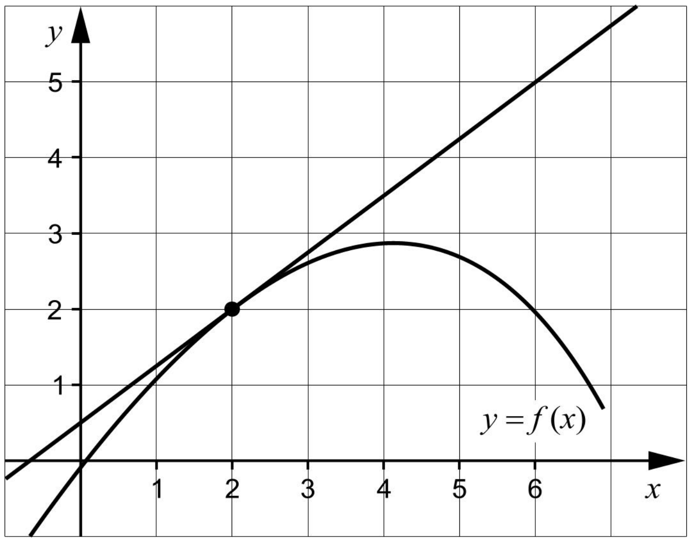
Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Find $f'(x)$ if $f(x) = x^3 - x^2$ *Only answer is required* (1/0)

2. The figure shows the graph to a function $y = f(x)$ and the tangent at the point on the curve where $x = 2$

Calculate $f'(2)$ *Only answer is required* (1/0)



3. Solve the equations. Give exact answers.

a) $10^x = 0.6$ *Only answer is required* (1/0)

b) $\lg x = 0.6$ *Only answer is required* (1/0)

4. The graph to the function $f(x) = x^4 - 4x$ has only one extreme point. The x-coordinate of the extreme point is 1.

Decide whether this extreme point is a maximum or a minimum. (1/0)

5. Solve the equation $5x^3 - 45x = 0$ (2/0)

6. The weight *y* of a growing pumpkin is a function of time *t*, that is $y = f(t)$
The weight *y* is measured in kg and the time *t* in days.

What do you find out by calculating $f'(10)$?
Choose one of the alternatives A-E. *Only answer is required* (0/1)

A The weight of the pumpkin in kg at the time 10 days.

B The increase in weight of the pumpkin in kg during 10 days.

C The time it takes for the weight of the pumpkin to increase by 10 kg/day.

D The time it takes for the weight of the pumpkin to increase to 10 kg.

E The increase in weight of the pumpkin in kg/day at the time 10 days.

7. A mirror is going to be made of a rectangular shaped mirror glass surrounded by a wooden frame.

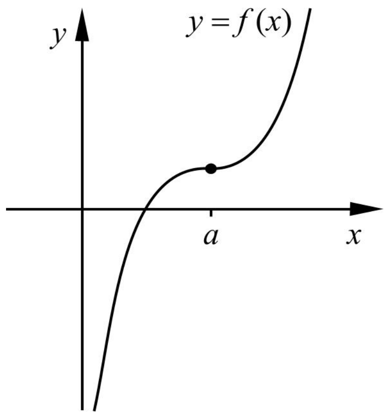
The frame should be made of a 0.5 dm wide strip of wood. In total, 60 dm of wooden strips will be used. The frame should have double wooden strips as top and bottom pieces, while the side pieces should consist of single strips, see figure.

The area of the mirror glass *A* dm^2, as a function of the length of the top piece *x* dm is given by
$A(x) = -2x^2 + 32x - 30$

a) Use derivatives to determine which value of the length *x* that gives the maximum area of the mirror glass. (3/0)

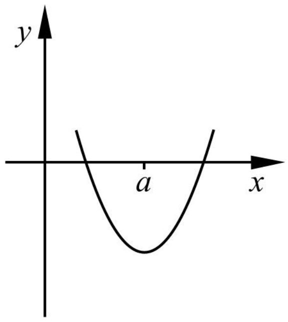
b) Show that the area of the mirror glass *A* can be written
$A(x) = -2x^2 + 32x - 30$ (0/2/¤)

8. The figure below shows the graph to the cubic polynomial $y = f(x)$
The graph has a terrace point for $x = a$.

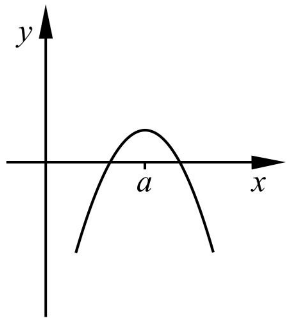


Which of the functions A-F shows a graph of the function $y = f'(x)$?
Motivate your choice. (0/1/¤)

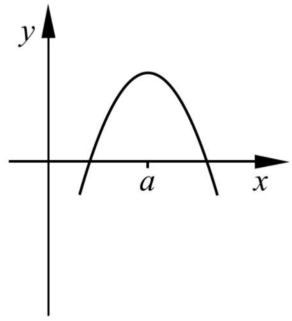
A



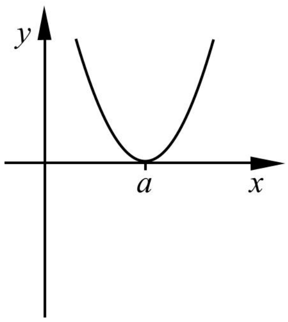
B



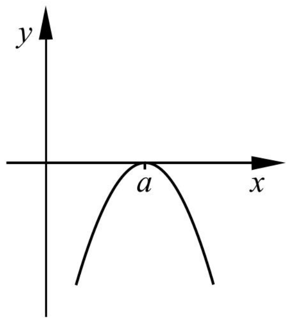
C



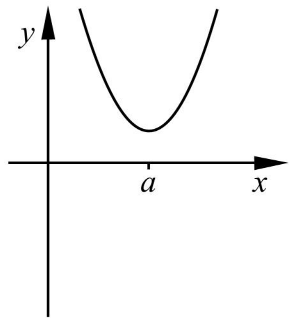
D



E



F



Part II

This part consists of 11 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

9. Calculate

a) $\lg 2 + \lg 5$ *Only answer is required* (1/0)

b) $\lg 5^2 + (\lg 5)^2$ *Only answer is required* (1/0)

10. Write down a problem about a real situation that can be solved with aid of the equation $5000 = 10000 \cdot 0.6^x$ (2/0)

11. The number of sent SMS text messages has increased in Sweden:

In 1998, 44 million SMS text messages were sent.

Five years later, in 2003, 1816 million SMS text messages were sent.

(Source: The National Post and Telecom Agency)

a) Calculate the yearly average increase in the number of sent SMS text messages. (1/0)

b) Calculate the yearly percentage increase in the number of sent SMS text messages. (0/2)

12. Differentiate the following functions

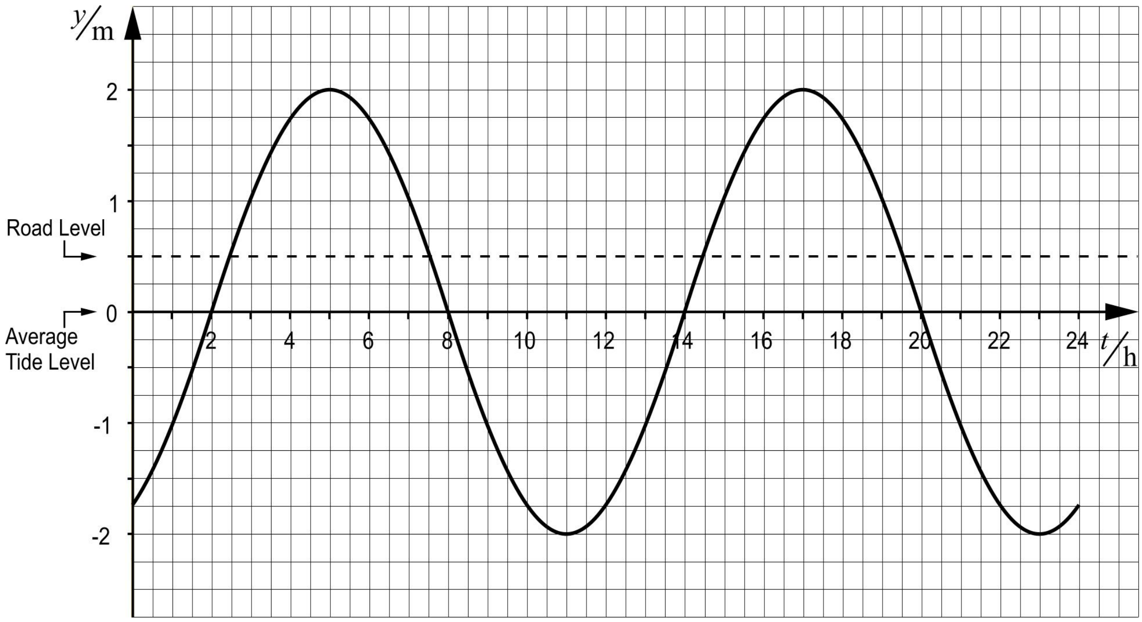
a) $f(x) = e^x$ *Only answer is required* (1/0)

b) $f(x) = ex$ *Only answer is required* (1/0)

c) $f(x) = x^e$ *Only answer is required* (0/1)

13. In North Eastern England there is an island, "The Holy Island of Lindisfarne", which is only accessible by a "causeway". This is a road that cannot be used during some parts of the day since the tide then covers the road.

The tide level varies around the average water level according to the diagram below. The tide level y is measured in metres and the time t in hours. The time is counted from midnight. The road to Holy Island is 0.5 m above the average water level.



a) When at its highest, how high above the roadway is the tide level?
Only answer is required (1/0)

b) At what time in the afternoon does the water start to rise above the roadway, and at what speed does this happen? (1/1)

14. A rational expression is the quotient of two polynomials.
What conditions must be satisfied for a rational expression to be defined? (0/1)

15. Simplify the expressions as far as possible.

a) $(x^2-6)(x^2+6)-x^4$ (1/0)

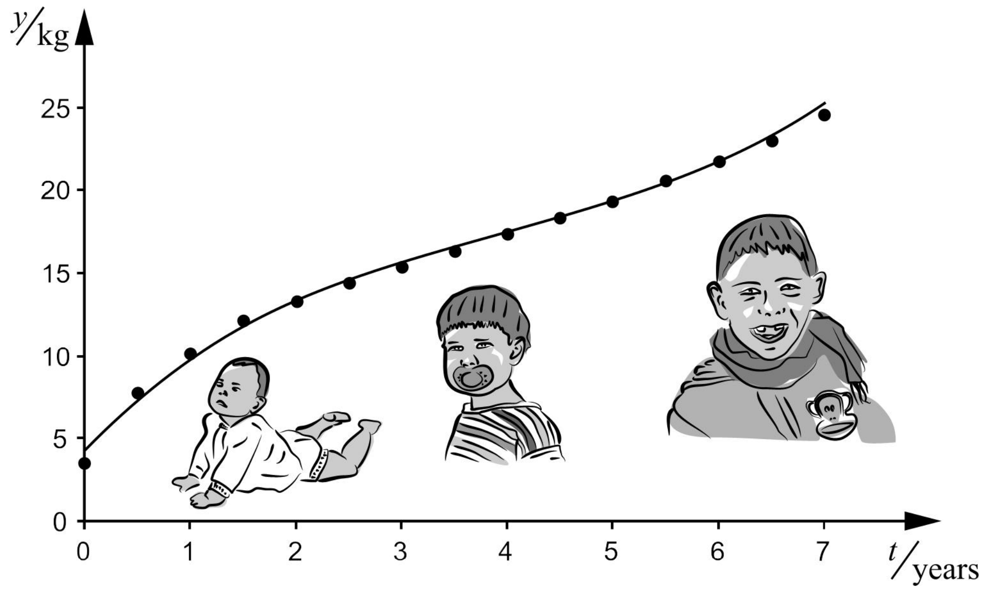
b) $\dfrac{2a^2-8a}{4-a}$ (1/1)

c) $\dfrac{3^x \cdot 3^x}{3^x+3^x+3^x}$ (1/1)

16. The derivative of a function $y=f(x)$ is $f'(x)=x^2+2x-5$

Determine the values of x for which the graph of $y=f(x)$ has a tangent with gradient 3. (0/2)

17. The points in the diagram below show how Olle´s weight increases during his first seven years in life. The diagram also shows a cubic curve that has been adjusted to the points.



The cubic curve is given by $y=0.10t^3-1.2t^2+6.5t+4.4$ where Olle´s weight y kg is a function of the time t years.

a) Find a function p that shows at what speed (kg/year) Olle´s weight increases. *Only answer is required* (0/1)

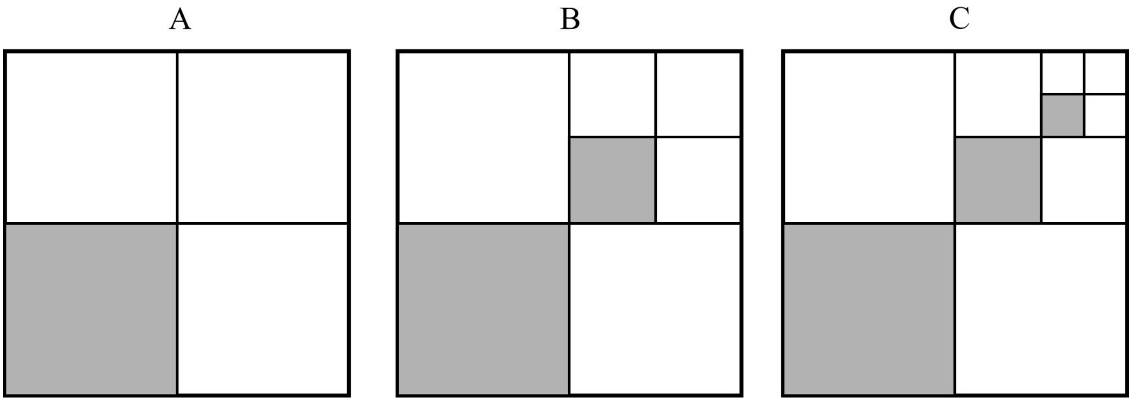
b) Calculate the lowest speed (kg/year) at which Olle´s weight increases. (0/2)

18. Determine $T'(2)$ when $T(x+h)=T(x)+h$ (0/1/¤)

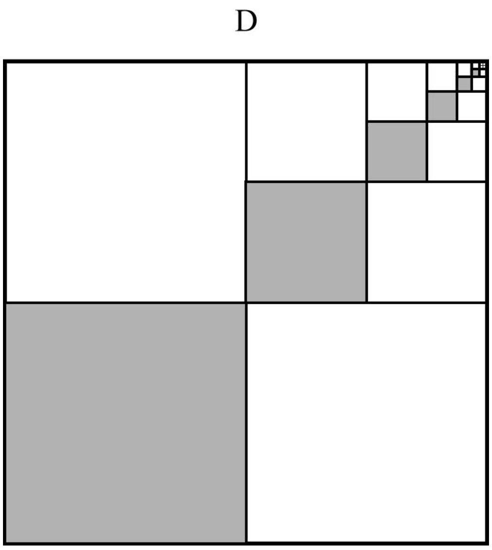
When assessing your work with the following problem, the teacher will take into consideration:
- How well you carry out your calculations
- How general your solution is
- How systematic you are in your investigation
- How well you justify your conclusions
- How well you present your work
- How well you use the mathematical language

19. The figure below shows three squares A, B and C which each consists of a number of smaller squares. Some of the squares have been shaded.

- Determine the *proportion* of the shaded area of each of the squares A, B and C.



Now take a look at square D. Assume that we introduce more and more squares of decreasing size into this square and that we shade some of them according to the same principle as above.



- Investigate and describe what then happens to the *proportion* of the shaded area in square D. Use your knowledge of geometric sums. (2/3/¤)