NATIONAL TEST IN MATHEMATICS COURSE C

AUTUMN 2008

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 120 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.*
	Part II: Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions.
	Write your name, the name of your education programme/adult education on all sheets of paper you hand in.
	Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.
The test	The test consists of a total of 17 problems. **Part I** consists of 9 problems and **Part II** consists of 8 problems.
	For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.
	Problem 9 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.
	Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points.
	The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	25 points of which at least 6 ”Pass with distinction” points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

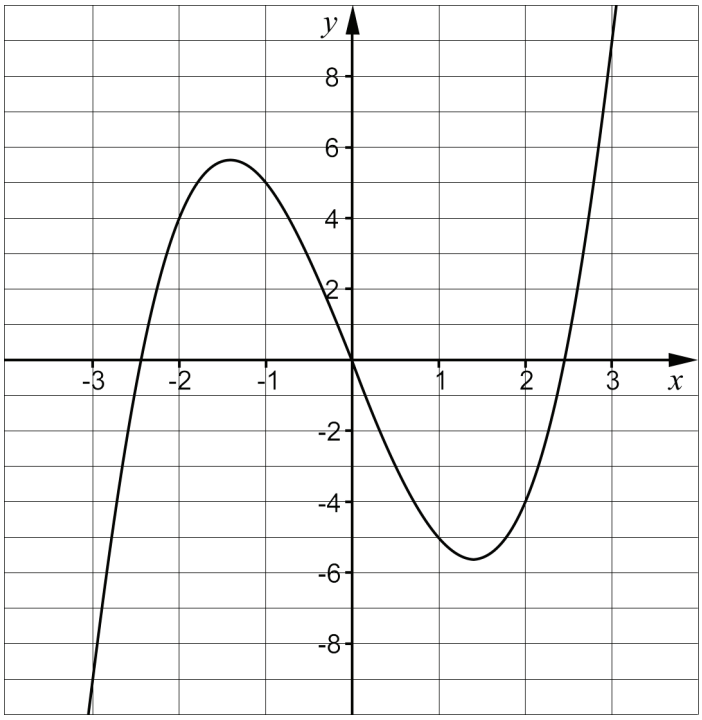
This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = 4x^2 + 7x + 5$ *Only answer is required* (1/0)

b) $f(x) = 3e^{2x}$ *Only answer is required* (1/0)

2. The figure shows the graph for $f(x) = x^3 - 6x$



Give the coordinates for a point on the graph where the derivative is positive.
Only answer is required (1/0)

3. Solve the equations, and give exact answers.

a) $x^7 = 14$ *Only answer is required* (1/0)

b) $7^x = 14$ *Only answer is required* (1/0)

4. Simplify the following expressions as far as possible.

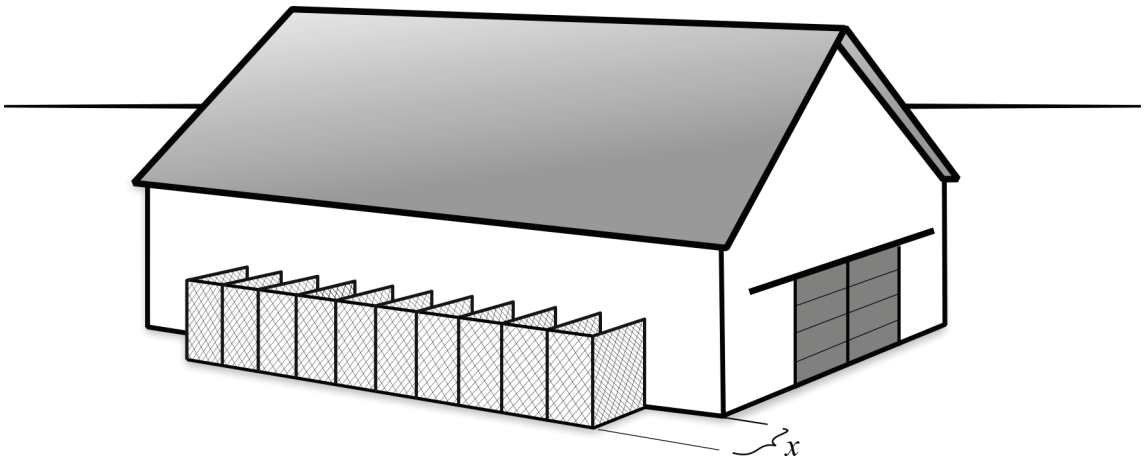
a) $\dfrac{5x^7+10}{x^7+2}$ (1/0)

b) $(a+1)(a-1)(a^2+1)$ (1/0)

c) $\dfrac{x}{x-1}+\dfrac{1}{1-x}$ (0/1)

5. Solve the equation $x^3-7x=0$
Give an exact answer. (2/0)

6. Karin runs a kennel and is going to build 10 rectangular exercise pens for her dogs. The exercise pens will all be the same size and attached to a barn. Karin has 44 m of fencing to use when she builds the exercise pens.



The area for an exercise pen A m^2 as a function of the length of the exercise pen's side x m (see figure) is then given by
$A(x)=4.4x-1.1x^2$ where $0<x<4$

a) Calculate, by using differentiation, the value of x which gives each exercise pen the largest possible area. (3/0)

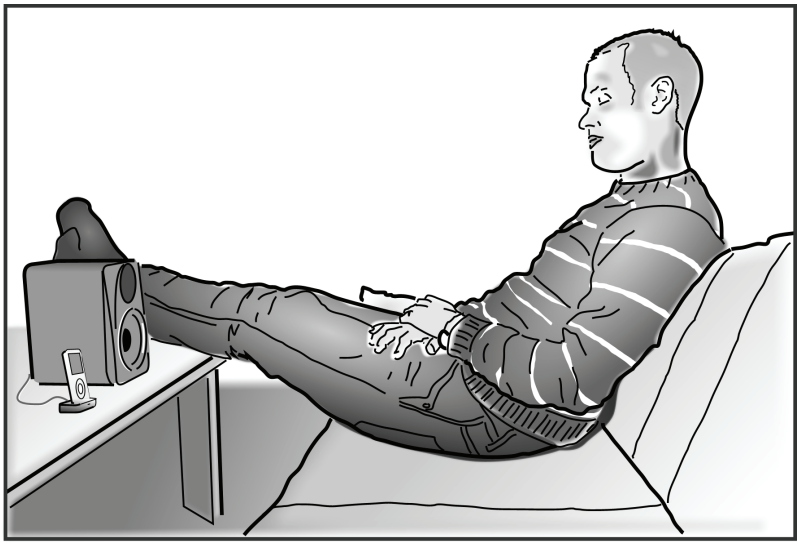
b) Show that the area A m^2 can be written as $A(x)=4.4x-1.1x^2$ (0/2)

7. For the function f it holds that $f(x) = Ax^3$ where A is a constant.
Use the definition of the derivative to determine $f'(x)$ (0/2/¤)

8. Sound level is a measure of how loud a sound is and is measured with the unit decibel (dB). Lucas is sitting and listening to music in his room. A simplified model of sound level L dB from his loudspeaker can be written

$$L = 90 + 10 \cdot \lg P$$

where P Watt (W) is the power that the loudspeaker gets from the amplifier.



a) Calculate the sound level when the power is 1 W.
Only answer is required (1/0)

b) Lucas has read that, under certain conditions, a sound level of 100 dB can damage his hearing.
At what level of power from the amplifier will the sound level be harmful? (2/0)

In general it is true that an increase in the sound level of 10 dB is perceived by the ear as if the sound has become twice as loud.

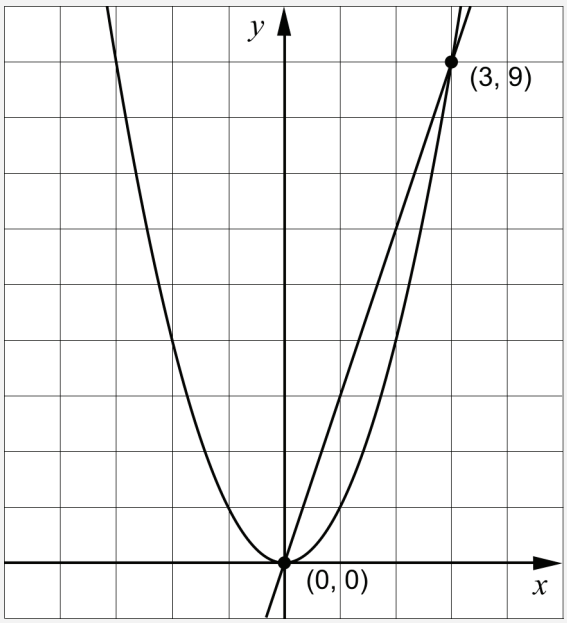
c) How many times greater must the power be in order for the ear to perceive that the sound has become twice as loud? (0/1/¤)

When assessing your work with the following problem your teacher will take into consideration:

- How well you carry out your calculations
- How close you come to a general solution
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

9. Kalle and Stina have received the following mathematical problem from their teacher:

In the figure the graph for $f(x) = x^2$ is shown and a secant that passes through the points (0, 0) and (3, 9). A secant is a straight line that passes through two points on a curve. Calculate the slope of the secant.



In order to calculate the slope of the secant they set up the following expressions:

Kalle: $\dfrac{f(3) - f(0)}{3 - 0}$ Stina: $\dfrac{f'(0) + f'(3)}{2}$

They calculate the value of their expression and discover that they both get the value 3. Kalle believes that Stina has been lucky to get the correct answer. Therefore, they decide to redo the same type of calculation for another secant and choose a secant that passes through the points (1, 1) and (5, 25). They set up the following expressions:

Kalle: $\dfrac{f(5) - f(1)}{5 - 1}$ Stina: $\dfrac{f'(1) + f'(5)}{2}$

- Calculate the values of Kalle's and Stina's new expressions.
- Draw a conclusion regarding Kalle's and Stina's methods based on the values of the four expressions above.
- Investigate if the same conclusion can be drawn if you instead study all the possible types of quadratic functions. (3/3/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. The athlete Susanna Kallur competes in the 100 metre hurdle event. In hurdle racing she is part of the world élite. At the 2007 World Championships in track and field she came in fourth with a time of 12.51 s.

By examining the replay of the race the time at which she passes each hurdle, respectively, has been determined.

Hurdle nr	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Distance run (m)	8.5	17	25.5	34	42.5	51	59.5	68	76.5	85
Time (s)	2.2	3.2	4.2	5.2	6.1	7.1	8.1	9.1	10.1	11.1

a) What average speed did Susanna have between the first hurdle and the tenth hurdle? (1/0)

The distance $s(t)$ metres that Susanna ran is a function of time t seconds after the start.

b) For Susanna's hurdle event it is true that $s'(11.6) = 10.6$
Interpret what this means. (0/1)

11. Julia's grandmother makes regular deposits of money for her grandchild. Her grandmother deposits SEK 1000 into a bank account each birthday from the year that Julia turns 1 and until the year she turns 18.

How much can Julia withdraw from the bank account the day she turns 18, right after the last deposit has been made, provided that the annual interest is 4 %? (2/0)

12. Sketch the graph of a function f for which it holds true that
$f(10) = 25$ and $f'(10) = 0$ (2/0)

13. For which value of x is the expression $\dfrac{x-1}{e^x - 1}$ not defined?

Only answer is required (1/0)

14. The bacteria *Clostridium perfringens* can cause serious food poisoning. If food that contains this bacteria is allowed to cool at room temperature then the number of bacteria increases by 5.9 % per minute. That is why one should always quickly cool food after cooking. It takes about 100,000 bacteria per gram food for a person to get food poisoning.
(Source: Smittskyddsinstitutet-Swedish Institute for Infectious Disease Control)

After cooking, in a piece of poached salmon there are 100 bacteria per gram. The poached salmon is cooled at room temperature. How long does it take before there are so many bacteria per gram of salmon so that the person who eats it gets food poisoning? (0/2)

15. In a geometric series with 10 terms there is a term 40.5 and after that the following term 121.5. Calculate the value of the first term if the sum is 14762 (0/2)

16. For the function f it holds that $f(x) = \frac{x^3}{3} + 2x^2 - 250x$

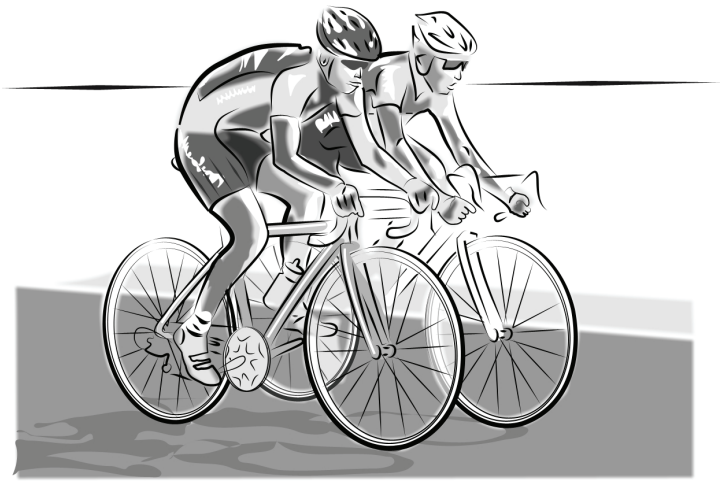
a) How many points on the graph of the function have a tangent with a gradient of 2? (0/2)

b) The graph of the function f has tangents at all points. Which values can the gradient of these tangents have? (0/2)

17. Fredrik and Gustav participate in the same bicycle race. The race is over a distance of 90 km. Fredrik maintains a constant speed throughout the race while Gustav's speed varies. Simplified, the distance (in km) they have cycled can be described by the functions:

$f(t) = 30t$ and $g(t) = t^3 - 6t^2 + 37.8t$ where t is the time in hours after the start.

Fredrik and Gustav start at the same time. Fredrik finishes first. He crosses the finish line exactly 3 hours after the start.



How long after the start is the distance between Fredrik and Gustav the greatest and how great is the distance between them at that time? (0/2/¤)