NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2009

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 90 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 8 problems and **Part II** consists of 10 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	26 points of which at least 7 ”Pass with distinction” points.
Pass with special distinction:	26 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = x^5 - 12x$ *Only answer is required* (1/0)

b) $f(x) = 10$ *Only answer is required* (1/0)

c) $f(x) = (3x)^2$ *Only answer is required* (1/0)

2. At the end of each year, Matilda has made deposits to her bank account, which has a fixed rate of interest. She writes down an expression giving the account balance (measured in SEK) immediately after the latest deposit:

$$\frac{1000(1.02^6 - 1)}{1.02 - 1}$$

a) What rate of interest does the account give? *Only answer is required* (1/0)

b) How many deposits has Matilda made? *Only answer is required* (1/0)

3. Solve the equations and give an exact answer

a) $x^5 = 25$ *Only answer is required* (1/0)

b) $e^x = 25$ *Only answer is required* (1/0)

4. Determine the number of (real) zeroes of the function f where $f(x) = x^3 + 100x$ (1/1)

5. When transporting goods, containers are often used. In order to use the space in a container as effectively as possible, the goods are packed as tightly as possible. The sofa ”Torulf” is packed for transport in one corner of the container, see Figure 1.

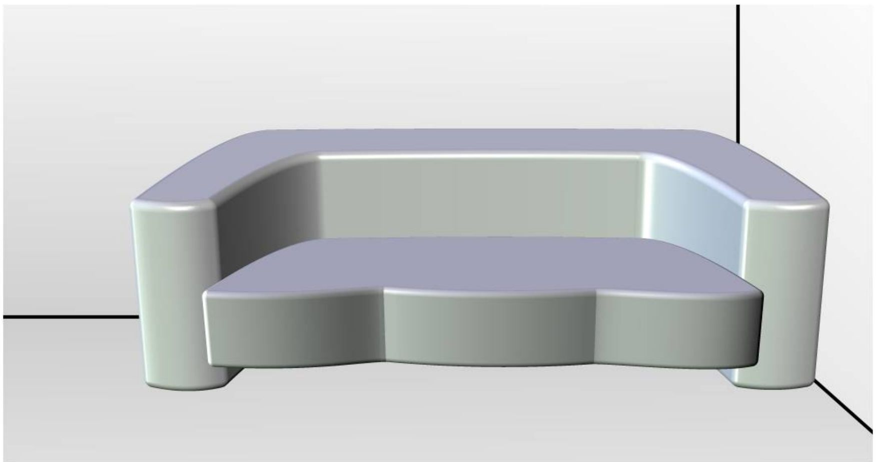


Figure 1. Sofa placed in the container

In the space left between the corner of the container and the sofa, a box can be placed. The box has the shape of a cuboid. To determine the possible dimensions of the box, one only has to investigate the base area of the box, see Figure 2.

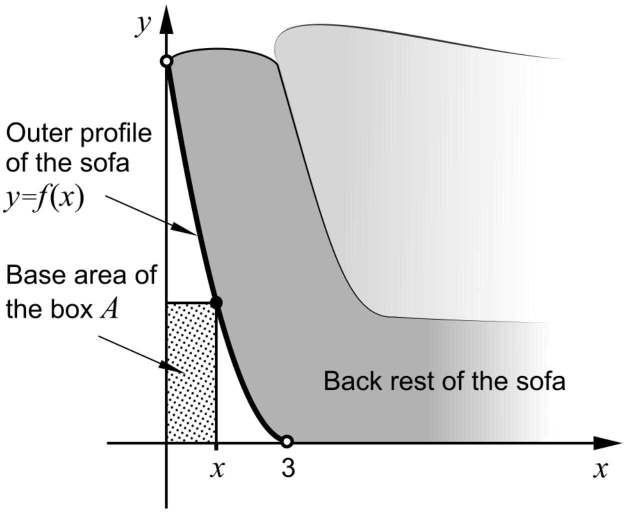


Figure 2. Sofa viewed from above

The base area A dm^2 can be described by $A(x) = x^3 - 6x^2 + 9x$ where x dm is the width of the box, see Figure 2.

a) What value of x maximizes the base area of the box? (3/0)

In Figure 2, the outer profile of the sofa facing the corner of the container is marked by a thick black line. The outer profile of the sofa is described by the function f where $y = f(x)$

b) Determine the expression $y = f(x)$ which describes the outer profile of the sofa. (0/1)

6.

a) For which values of x is the expression $\dfrac{4}{x-2}-\dfrac{8}{x(x-2)}$ undefined?

Only answer is required (1/0)

b) Simplify $\dfrac{4}{x-2}-\dfrac{8}{x(x-2)}$ as far as possible. (2/0)

c) Solve the equation $\dfrac{4}{x-2}-\dfrac{8}{x(x-2)}=2$ (0/1)

7. A group of people greet each other by shaking hands. The number of handshakes H in the group is given by $H=\dfrac{n(n-1)}{2}$ where n is the number of people.

Assume that the group A consists of a number of people, and that the group B consists of twice the number of people of group A. The people in group A greet each other and the people in group B greet each other.

Write down an expression describing the difference between the number of handshakes in the two groups. Then simplify the expression as far as possible. (0/2/¤)

8. The derivative of the function f is $f'(x)=(x-a)(x-b)^2$ where a and b are constants satisfying $0<a<b$

Investigate for which x the function f is increasing. (0/2/¤)

Part II

This part consists of 10 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

9. For children aged between 5 and 13, there is a model describing the relationship between the child's weight y kg and height x m. According to this model
$y = 2.4 \cdot 10^{0.8x}$

Using this model, answer the following questions:

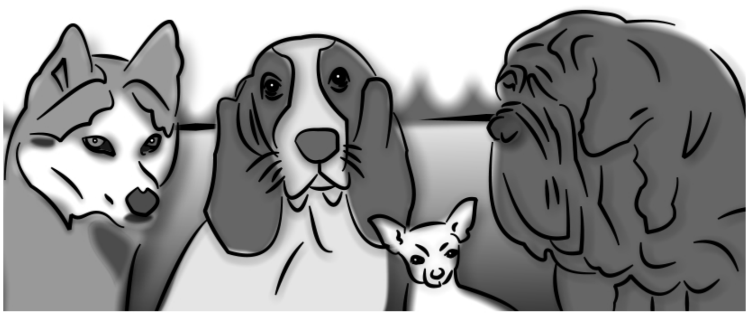
a) How much does a child weigh, who is 1.2 m tall? (1/0)

b) How tall is a child whose weight is 32 kg? (1/0)

10. Since 2001, all dogs in Sweden must be registered by law. Since then, the number of registered dogs has increased each year. The table below shows the number of registered dogs at the end of the years 2001-2006.

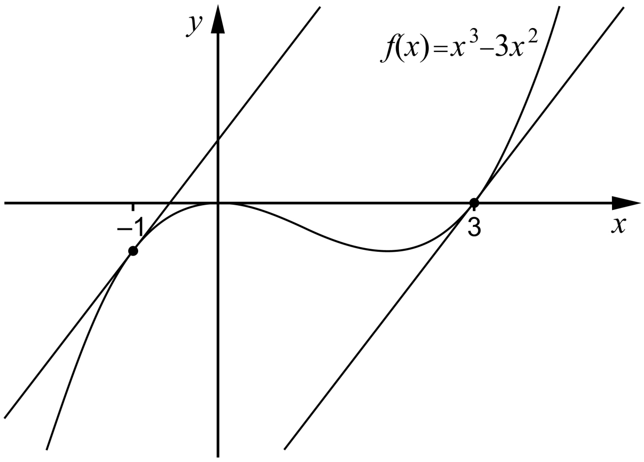
Year	Number of dogs
2001	159 108
2002	221 560
2003	295 521
2004	338 203
2005	387 884
2006	452 676

Source: Swedish Board of Agriculture



Calculate the average increase in the number of registered dogs per year between 2001 and 2006. (2/0)

11. The figure shows the graph of the function f where $f(x) = x^3 - 3x^2$
At the points with x-coordinates -1 and 3 respectively, the tangents to the curve have been drawn.



In the figure the tangents appear parallel. Investigate whether they are parallel. (2/0)

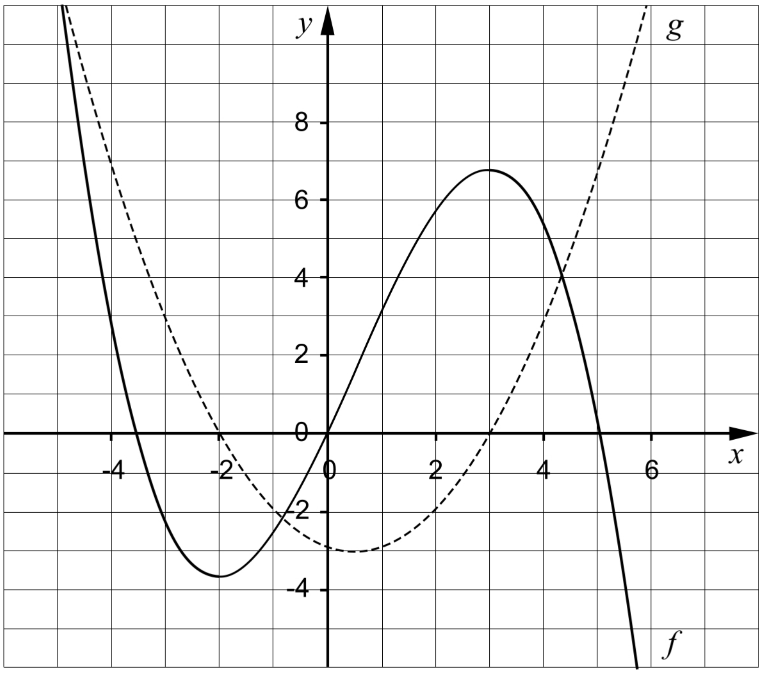
12. There are several functions for which it is true that $f(0) = 20$ and $f'(0) = 20$
Determine one such function. (1/1)

13. In the year 2001, a terrace house in Umeå was bought for 1.23 million SEK. Seven years later, the house was sold for 2.49 million SEK. Assume that the increase in price between these years was exponential.

Calculate the yearly percentage increase in price. (0/2)

14. For the function f it holds that $f(x) = x^4 - 420x^2 + 16x$
How many points on the graph of this function have a tangent with gradient 16? (0/2)

15. The figure shows the main characteristics of the graphs of the two functions *f* and *g*.



Sven claims that the function g is the derivative of the function f.
Investigate if his claim is correct. (0/2)

16. Let g and f be two functions. The graph of the function g is tangent to the graph of the function f at the point where $x = a$

Which **two** of the alternatives A - F below must then always be satisfied?

A $f'(a) = g(a)$	B $f'(a) = 0$	C $f'(a) = g'(a)$
D $f(a) = g'(a)$	E $f(a) = g(a)$	F $g'(a) = 0$

Only answer is required (0/1)

17. Moa and Gustav each investigate a different cubic function $y = f(x)$. Each cubic function has two turning points, in $x = 2$ and $x = 6$
Their teacher asks them to determine the largest value of their functions in the interval $0 \leq x \leq 3$. Moa claims that the largest value of her function is $f(2)$ and Gustav claims that his function's largest value is $f(0)$. The teacher tells them that they are both right.

Investigate how they can both be right. (0/2/¤)

When assessing your work with the following problem, the teacher will take into consideration

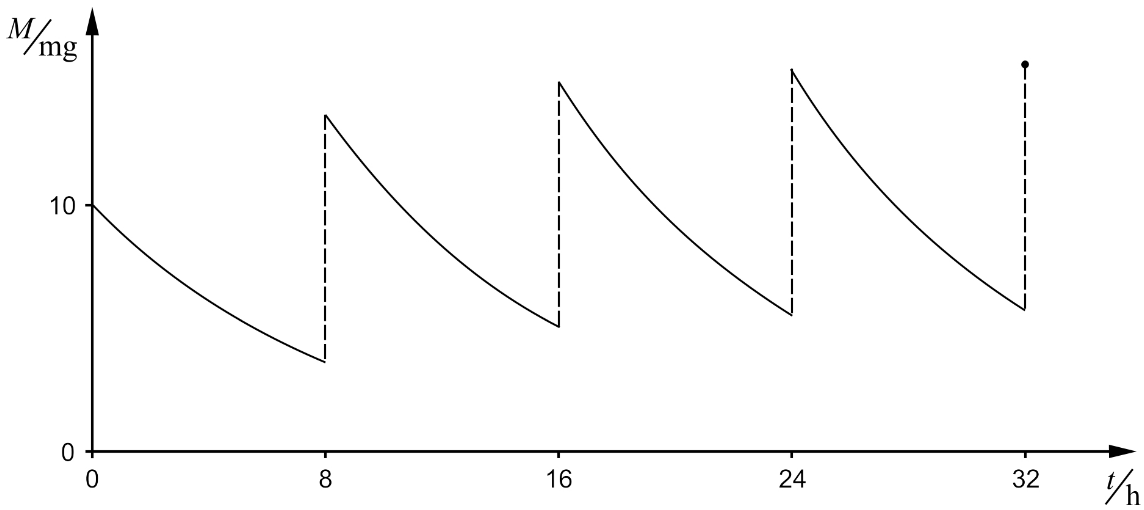
- How general your solution is
- How well you justify your conclusions
- How well you carry out your calculations
- How well you present your work
- How well you express yourself mathematically

18. Drugs injected directly into the bloodstream start to take effect immediately, but it may take one or two days before full effect is achieved. Patients are therefore injected with equal doses of the drug at equal time intervals during a period of time. For a certain drug, the amount y mg in the bloodstream, t hours after the first injection of 10 mg, is described by:

$$y = 10e^{\frac{-t}{8}}$$

- How many mg of the drug are still in the bloodstream 5 hours after the first injection?

- After 8 hours, the patient receives a second dose of the drug. In total, how many mg of the drug are there in the bloodstream immediately after the second injection?



The graph above shows a simple model for how the total amount M mg of the drug in the bloodstream of the patient varies with time, t, measured in hours, until the point in time when the patient has just received the fifth dose.

- Write down an expression describing the total amount of the drug in the blood-stream immediately after the fifth injection. Calculate this amount.

Assume that a patient continues to receive drug doses in accordance with the model given above over a longer period of time. The total amount of the drug in the bloodstream will increase, but cannot increase indefinitely.

- Determine the upper limit for the total amount of the drug in the bloodstream. (2/5/¤)