NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2010

Directions

Test time
240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.**

Resources
Part I: ”Formulas for the National Test in Mathematics Course C.”
Please note that calculators are not allowed in this part.

Part II: Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”

Test material
The test material should be handed in together with your solutions.

Write your name, the name of your education programme/adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test
The test consists of a total of 17 problems. **Part I** consists of 9 problems and **Part II** consists of 8 problems.

For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 9 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels
The maximum score is 44 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

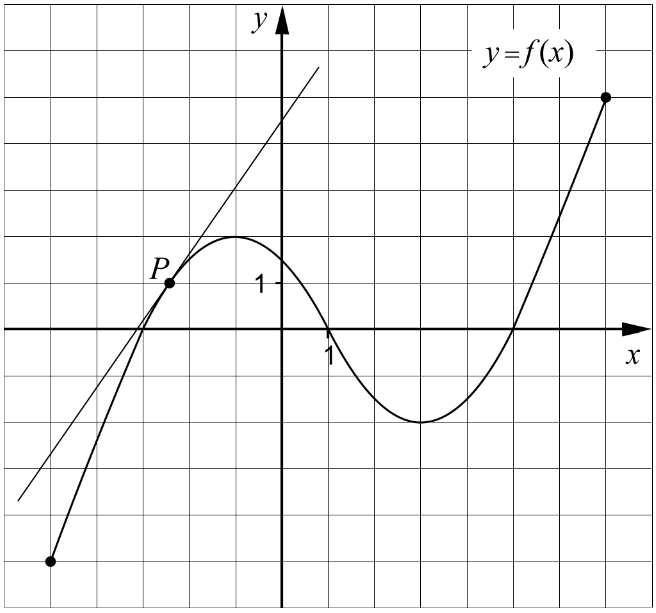
Lower limit for the mark on the test

Pass:	12 points.
Pass with distinction:	25 points of which at least 7 ”Pass with distinction” points.
Pass with special distinction:	25 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. The figure shows the graph of a function *f*. Solve the following problems by using the graph.



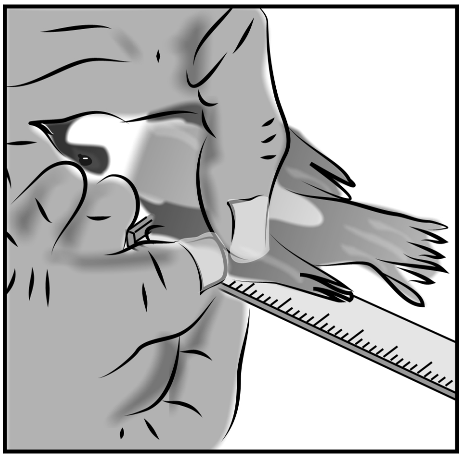
a) The point *P* lies on the curve. A straight line with the same gradient that the curve has in *P* passes through *P*. What is the name of such a line?
Only answer is required (1/0)

The function f is defined within the interval $-5 \le x \le 7$

b) For what x is it true that $f'(x) = 0$? *Only answer is required* (1/0)

c) What is the greatest value of the function? *Only answer is required* (1/0)

2.



When a bird is ringed it is often weighed and its wingspan measured. A biologist has ringed a number of Penduline Tits by the Lake Tåkern in Östergötland. The data collected by the biologist lead to the following model between the weight and the wingspan of the birds:

$y = -6x^2 + 360x + 5000$ where $10 \le x \le 40$

y is the weight in milligrams and x the wingspan in millimetres.

Use the derivative to calculate the maximum weight of a Penduline Tit according to this model. (4/0)

3. Calculate

a) $\lg 10000 + \lg 100$ (1/0)

b) $\ln \mathrm{e}^4 + \mathrm{e}^{\ln 2}$ (1/0)

4. The function f has a saddle point at $(2, 3)$

Calculate $f'(2)$ *Only answer is required* (1/0)

5. Simplify as far as possible

a) $(1-x)(1+x+x^2)$ (1/0)

b) $\dfrac{(a-b)(a^2+2ab+b^2)}{(a+b)^3}$ (0/1)

6. Solve the following equations

a) $7^x - 5 = 0$ (1/0)

b) $10x^3 - 1000x = 0$ (2/0)

c) $\lg x + \lg 4 = 2$ (0/2)

7. Lovisa wants to calculate

$$12345678 \cdot 12345678 \cdot 12345678 - 12345678 \cdot 12345677 \cdot 12345679$$

She uses her calculator and gets the answer 0. Lovisa is convinced that the calculator has given her the wrong answer and believes that it is possible to use algebra instead to find the correct answer. Help Lovisa find the correct answer by using algebra. (0/2)

8. Use the definition of the derivative to determine the derivative of $f(x) = \frac{A}{x}$ where A is a constant. (0/2/¤)

When assessing your work with this problem, the teacher will take into consideration:
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

9. In this problem you are going to investigate what value of x gives the greatest vertical distance between two curves in the interval $0 < x < 1$

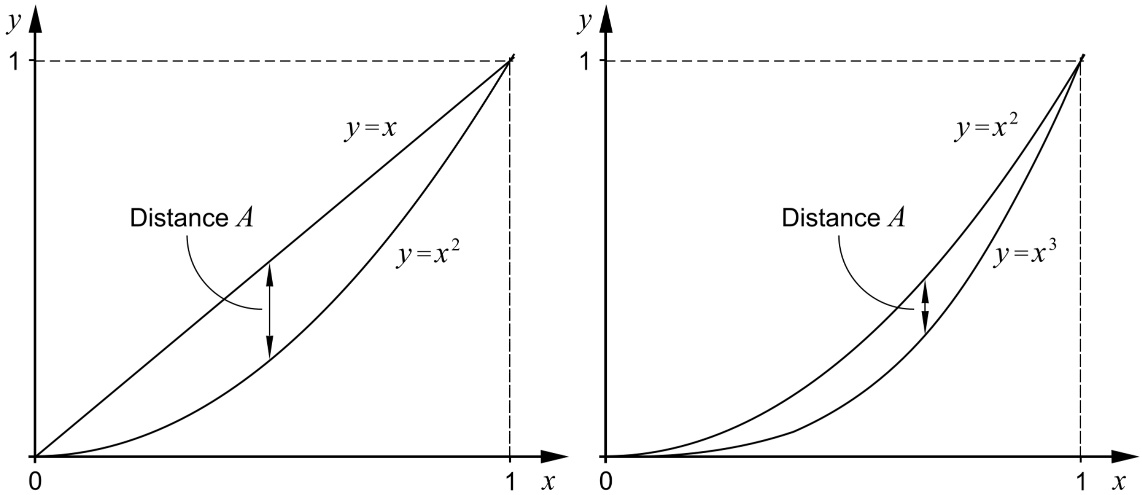


Figure 1 Figure 2

Figure 1 shows the curves $y = x$ and $y = x^2$. The distance between the curves is given by $A(x) = x - x^2$

- What value of x gives the greatest distance in figure 1?

Figure 2 shows the curves $y = x^2$ and $y = x^3$

- What value of x gives the greatest distance in figure 2?

In the general case, the two curves are given by $y = x^n$ and $y = x^{n+1}$ where $n = 1, 2, 3...$
Even in the general case there is a value of x in the interval $0 < x < 1$ where the distance between the curves is as great as possible. We call this x-value x_{max}

- Investigate what happens to the value of x_{max} as n grows larger and larger. (3/3/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. Calculate $f'(0.5)$ when $f(x) = 5e^{2x}$
Round to the nearest integer. (1/0)

11. The eye's ability to adjust the size of the pupil decreases with age. This means that there is an increased sensitivity to glare as you get older. The ability to tolerate glare as a function of age can be described by the relation

$T(x) = 283 \cdot e^{-0.0578x}$ where $x \geq 18$

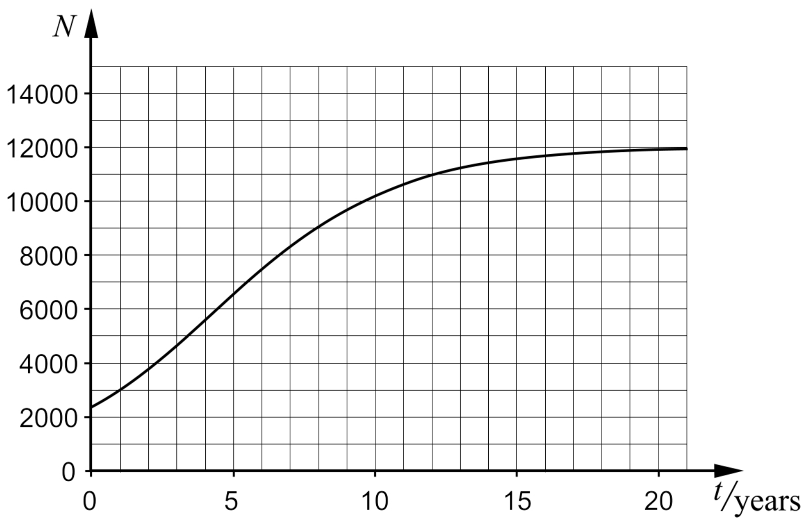
where $T(x)$ is the ability to tolerate glare * and x is the age in years.

a) Calculate the ability to tolerate glare for a person who is 18 years old. (1/0)

b) At what age is the ability to tolerate glare half the size of what it is at age 18? (2/0)

(*You can ignore the unit of the ability to tolerate glare.)

12. A scientist studied the number of seals in one population. The figure below shows the number of seals N as a function of time t years.



a) Use the graph to determine an approximate value of $N'(10)$ (1/0)

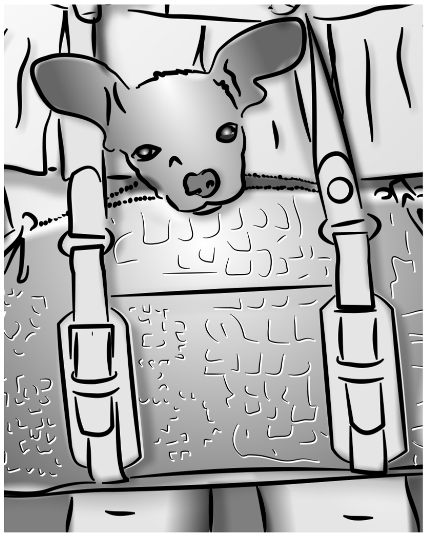
b) Give your interpretation of what $N'(10)$ means in this context. (0/1)

13. The Chihuahua is the smallest breed of dog in the world. During the 2000s the breed has become increasingly popular.

In 2000 there were 306 dogs of the breed registered and 2220 dogs of the breed were registered in Sweden, in 2008.

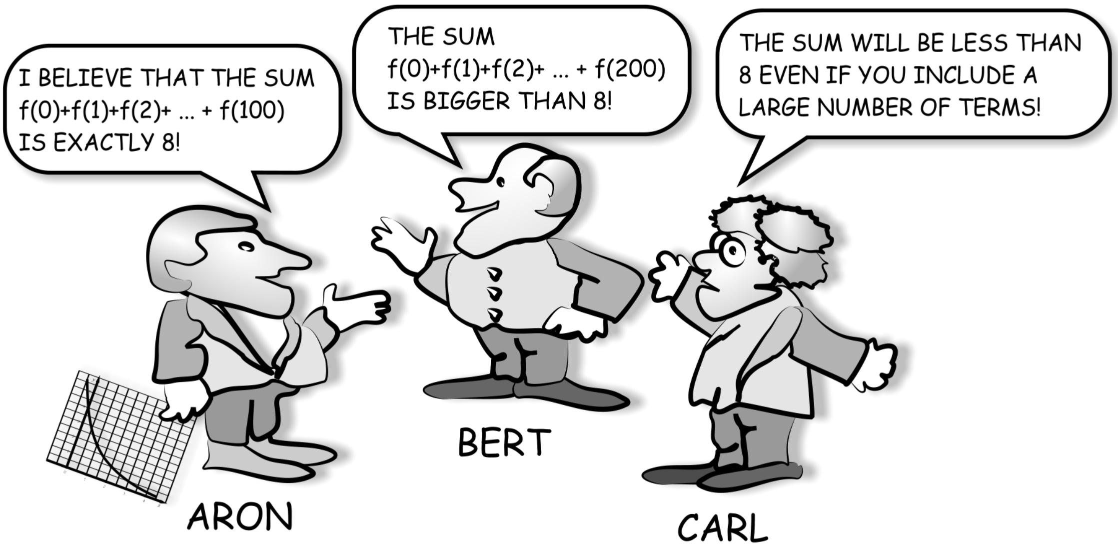
Assume that the increase has been exponential. By what yearly percentage has the number of registered dogs of the breed Chihuahua increased?

(Source: Svenska Kennelklubben)



(0/2)

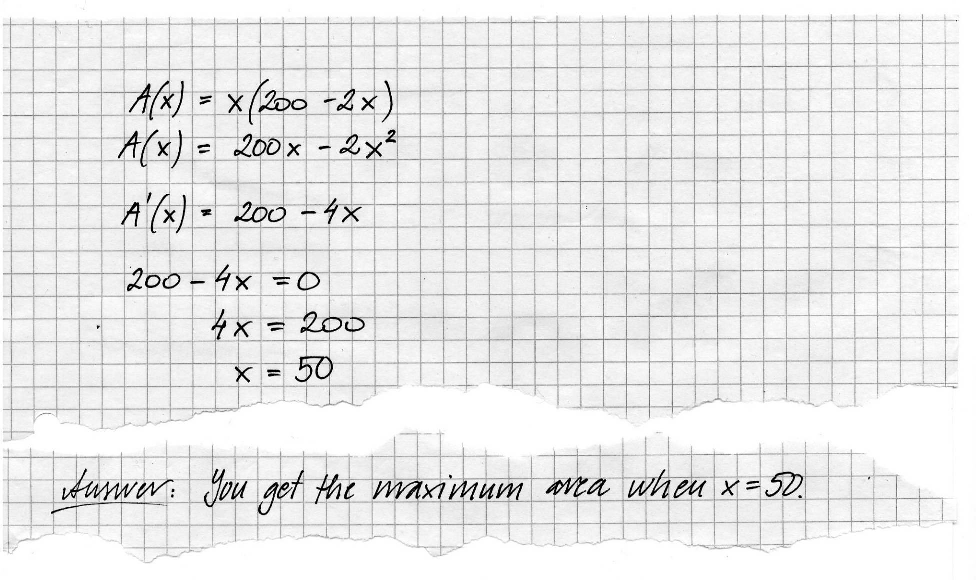
14. Aron, Bert and Carl look at the graph of the function $f(x) = 4 \cdot 0.5^x$ and discuss the sum they get if they add $f(0) + f(1) + f(2) + f(3) + ...$
They have different opinions on the value of the sum:



Decide who is/are right. Justify your answer. (0/2/¤)

15. It is true for the function f that $f(x) = x^2$. Give another function g for which it is true that $g'(x) < f'(x)$ for all $x > 0$ *Only answer is required* (0/1)

16. Frida has solved a problem in her maths book. One part of her solution looks like this:



$A(x) = x(200 - 2x)$

$A(x) = 200x - 2x^2$

$A'(x) = 200 - 4x$

$200 - 4x = 0$

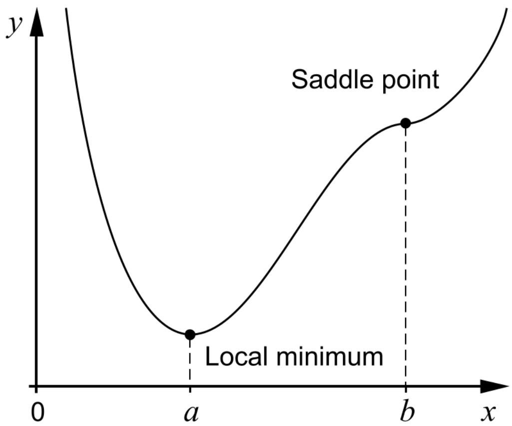
$4x = 200$

$x = 50$

Answer: You get the maximum area when $x = 50$.

Write down your own text that suits Frida's solution. The text should be about a real situation where an area A is investigated. It should be clear from your text what x and 200 represent. (0/2/¤)

17. The figure below shows the fundamental features of the graph to a quartic function f where $y = f(x)$. The graph has a local minimum where $x = a$ and a saddle point where $x = b$



a) Draw a sketch that shows the graph of the derivative of the function. (0/2)

The derivative of the function can be written in the form
$f'(x) = k(x-a)(x-b)^2$ where k is a positive constant.

b) Study the expression of the derivative, $f'(x) = k(x-a)(x-b)^2$ and the graph of the function f when $x > a$. Explain the relation between the appearance of the graph and the fact that the factor $(x-b)$ is squared in the expression for the derivative. (0/2/¤)