NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2011

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 8 problems and **Part II** consists of 10 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 8 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points. Pass with distinction: 25 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: 25 points of which at least 15 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Determine $f'(x)$

a) $f(x) = 2x^4 - 8$ *Only answer is required* (1/0)

b) $f(x) = 5e^{3x}$ *Only answer is required* (1/0)

c) $f(x) = x^{n+1}$ *Only answer is required* (0/1)

2. The following expression is given: $3x^4 - 9x - 2x^4 + x$

a) Simplify the expression as far as possible. *Only answer is required* (1/0)

b) Factorise the simplified expression. *Only answer is required* (1/0)

c) Solve the equation $3x^4 - 9x - 2x^4 + x = 0$ *Only answer is required* (1/0)

3. a) Solve the equation and give an exact answer.

$x^5 = 6$ *Only answer is required* (1/0)

b) Which of the following intervals contains the solution to the equation $x^5 = 6$?

Only answer is required (1/0)

A. $-2 \le x < -1$

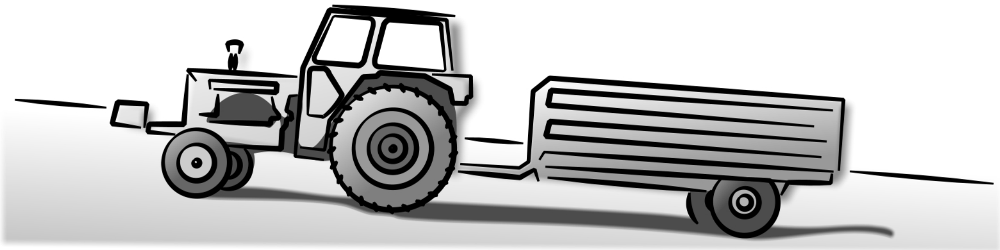
B. $-1 \le x < 0$

C. $0 \le x < 1$

D. $1 \le x < 2$

E. $2 \le x < 3$

4. The fuel consumption of a tractor depends, among other things, on the velocity of the tractor, the inclination of the road and the weight of the tractor's load.



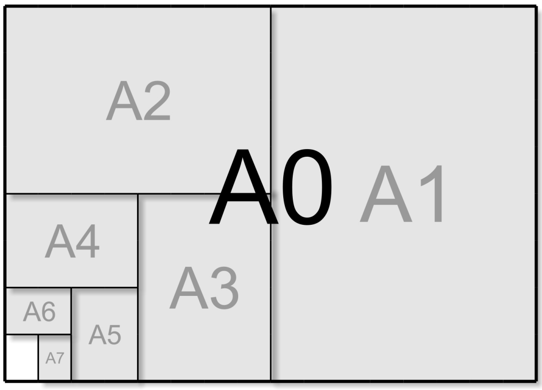
At a certain inclination and weight, the fuel consumption of the tractor is B (litres/km) and it depends on the velocity of the tractor v (km/h). According to a simple model, the fuel consumption of the tractor can in this case be described by the relationship:

$$B(v) = 0.001v^2 - 0.04v + 0.9$$

Use the derivative and show that the fuel consumption is at its lowest at the velocity 20 km/h. (3/0)

5. Calculate the exact value of the expression $\dfrac{x^2-4}{x-2}$ when $x = 2.0123456789$ (0/1)

6. The A format is the most common paper size standard in Europe. The base format A0 has an area of 1 m^2. The remaining formats A1-A7 are all received by a repeated halving of the A0 format, see figure.

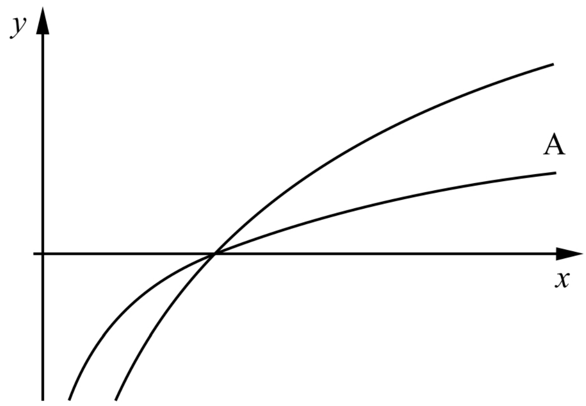


a) What is the area (in m^2) of a paper of A3 format?
Only answer is required (1/0)

b) Interpret what is being calculated with the expression $\dfrac{0.5\cdot(0.5^6-1)}{0.5-1}$ in this context, see figure above. (0/2)

c) What value does the expression $\dfrac{1\cdot(0.5^n-1)}{0.5-1}$ approach as n gets larger and larger? Justify your answer. (0/2/¤)

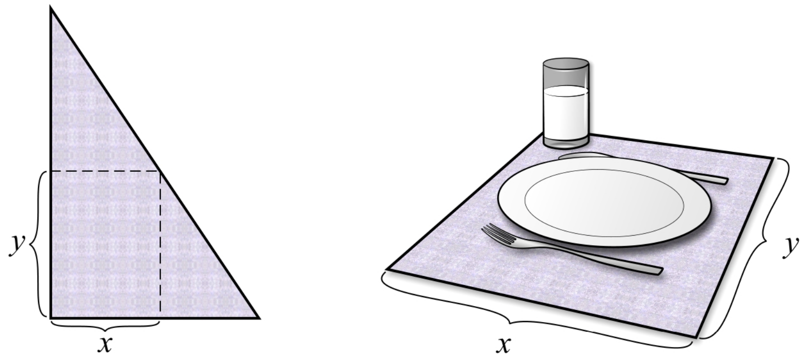
7. The figure shows the graphs of $y = \ln x$ and $y = \lg x$
Which of these functions has been marked A? Justify your choice. (0/1/¤)



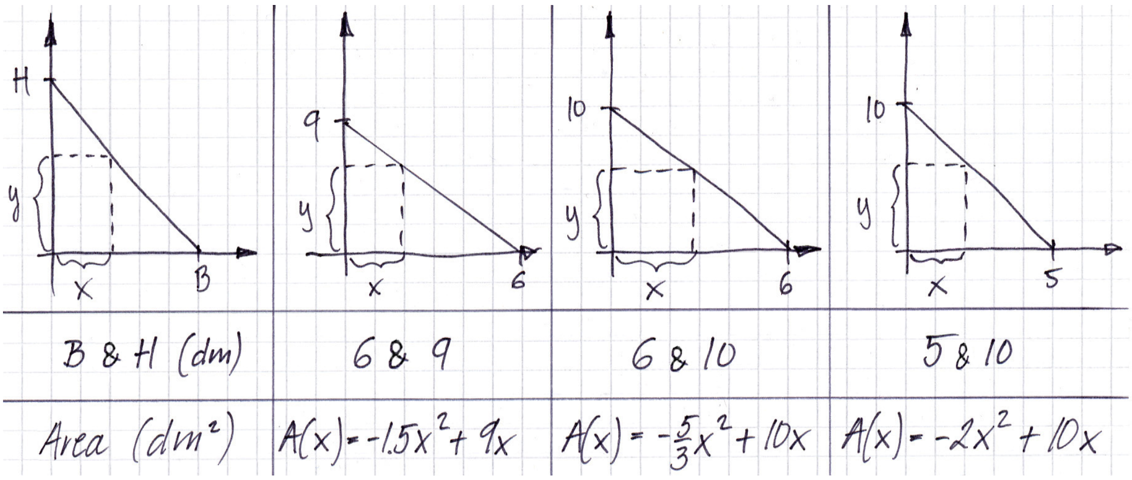
When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

8. Kim has started a business within ”Junior Achievement” (JA). He will receive left-over pieces of fabric from a factory and he is going to make table mats. He does not know the exact measurements of the pieces, but the factory can guarantee that the pieces have the shape of a right-angled triangle. Out of these pieces, Kim will cut rectangular table mats with width *x* and length *y*, see figure.



Kim wants to investigate how to cut so that the area of the table mats is as large as possible. He starts by investigating three different kinds of pieces of fabric with possible measurements. Kim denotes the height of the triangular shaped pieces *H* and the base *B* and comes to the following conclusion:



- Use the derivative and calculate the width *x* and the length *y* of the table mats that give the largest area *A* for each of the three cases above.

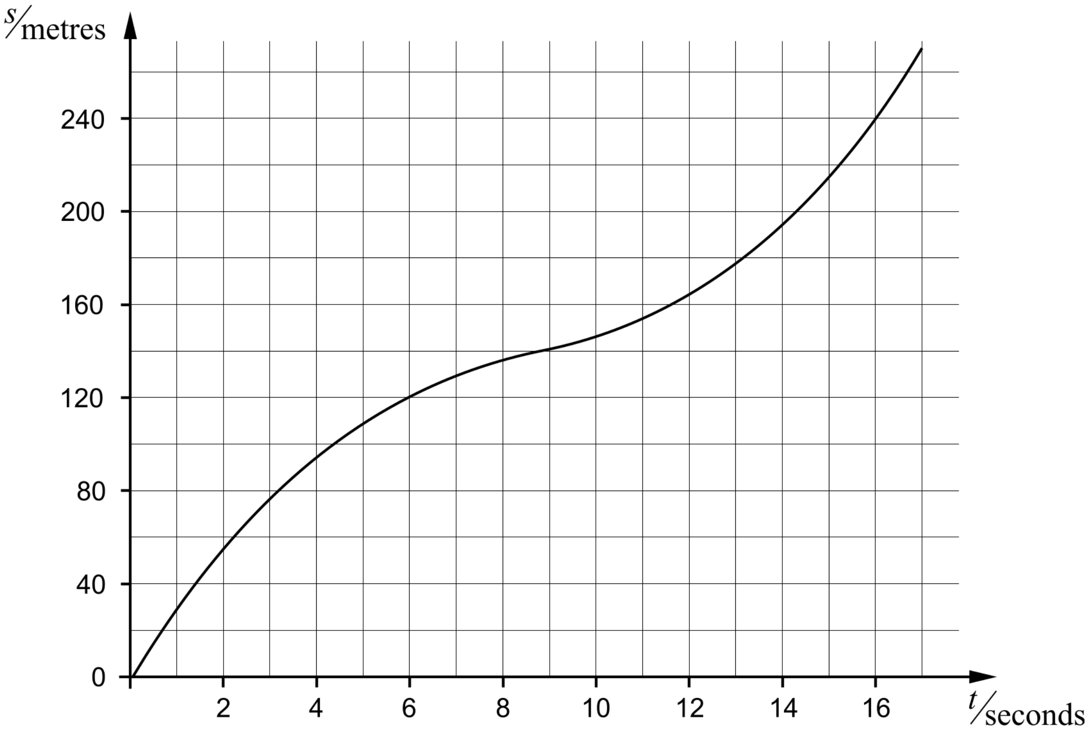
- Study the measurements of the width *x* and length *y* that you have calculated in the three cases above. Formulate a conclusion as to where Kim should cut at the base *B* and height *H* to get the largest possible area of the table mats.

- Assume that the pieces of fabric still have the shape of a right-angled triangle with base *B* and height *H*. Investigate whether your conclusion is true regardless of the measurements of the pieces of fabric. (3/3/¤)

Part II

This part consists of 10 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

9. The graph in the diagram below shows how a car travels through a curve. After time t seconds the car has travelled the distance $s(t)$ metres.



Calculate the average velocity of the car in the time interval $6 \leq t \leq 16$ seconds. (2/0)

10. Petra considers taking an SMS loan of SEK 2000. The total loan cost (interest, fees etc) for the first month is 25 % of the loan sum. Petra assumes that her debt increases with the same percentage every month and writes down the relationship:

$S = 2000 \cdot 1.25^x$ where S is the debt in SEK after x months.

Calculate how many months it would take before Petra's debt is more than SEK 1 million. (2/0)

11. Which of the alternatives A-E is a geometric progression?

Only answer is required (1/0)

A. 3, 4, 6, 9, 13

B. 2, 4, 6, 8, 10

C. 1, 1, 2, 3, 5

D. 4, 9, 16, 25, 36

E. 2, 4, 8, 16, 32

12. It holds for the function f that $f(x) = 0.01x^3 - 0.024x^2 + 2$

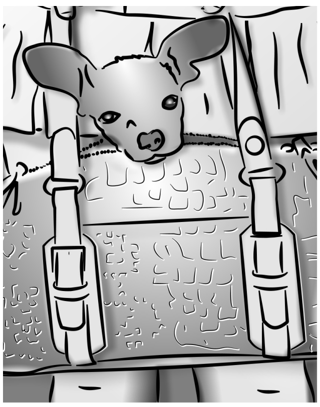
Calculate the global maximum and minimum of the function within the interval $-0.5 \leq x \leq 2.5$ (3/0)

13. a) Give an example of a function g for which it holds that $g'(3) = 2$

Only answer is required (0/1)

b) Give an example of a function h for which it holds that $h'(x) = h(x)$ for all x and $h(x) \neq 0$ *Only answer is required* (0/1)

14. The Chihuahua is the smallest breed of dog in the world. Over the last years the breed has become increasingly popular.

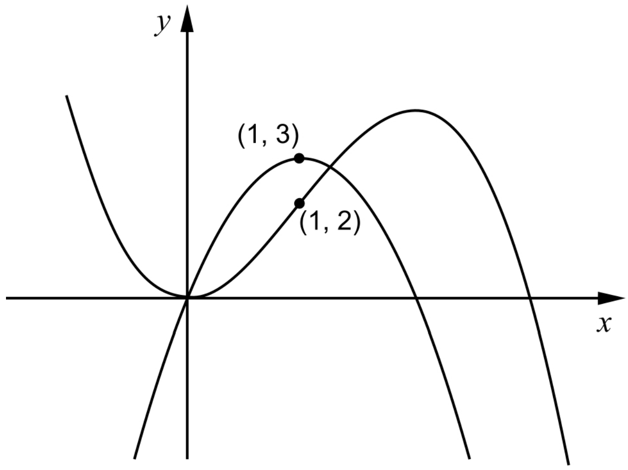


In 2000 there were 306 dogs of the breed registered in Sweden and in 2010 there were 2372 dogs of the breed registered in Sweden.

Assume that the increase has been exponential.

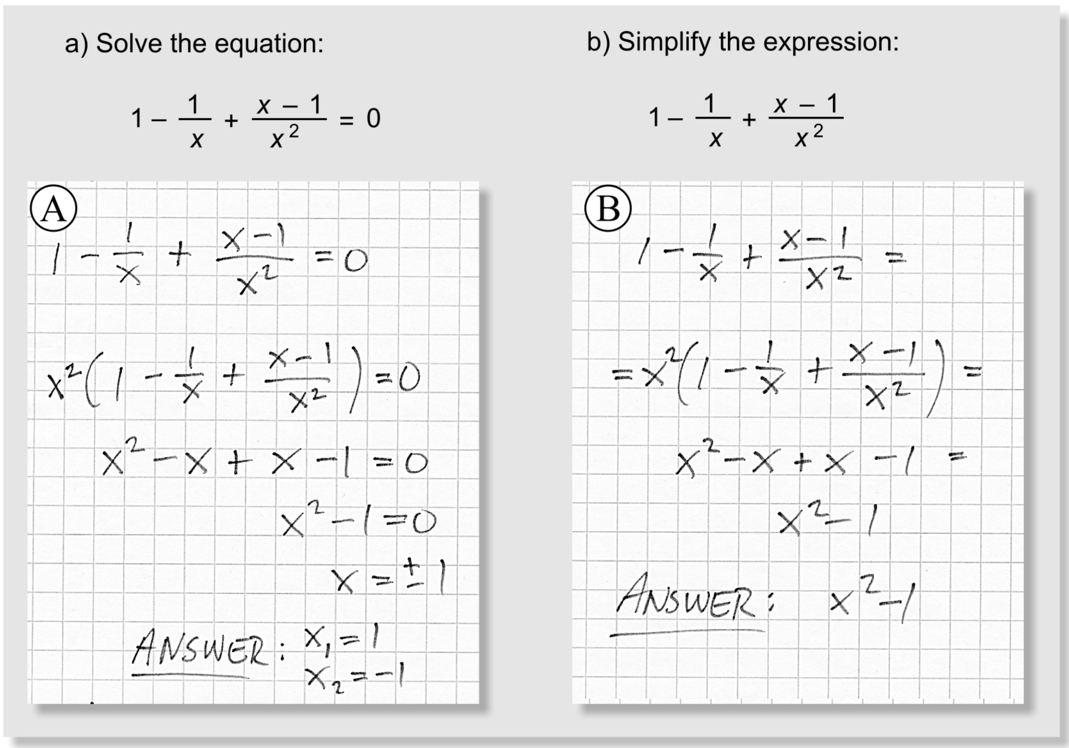
By what yearly percentage has the number of registered dogs of the Chihuahua breed increased? (0/2)

15. The graph of the function f and its derivative f' are drawn in the figure below.



Determine the equation of the tangent to the curve $y = f(x)$ at the point $(1, 2)$ (0/2)

16. Ove has solved two problems in his maths book, see below.



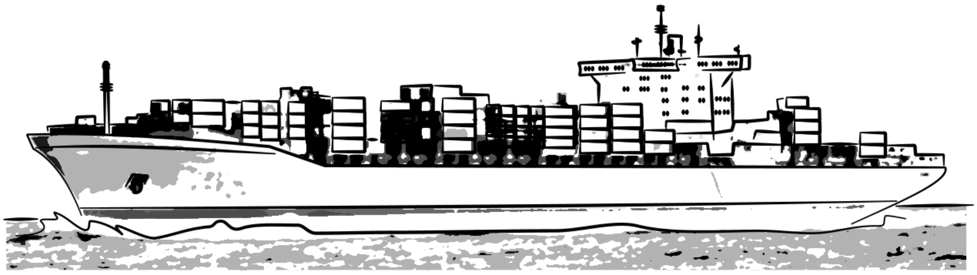
The solution to one of the problems is wrong. Which of the solutions A or B is incorrect? What mistake does Ove make and why is it not correct to do so? (0/1/¤)

17. The point P lies on the graph of a function f. The slope of the graph at point P is:

$$\lim_{h \to 0} \frac{((2+h)^4 + 3(2+h)) - (2^4 + 6)}{h} = 35$$

What are the coordinates of P? (0/2)

18. A cargo liner is going to make a 24000 km long journey from Gothenburg to Singapore.



a) Assume that the cargo liner maintains a constant velocity throughout the trip and that the trip takes t days. Write the time t in days as a function of the velocity v km/h.

Only answer is required (0/1)

Let us assume that the main costs for driving the ship are fuel costs and the crew's salaries. According to a simplified model, the cost K SEK/days be described as

$$K = 0.15v^3 + 25000 \text{ (SEK/days)}$$

b) At what velocity is *the total cost for the whole trip* the lowest? (0/3/¤)