NATIONAL TEST IN MATHEMATICS COURSE C
AUTUMN 2012

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 90 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 10 problems and **Part II** consists of 7 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points.
Pass with distinction:	25 points of which at least 7 ”Pass with distinction” points.
Pass with special distinction:	25 points of which at least 15 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = 2$ *Only answer is required* (1/0)

b) $g(x) = \dfrac{x^3}{4}$ *Only answer is required* (1/0)

2. Solve the equations. Give exact answers.

a) $x^9 = 20$ *Only answer is required* (1/0)

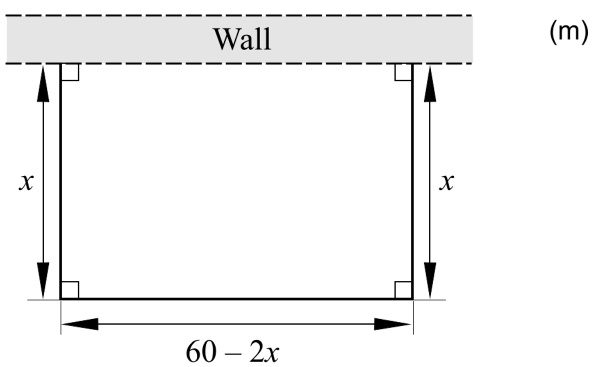
b) $e^x = 9$ *Only answer is required* (1/0)

c) $\lg x = 9$ *Only answer is required* (1/0)

3. For what value of x is the expression $\dfrac{3x-9}{x-5}$ not defined?

Only answer is required (1/0)

4. Leila has bought 60 metres of fencing to build a sheepfold, where a wall constitutes one of the sides. She investigates how long the sides of the sheepfold should be in order to get the largest possible area. Leila draws a picture and denotes the width of the sheepfold by x m. The length is then $(60-2x)$ m. See figure.



a) Write an expression for the area of the sheepfold.

Only answer is required (1/0)

b) Use differentiation to determine the maximum area of the sheepfold. (3/0)

5. For the function f it holds that $f(x) = x^2 + 8x$
For what value of x does the graph of f have a gradient of 12? (2/0)

6. Simplify the expression $\frac{(x-3)(x+2)}{2x-6}$ as far as possible. (1/0)

7. Simplify

a) $\ln e^{5x}$ *Only answer is required* (1/0)

b) $\lg(1000x) - 3$ (0/1)

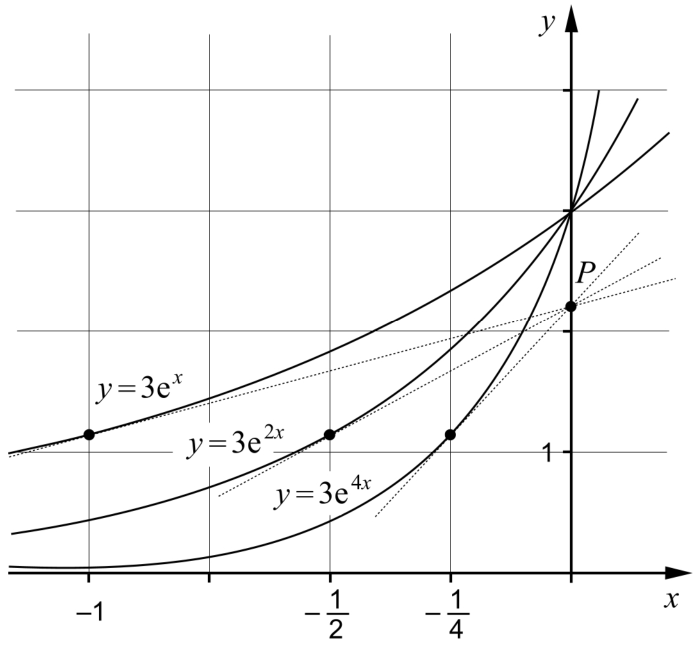
8. a is a constant in the equation $x^3 + ax = 0$. The number of real solutions to the equation differs depending on the value of a.

a) Determine a value of a so that there is *only one* real solution to the equation $x^3 + ax = 0$ (0/1)

b) Investigate for what values of a there are *three different* real solutions to the equation $x^3 + ax = 0$ (0/1/¤)

9. For the function f it holds that $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 7 - (x^2 + 7)}{h}$
Find $f'(3)$ (0/2)

10. For the function f it holds that $f(x) = 3e^{ax}$ where $a \neq 0$
In the figure below you can see the graphs of the function when $a = 1,\ 2$ and 4
For each graph, the tangent at $x = -\dfrac{1}{a}$ is drawn.



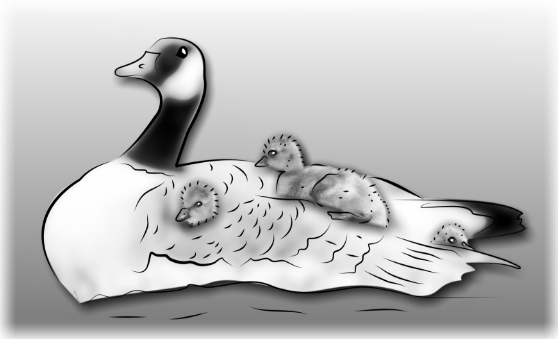
When $a = 1, 2$ and 4 the tangents and the y-axis intersect at the same point P.
Investigate if this holds for all values of a where $a \neq 0$ (0/3/¤)

Part II

This part consists of 7 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

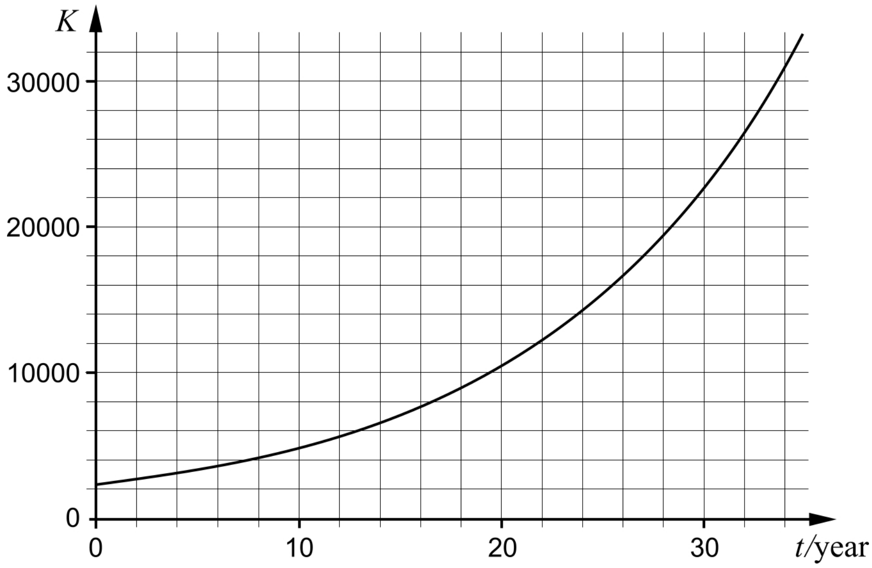
11. Calculate $f'(5)$ when $f(x) = 6e^{0.7x}$
Round your answer to the nearest integer. (1/0)

12.



The Canada Goose was introduced into Sweden in the 1930s. The population has increased ever since. Every year, at the same time, there is a survey of the number of Canada Geese. The growth of the population can be described by an exponential model.

The diagram below shows the number of Canada Geese K as a function of time t years, where $t = 0$ corresponds to the year 1977.



a) Use the graph and determine an approximate value of $K'(30)$. (1/0)

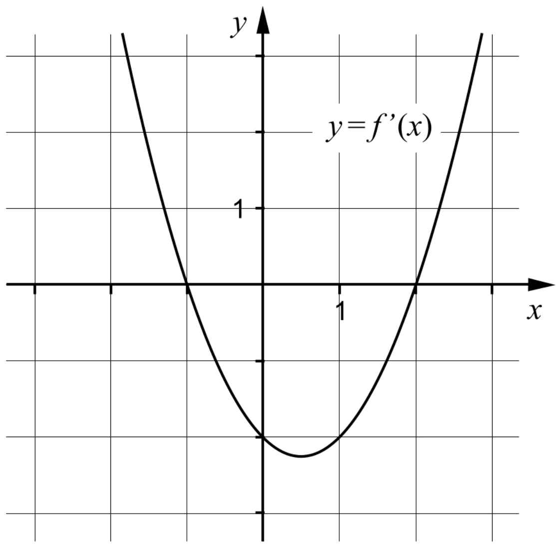
b) Interpret what $K'(20) = 800$ means to the number of Canada Geese in this context. (0/1)

13. Kim is going to solve the following problem from a maths book:

> The function $g(x) = x^3 + 1.5x^2 - 6x$ is defined on the interval $-3 \leqslant x \leqslant 3$
> In the interval the function has two extremums, (–2; 10) and (1; –3.5)
> Calculate the largest value of the function.

Kim claims that the largest value of the function is 10.
Investigate whether Kim is right. (2/0)

14. The figure below shows the graph of the derivative f' of a cubic function f.



a) For what values of x is f decreasing? *Only answer is required* (0/1)

b) For what value of x does f have a minimum point? (0/2)

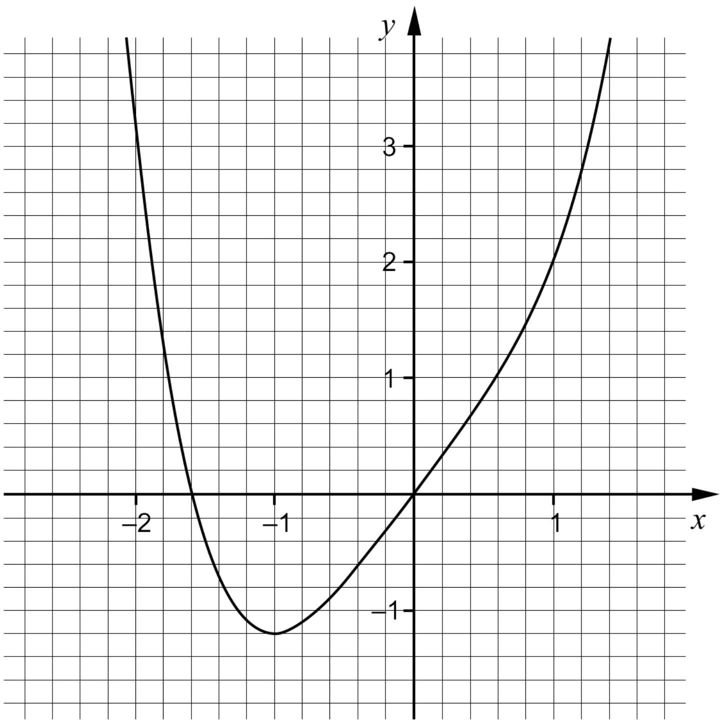
c) Find the equation of the tangent to the curve f at the point where $x = 0$
if $f(0) = 1$ (0/2)

15. In June 2001 there were 6.7 million mobile phone subscriptions in Sweden and in June 2011 the number had increased to 13.1 million. Assume that the yearly percentage increase has had the same size during the whole period of time.

In what year were there 9.0 million mobile phone subscriptions? (0/3)

16. Kajsa is going to solve the equation $0.4x^4 + 1.6x = 1$
She cannot solve the equation algebraically and therefore plans to solve it graphically.

Kajsa starts by drawing the graph of $y = 0.4x^4 + 1.6x$



a) With the help of this graph Kajsa can find two solutions to the equation $0.4x^4 + 1.6x = 1$

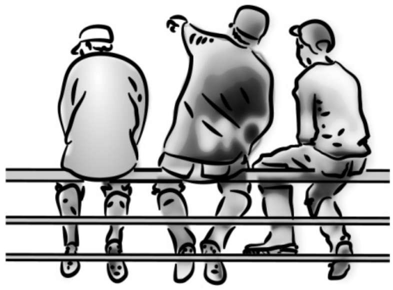
What are these solutions? *Only answer is required* (1/0)

b) Kajsa is unsure whether there are more real solutions to the equation. Show her how she can, without solving the equation, be sure that there are not more than two real solutions. (0/2/¤)

When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

17. Oskar, Martin and Johan, all born in 1994, are discussing regular deposits and which alternative gives the most interest on the money.



Each one of them is going to deposit a total of SEK 10000 into their own account with an interest rate of 2%.

Martin is going to deposit SEK 2000 every year.
Oskar is going to deposit SEK 1000 every year.
Johan is going to deposit SEK 2000 every second year.

All deposits are made at the end of the year. The table below shows when the three boys plan to make their deposits.

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Martin	2000	2000	2000	2000	2000					
Oskar	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Johan		2000		2000		2000		2000		2000

- How much money does Martin and Oscar have in their respective accounts immediately after their last deposits?

- Martin keeps his money in his account after his last deposit. He claims that he will have more money in his account than Oskar will have in his immediately after Oskar has made his last deposit.
  Investigate whether Martin is right.

- Calculate how much money Johan has in his account immediately after he has made his last deposit.

Now study Oskar's and Johan's strategies for saving based on the following premises:

The interest rate is 2% on both accounts.
Oskar deposits SEK b every year, starting in the year 2012. He makes n deposits.
Johan deposits twice the amount every second year, starting in the year 2013.
Totally, they deposit the same amount of money in their accounts and all deposits are made at the end of the year.

- Show that Oskar's strategy for saving always gives more money than Johan's strategy, regardless of the values of b and n. (4/3/¤)