

This material is confidential until the end of March 1998

Directions

Test period	3 - 18 December, 1996.
Test time	180 minutes without a break
Resources	Calculator and formula tables. The formula sheets are attached to the examination paper.
Test material	The test material should be handed in together with your solutions. Write your name, gymnasium program/adult education and date of birth on the papers you hand in.
Test	The test is made up of 13 exercises. Most of the problems are of long-answer type. With these problems, it is not enough with just a short answer, it requires • that you write down what you do • that you explain your train of thought • that you draw figures when needed • that you show how you use your calculator in numerical and graphical problem solving. For some exercises, (where it says "<i>Only an answer is required</i>") only the answer needs to be given. Try all of the problems. It can be relatively easy, even at the end of the test, to earn some points for a partial solution or presentation.
The grading levels	The teacher responsible will explain the grade levels which are required for "Passed" and "Passed with Distinction". It is possible to earn a maximum of 42 points in the test.

1. Let $y = x^3 + 5x$

a) Find y'

Only an answer is required (1p)

b) Calculate $y'(3)$

Only an answer is required (1p)

2. Sort the following into size order. Start with the smallest.

e^5

$10^{2.5}$

$\lg 10000$

Only an answer is required (1p)

3. The function f has the graph shown in the figure.

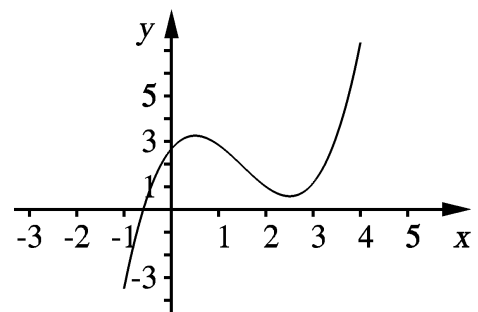
Sketch the graph on your paper and mark

a) a point on the graph where the derivative is zero

(1p)

b) a point on the graph where the derivative is negative

(1p)



4. In 1996, a family bought a small holiday home on a river in the north.

However the plot on which the house stands has to be rented. The rent contract states that the year's rent was set at 1420 SEK for the year 1991 and thereafter according to the consumer price index for January.

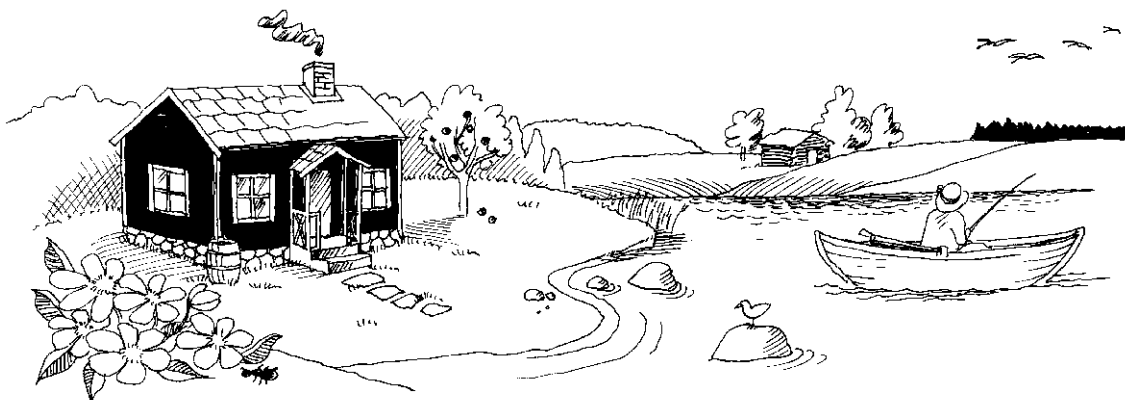
What is the rent for the year 1996?

Year	1991	1992	1993	1994	1995	1996
CPI (January)	218.9	230.2	241.0	245.1	251.3	255.6

(The information in the table is obtained from the Central Statistics Office.

CPI = Consumer Price Index)

(2p)



5. Marie bought a computer for 15 000 SEK. Three years later she sold it for 6 000 SEK.

How large a yearly percentage decrease does this correspond to? (3p)

6. Two sisters who are horse riding enthusiasts have long wished for their own paddock (a fenced area for horses) at home in the yard.

The paddock should be rectangular in shape and be situated so that two of its corners lie against the gravel roads and a longer side lies along the asphalt road as shown in the figure.

The sisters want their paddock to have as large a riding area as possible. Therefore they found the relationship between the area and the length of the paddock.

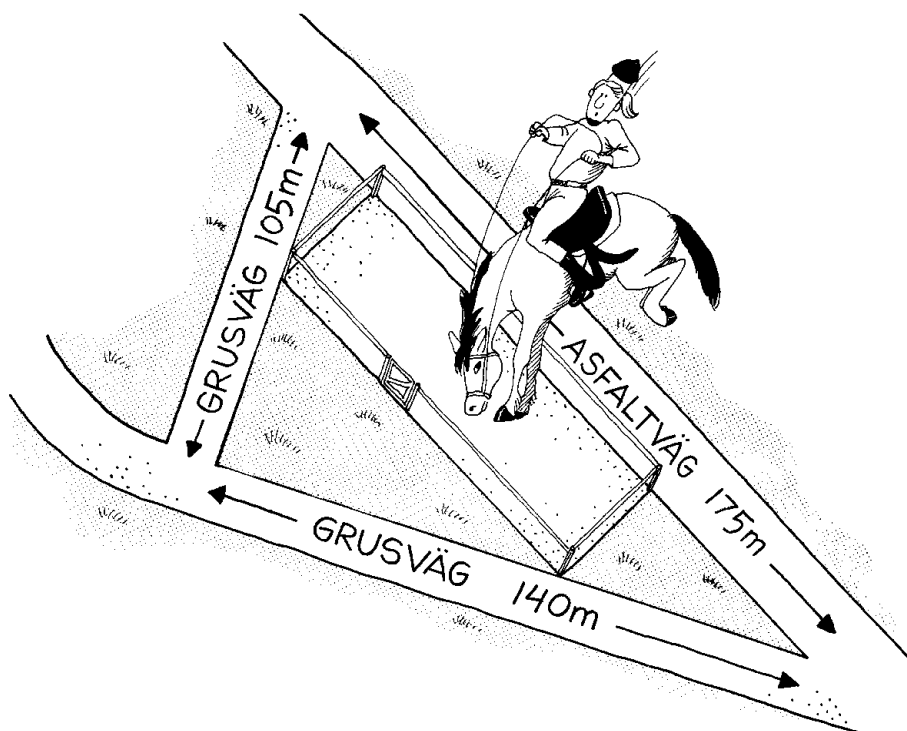
The relationship satisfies the following:

$$A(x) = 84x - 0.48x^2$$

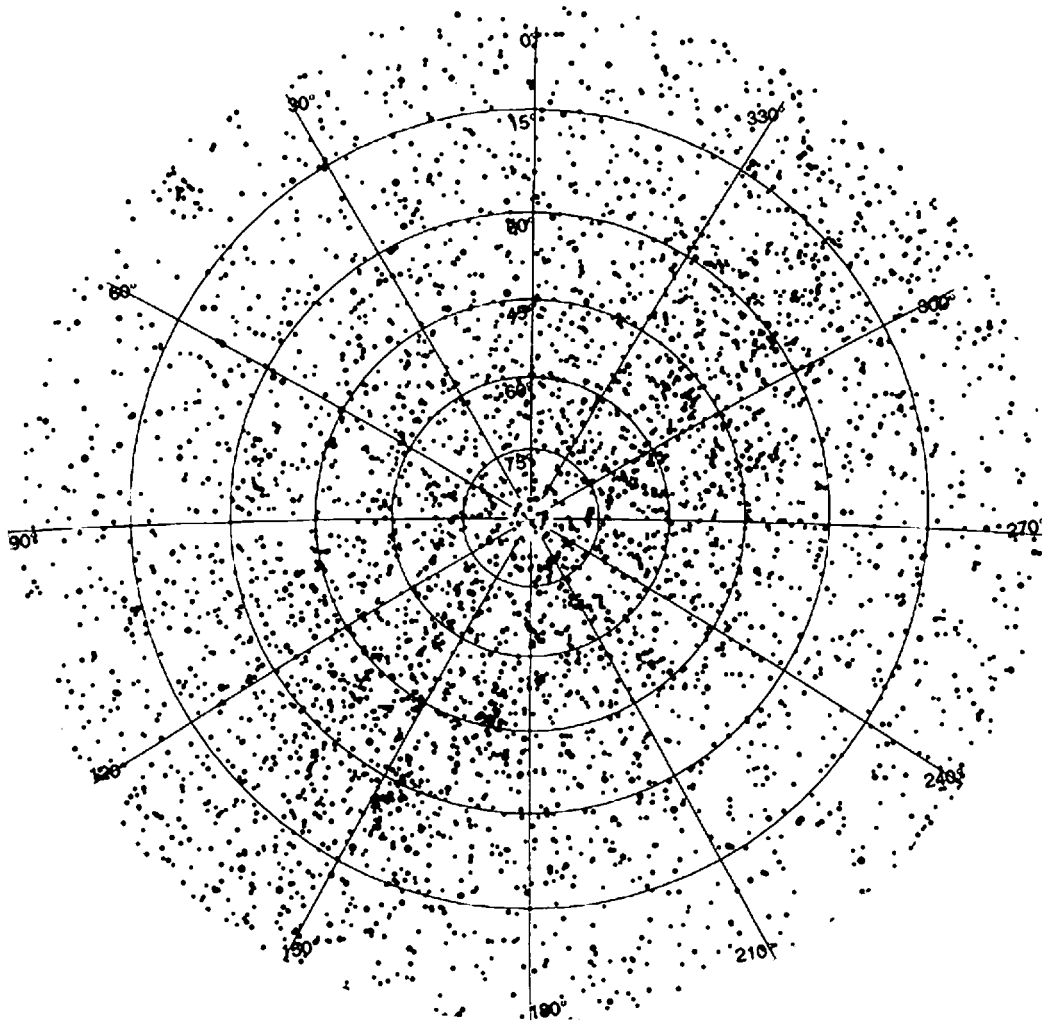
where $A(x)$ is the area of the paddock in square meters and x is the length in meters.

- a) Calculate the largest possible area of the paddock. (3p)
- b) When the sisters calculated the largest possible area of the paddock, they thought that the length was too great when compared with width. The sisters decided that the length of the paddock should be 70 meters.

What is the width of the paddock? (2p)



7. a) Give an example of a function which is increasing when $x = 2$. (1p)
- b) Calculate the slope of the graph to the function at $x = 2$. (1p)
8. The star map below shows the position of the stars as they would be seen from a place outside Uppsala on a clear night in December 1996. There are various different ways of estimating the number of stars on the star map without counting every star.
- a) Estimate the number of stars on the star map in some such manner. (3p)
- b) Give an advantage and a disadvantage of your method of estimating the number of stars compared with counting them all. (1p)



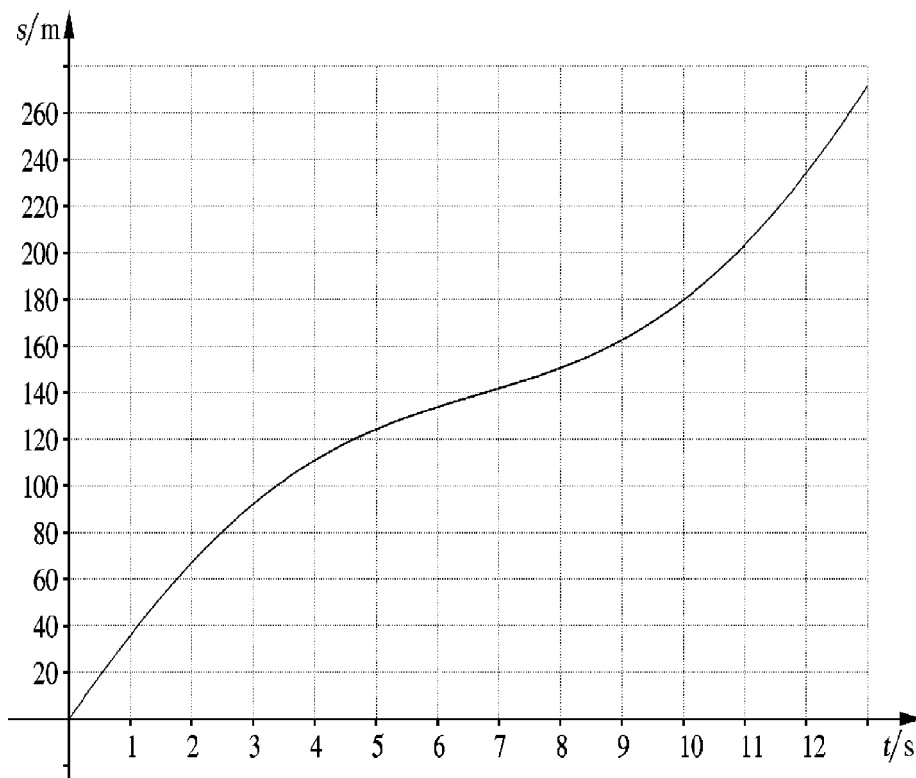
9. The graph in the diagram below shows a driving stretch for a rally car during part of a competition.

After t seconds the car has travelled $s(t)$ meters.

With the help of the diagram, calculate the following two expressions and explain what the values say about the movement of the car.

a) $\frac{s(12) - s(10)}{12 - 10}$ b) $s'(5)$ (4p)

- c) What do you think the part of the rally track which corresponds to the diagram looks like?
Motivate your answer. (1p)



The diagram shows how the distance s meters is dependent upon the time t seconds.

10. On an island in Skeppsvik archipelago outside Umeå, a plant called the sea aster grows. This plant is attacked by parasitic flies. The flies prevent the flowers from being able to produce seeds and to grow the next year.

In order to be able to estimate the risk that the sea aster will not be able to produce seeds, the number of flies on the islands where the plants grow must be known.

To make such an estimation, a biologist took a sample on one of the islands.

The result of the investigation was that she captured 93 flies on the island and marked them. Afterwards the flies were released.

A week later she visited the island again and captured a sample consisting of 37 flies. Of these, 9 were seen to be marked.

- a) Using this investigation, estimate the number of flies there are on the island. (2p)

When the percentage of marked flies on the island is calculated, using the percentage of marked flies in the sample, the value has a known uncertainty, called an error margin. A formula for estimating this statistical error margin f can be seen below

$$f = 1.96 \cdot \sqrt{\frac{p(100 - p)}{n}}$$

In the formula

- n is the size of the sample
- p is the fraction (**in %**) of marked flies in the sample.
- f is the error margin in percentage units.

- b) Give a way of decreasing the error margin and making the estimate more certain. 1p)

If the sample was random and if x is the percentage of marked flies on the island, then with 95% certainty

$$p - f \leq x \leq p + f$$

- c) Find the interval which with 95% certainty contains the number of flies on the island. (2p)



11. In protest against the French nuclear weapon testing, Lars started a chain letter. It worked in the following way:

The first week, he sent a protest letter to the French president. At the same time he sent a letter to two friends in Sweden with an invitation to do the same the next week, that is to send a protest letter to the French president and a letter to two friends. The letter to the friends, which also contains a suggestion for the protest letter can be seen below.

Suppose that the chain functions perfectly up to the 20th week.

- a) How many such protest letters were sent to the French president during the 20th week? (2p)
- b) How many such protest letters were sent in total to the French president during the 20 weeks? (2p)

Dear Friend,

In order to protest against the French nuclear weapon tests, this chain letter is being sent around Sweden. I have just sent a protest letter to the French president and would like you to send him such a protest letter next week as well. I would also like you to copy this letter and send it to two of your friends next week. Then, the week after, they can do the same thing as we have done. If this works, the French president will receive many protest letters.

Sincerely

Dear Mr. President,

I would hereby like to strongly protest against France's decision to carry out nuclear tests in the Pacific Ocean near Mururoa. These tests will have serious consequences for the wildlife and people of the region.

Yours sincerely

Monsieur le Président,

Je tiens ici à protester avec force contre la décision de la France pour la reprise des essais nucléaires dans le Pacifique à Mururoa. Ces essais auront des conséquences graves pour les habitants et pour l'équilibre écologique de la région.

Salutations distinguées

12. A wolf has been shot by a poacher. You, who have the nickname Skärlock Holm, have been invited to investigate the case. The three suspects for the crime, Darth Vader, The Joker and Al Capone all have alibis for the day except for the following times.

Darth does not have an alibi for 8.00-11.00 on the day in question.

The Joker does not have an alibi for 11.00-15.00 on the day in question.

Al does not have an alibi for 15.00-21.00 on the day in question.

The suspects can only have committed the crime during the period they have no alibi.

Your mission, which you choose to accept, is to determine the time of the crime and answer the question as to which suspect could have committed the crime.

To calculate the time of the wolf's death, you take the temperature of the body at two different times. The first measurement is taken at 21.00 the day the wolf was shot, and the wolf's temperature is found to be 28.0°C . Three hours later you find the wolf's temperature is 25.6°C . You suppose that the temperature of the body after the wolf's death decreases exponentially with time and that a live wolf's temperature is 36.9°C .

Which of the suspects could have committed the crime? (4p)

Because of the gravity of the situation, it is of course important that you show your calculations and motivate your answer.

13. In a formula sheet it says that the function $f(x) = \ln x$ has the derivative $f'(x) = \frac{1}{x}$ for all x .

Investigate whether this derivation rule really seems to be correct. (3p)
You do not need to provide a proof.