

Concerning test material in general, the Swedish Board of Education refers to the Official Secrets Act, the regulation about secrecy, 4th chapter 3rd paragraph. For this material, the secrecy is valid until the expiration of December 2009.

## Directions

Test time	180 minutes without a break.
Resources	Calculator and table of formulae. A formulae sheet is enclosed to the test.
Test material	<p>The test material should be handed in together with your solutions.</p> <p>Write your name, the name of your education programme / adult education, and your date of birth on all the sheets you hand in.</p>
The test	<p>The test consists of 13 problems.</p> <p>In most of the problems it is not enough to give short answers, they require</p> <ul style="list-style-type: none"><li>• that you write down what you do</li><li>• that you explain your train of thought</li><li>• that you, where necessary, draw figures</li><li>• that you show how you have used your resources when you have solved problems numerically/graphically</li></ul> <p>For some problems (where it says <i>Only an answer is required</i>) you only need to give the answer.</p> <p>Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for a partial solution or presentation.</p>
The score levels	The teacher responsible will inform you about the scores required for "Passed" and "Passed with Distinction". The maximum score is 38 points.

1. Solve the equations

a)  $10^x = 25$  *Only an answer is required* (1p)

b)  $x^9 = 7$  *Only an answer is required* (1p)

2.  $f(x) = x^5 + 7x^2 - 5x + 3$

a) Differentiate  $f(x)$  *Only an answer is required* (1p)

b) Give an example of another function that has the same derivative as the given function.  
*Only an answer is required* (1p)

3. The function  $f(x) = e^{2x}$  is given

a) Calculate  $f(4)$  *Only an answer is required* (1p)

b) Find  $f'(x)$  *Only an answer is required* (1p)

4.



On account of the above headline, a journalist wants to investigate the population growth in Sweden during a longer period of time.

In the Statistical Yearbook of Sweden she reads that on January 1, 1900 the population was 5.1 million people, and on January 1, 1999 8.9 million people.

On average, how many per cent has the population increased each year between January 1, 1900 and January 1, 1999? (3p)

5. Which of the below alternatives A-F shows the graph to a function  $y = f(x)$

a) where  $f'(2) = 0$

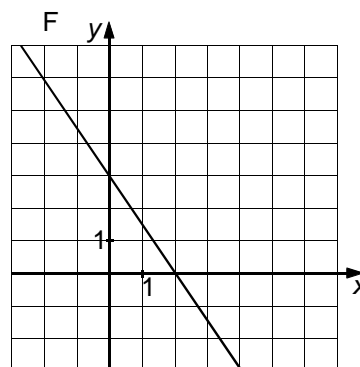
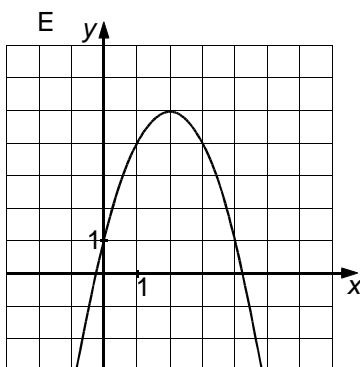
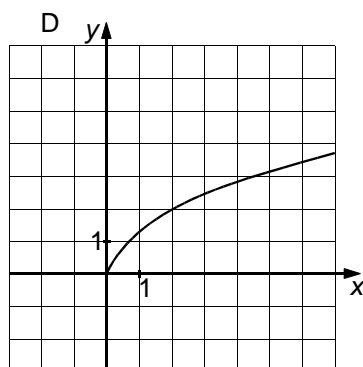
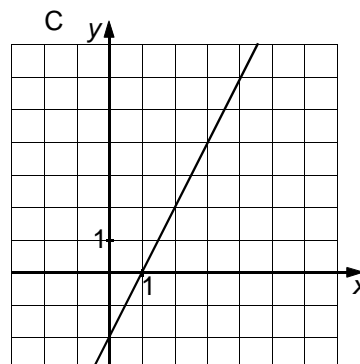
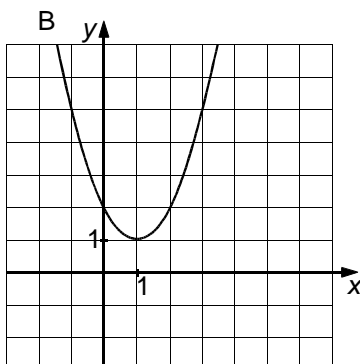
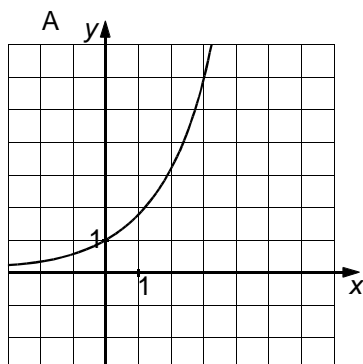
Only an answer is required

(1p)

b) where  $f'(1) < 0$

Only an answer is required

(1p)



6.  $1200 + 1200 \times 1.03 + 1200 \times 1.03^2 + \dots + 1200 \times 1.03^{19}$

a) Calculate the sum of the geometric series.

(2p)

b) Formulate a problem about a real situation. The problem is to be solved by calculating the sum of this geometric series.

(2p)

7. A student wants to compare the development of the study grants in Sweden and Norway. In a journal he reads that in Sweden the study grants have increased by 1.1% during the years 1995-1998. When calculating this increase, the CPI has been taken into consideration.

To be able to do the comparison, the student picks out the following information about Norway:

Year	1995	1998
Study grant (in Norwegian crowns)	1087	1462
CPI (Norway)	260.0	276.2

How many per cent did the study grant in Norway increase between 1995 and 1998, if the CPI is taken into consideration?

(2p)

8. The graph of the function  $y = x^3 - 45x^2 - 3000x + 1000$  has a minimum point. Use the derivative to find the co-ordinates of this point. (3p)

9. In the 1999 World Athletics Championships, Maurice Greene won the hundred-metre race in a time of 9.80 s. In a magazine, Billy finds a table of how Maurice Green ran the first part of the race:

Time in seconds	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Distance leapt in metres	4.5	8.0	12.1	16.6	21.6	26.9	32.5

Billy finds a function  $s(t) = 4.49t^{1.43}$  that goes well with the values in the table.  $s(t)$  is the distance leapt after the time  $t$  seconds.

- Use the rate of change to calculate Maurice Greene's speed 3.0 s after the start. (1p)
- Use the derivative to calculate Maurice Greene's speed 3.0 s after the start. (2p)
- Which of the two above methods is most suitable for calculating Maurice Greene's speed 4 s after the start? Justify your choice. (1p)
- Is the function  $s(t) = 4.49t^{1.43}$  a reasonable model that might be true for the whole hundred-metre race? Justify your answer. (1p)



10. The politicians in a municipality are interested in knowing the inhabitants' attitudes towards different school issues. A decision is taken that a random sample survey is to be done, and 800 people are randomly chosen. A questionnaire is sent out, where one of the questions is: "Do you think that all the upper secondary school pupils in this municipality should be able to borrow a portable computer during their time of study?"

The following answers were obtained:

Yes	No	Don't know
325	220	20

- a) Suppose that the opinions among those who didn't answer are distributed in the same way.

How many per cent of the inhabitants in the municipality thinks that the pupils should be able to borrow a portable computer?

(1p)



- b) The politicians want a more reliable result from the survey, and therefore they discuss the two following alternatives:

Alternative 1: Complete the survey with 800 new randomly chosen people.

Alternative 2: Complete the survey with a non-response survey to see what those who had been asked but did not answer think.

Explain why it may be suitable to choose alternative 2.

(1p)

- c) They choose to do a non-response survey and call 50 people of those who didn't answer the question about borrowing a computer. They answered as follows:

Yes	No	Don't know
15	20	15

With a basis in the two surveys, how many per cent of the inhabitants in the community are for "borrowing a computer"?

(2p)

- 11.** The graph of a quadratic function has its maximum point at  $(-1, 4)$ .

In a coordinate system, sketch what the graph of the derivative to the function might look like.

(2p)

- 12.** Kajsa suffers from goitre. The treatment of the disease includes her drinking a solution containing radioactive iodine. Iodine is absorbed by the thyroid gland, which then emits radiation. The radioactivity of iodine decreases exponentially with time and is halved every 6 days. In the beginning of the treatment, the activity is 230 MBq (MBq is a unit of radioactivity).

Kajsa works at a day-care centre and is near the children most part of the day. Therefore, she has to be on the sick list until the activity has decreased to 75 MBq.

For how long does Kajsa at least have to be on the sick list?

(3p)

- 13.** A 30-cm long cord is cut into two pieces. The first part is formed into a circle, and the second part is formed into a square.

Show that the sum of the area of the circle and the square always exceeds  $30 \text{ cm}^2$ , no matter where the cord is cut.

(4p)