Concerning test material in general, the Swedish Board of Education refers to the Official Secrets Act, the regulation about secrecy, 4th chapter 3rd paragraph. For this material, the secrecy is valid until the expiration of December 2011.

NATIONAL TEST IN
MATHEMATICS COURSE C
SPRING 2001

Directions

Test time	240 minutes without a break.
Resources	Calculators and "Formulas for National Test in Mathematics Courses C, D and E".
Test material	The test material should be handed in together with your solutions.
	Write your name, the name of your education programme / adult education on all sheets of paper you hand in.
The test	The test consists of 15 problems.

To some problems (where it says *Only answer is required*) it is enough to give short answers.

For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 15 is a larger problem which may take up to one hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and
mark levels
The maximum score is 45 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1).

Lower limit for the mark on the test
Pass: 13 points.
Pass with distinction: 26 points of which at least 5 "Pass with distinction points".

Name: _____   School: _____

Education programme/adult education: _____

1. $y = 5x^2 + 3x$

   a)   Find y' *Only answer is required*   (1/0)

   b)   Find y'' *Only answer is required*   (1/0)

2.   Solve the equations

   a) $\lg x = 4$ *Only answer is required*   (1/0)

   b) $2^x = 6$ *Only answer is required*   (1/0)

3.

During the 1970's, Ted Gärdestad was a huge teenage idol. In 1972, during a tour in Sweden he was paid SEK 3000 for one performance.

Following the CPI, calculate how much this is in the year 2000.   (2/0)

Year	CPI
1972	47
2000	261

(The information in the table above is taken from Statistics Sweden. CPI = Consumer Price Index)

4.   The following problems concern depositing money into a bank. We shall neglect any effects from taxes.

   a)   Determine the final capital after 15 years, given that the initial capital deposited is SEK 4000 and the yearly interest rate is 3.0 %?   (1/0)

   b)   If you deposit SEK 10 000, what yearly interest rate will give you SEK 18 000 after 13 years?   (2/0)

5. Find the sum of the first 50 terms in the geometric progression:

1000, 900, 810, 729, ………. (2/0)

6. a) Differentiate $y = e^{2x} + \ln 5$ *Only answer is required* (1/0)

b) Evaluate $f'(-7)$ if $f(x) = \dfrac{x^3}{3} + x$ (2/0)

7. In a municipality there are two schools, East and West. The East school has 1350 students and the West has 520 students. In both schools it is forbidden to have a mobile phone switched on during the lessons.

To see if the rule has any support among the students, the students' councils in both schools have carried out a common survey. In each school some SP-classes were chosen and all the students were asked:
"Do you think that you should be allowed to have your mobile phone switched on during the lessons?"
The answers can be seen in the table below:

School	"No answer"	"Yes"	"No"
East	17	27	58
West	30	49	16

The students' councils summarised the survey as follows:
Total percent of "Yes"-answers: $\dfrac{27 + 49}{27 + 49 + 58 + 16} \approx 51\ \%$
Conclusion: The majority favor having their mobile phones switched on.

Some students critisized the survey and the conclusion the students' councils drew.
Give an example of two such critisisms. (1/1)

8. Let $f(x) = x^3 - 3x + 5$

Using derivatives investigate if the function $f(x)$ has a maximum in the interval
$-2 \le x \le 1$. (3/0)

9. Two students solved the following problem:
   In a culture of bacteria the number of bacteria increased from 300 to 900
   in 5 hours.
   Calculate the percentage increase in bacteria.

Ingela's answer was approximately 25 % and Elias' 200 %.

Explain in words and calculations how they might have reasoned. (2/1)

10. By burning fossil fuels, the concentration of carbondioxide in the air has
   increased. The graph below shows how the concentration of carbondioxide
   expressed in ppm has changed from the early 1800's until the present day. The
   values of the concentration of carbondioxide stem from measurements on ice
   cores from the inland ice in the Antarctic with entrapped air inclusions, and from
   atmospheric measurements over Mauna Loa, Hawaii.

   a) How many times greater is the average rate of change of the concentration
       of carbondioxide during the 1900's than during the 1800's? (2/0)

   b) Find the rate of change in the concentration of carbondioxide in 1980. (0/2)
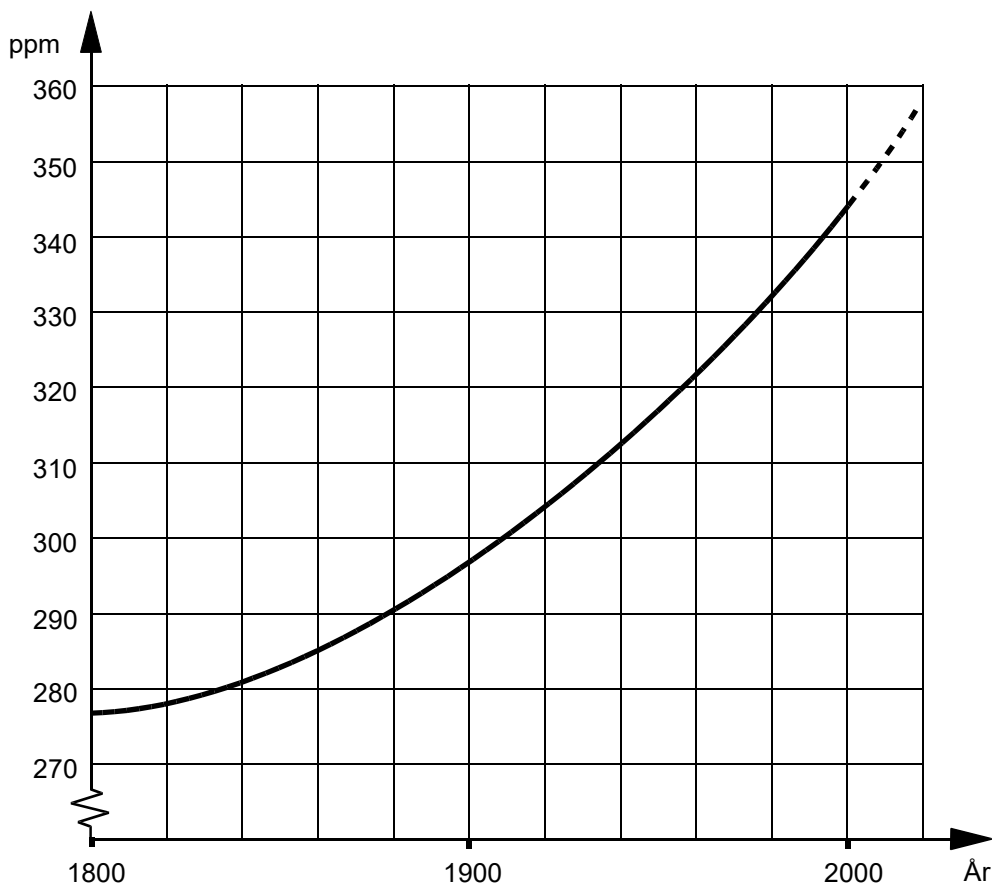
11. As a special project Anna has chosen to investigate the growth of a small cucumber. She finds out that during the first four days the weight of the cucumber grows according to the formula:

$$f(t) = 0.100 \cdot e^{0.223 \cdot t}, \qquad 0 \le t \le 4$$

where $f(t)$ is the weight of the cucumber in kilograms and t is the time in days from the start of the investigation.

During the following six days ($4 < t \le 10$) the weight of the cucumber $f(t)$ increases by 0.054 kilograms per day.
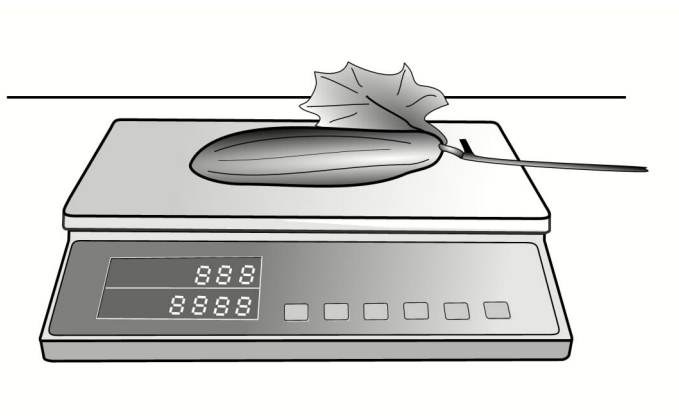
a) What is the weight of the cucumber after 2 days? (1/0)

b) Find $f'(3)$ and $f'(6)$. (1/1)

c) In words, describe what $f'(3)$ signifies about the growth of the cucumber. (0/2)

12. The figure below shows the graph of a function $y = f(x)$.

Arrange the following values according to size: $f'(a), f'(b), f'(c), f'(d)$
Start with the smallest and remember to justify your ordering. (0/2)
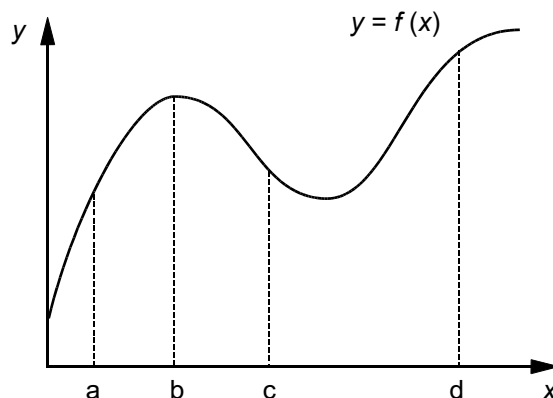
13. For a function $f(x)$ it is known that:

- $f(5) = 3$
- $f'(x) = 2$ for all x

Evaluate $f(10)$ (0/2)

14.

VISSEFJÄRDA

Lucky numbers gave 4.5 million

■ The 32-year-old industrial worker Jonas Modig, Vissefjärda, was SEK 4.5 million richer when he left the TV-studio yesterday morning, all thanks to a lottery ticket he bought at Bilisten, Vissefjärda in mid-December.

During a live appearance on the TV-program Nyhetsmorgon he made use of his lucky numbers when he picked two new lottery tickets, and it paid off. Jonas Modig can look forward to recieving SEK 15 000 per month during the next 25 years.

According to the newspaper article above, the winner should be congratulated on his winnings. However, the cost for Svenska Spel is not SEK 4.5 millions, partly because the sum is index-linked, which means that it should follow the CPI, and partly because the company can place the money that has not yet been paid in a savings account.

What is the cost today to Svenska Spel (the present value) of SEK 15 000 /month during 25 years? Assume that the interest rate is 0.5 % per month and that the CPI does not change during the period. (0/4)

15. This problem concerns the derivative of a quadratic function. We shall examine the graph of the derivative. A general quadratic function can be written as $y = ax^2 + bx + c$ where a, b and c are constants.

One can carry out the general investigation (point three below) or if one prefer, to solve the problem step by step as indicated.

- If $a = 1$, $b = 0$ and $c = 0$ then we have the function $y = x^2$. Draw the graph of y'.

- Choose your own values of a and draw the graph of the derivative of your new functions.
  How does your choice of a affect the graph of the derivative?

- Investigate in as detailed and complete manner as is possible how a, b and c affect the graph of the derivative. (2/4)

When evaluating problem 15 your teacher will look at:
- How close your solution is to a general solution
- How well you use the mathematical language
- The clarity of your explanations, justifications and conclusions