NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2004

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, and ”Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 8 problems and **Part II** consists of 8 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 27 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: In addition to the requirements for ”Pass with distinction” you have to show ”*Pass with special distinction*” *qualities in at least two* of the ¤-problems. You must also have at least 14 ”Pass with distinction”-points.

Name: ______________________________ School: ________________________________

Education programme/adult education: ____

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

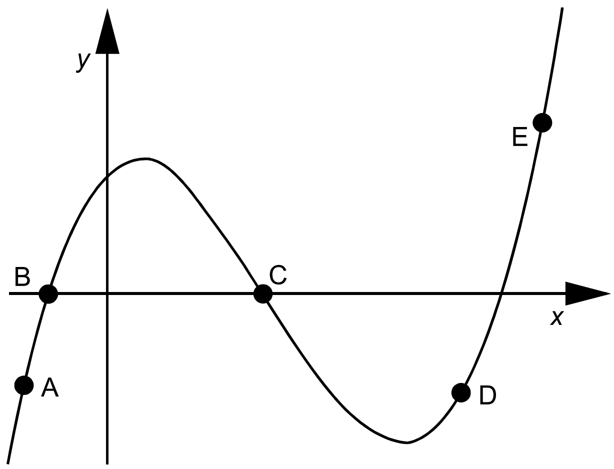
a) $f(x) = 6x^2 + 9x + 1$ *Only answer is required* (1/0)

b) $f(x) = 60e^{-2x}$ *Only answer is required* (1/0)

c) $f(x) = \dfrac{x+3}{4}$ *Only answer is required* (1/0)

2. Solve the equation $(x-2)(x+3)(x-5) = 0$ *Only answer is required* (1/0)

3. The picture below shows the main features of the graph of the function $y = f(x)$.



a) For which point or points (A, B, C, D and E) is the derivative negative?
Only answer is required (1/0)

b) How many solutions does the equation $f(x) = 0$ have?
Justify your answer from the look of the graph. (1/0)

c) How many solutions does the equation $f'(x) = 0$ have?
Justify your answer from the look of the graph. (1/0)

4. Solve the equations. Give exact answers.

a) $x^9 = 18$ *Only answer is required* (1/0)

b) $9^x = 18$ *Only answer is required* (1/0)

5. Sometimes it can be advantageous to rewrite an exponential function in another base.

a) Rewrite the function $y = 3^x$ in base e. *Only answer is required* (1/0)

b) Give one advantage of using base e in exponential functions rather than using other bases. *Only answer is required* (1/0)

6. A mirror is going to be made from a rectangle of mirror glass surrounded by a wooden frame. The frame will be made of a 0.5 dm wide strip of wood. In total, 60 dm of wooden strips will be used. The frame will have double wooden strips as top and bottom pieces, while the side pieces will consist of single strips, (see figure below).

The area of the mirror glass *A,* as a function of the length of the top piece *x* dm, is given by the relation: $A(x) = -2x^2 + 32x - 30$.

a) Determine which value of the length *x* that gives the maximum area of the mirror glass by using the derivative. (3/0)

b) Show that the area of the mirror glass *A* can be written as $A(x) = -2x^2 + 32x - 30$, where *x* is the length of the top piece. (0/2/¤)

7. Arrange the following four numbers according to size, from left to right. Start with the smallest number.

$\sqrt{10}$ $\quad$ e $\quad$ $\lg 1000$ $\quad$ $\ln \mathrm{e}^2$ *Only answer is required* (0/1)

8. In the expression $\dfrac{A}{Ax^m + x^{m+1}}$ m is an integer.
Determine the constant A so that the expression is not defined for $x = 2$ (0/2/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

9. Give an example of a fourth degree polynomial function and give the derivative of the function. (2/0)

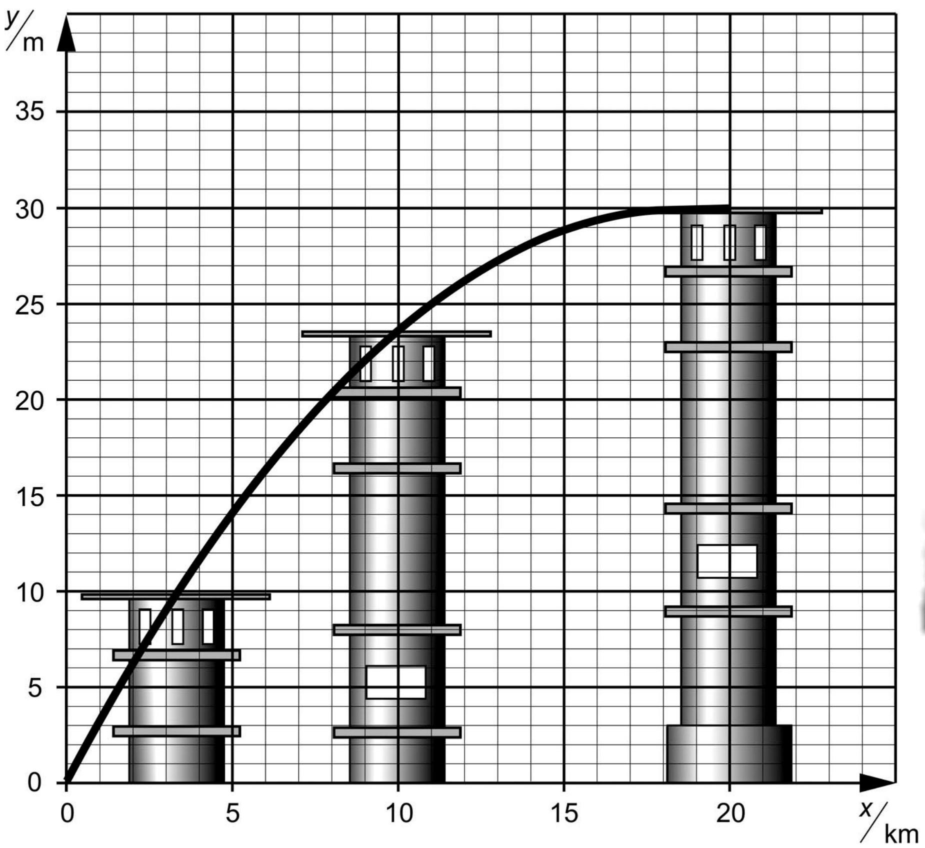
10. The first three terms in a geometric sum are $2 + 2.6 + 3.38 + ...$

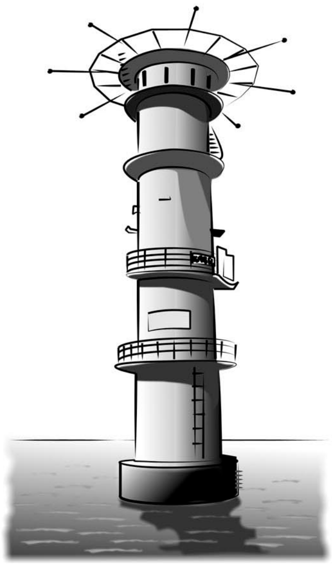
a) Determine the fourth term of the sum. (1/0)

b) Calculate the sum of the first 43 terms. (2/0)

11. Outside Falsterbo in southern Skåne stands the Falsterborev lighthouse firmly anchored to the bottom of the sea. If you come from the south by boat you can see the top of the lighthouse rise up over the horizon. The closer you come to the lighthouse the more you see of it.

Let x km be the distance travelled since you first caught a glimpse of the top of the lighthouse. Let y m be the height of the visible part of the lighthouse. The height y is then a function f of the distance travelled x. You can see the graph of the function below.





a) Use the graph to determine $f'(10)$. (2/0)

b) In words, explain the meaning of $f'(10)$ in this context. (0/2)

12. There are several functions for which it is true that $f(0) = 200$ and $f'(0) = 200$. Determine one such function. (1/1)

13. At the beginning of the 20th century there were approximately 10000 Lesser White-fronted Geese in Sweden. After that there was a dramatic decrease in the population of the Lesser White-fronted Geese and at the beginning of the 1980s they were almost extinct in Sweden.

Therefore, in 1981 the project ”Projekt Fjällgås” was started by the Swedish National Environmental Protection Agency to save the species. In brief, the aim with the project was to change the migration paths of the Lesser White-fronted Geese. The project has been successful and by 1999 the Swedish population was estimated to be 50 birds. By 2003 the population had increased and was estimated to be 90 birds.

© Photo: Lars Göran Lindström

a) Calculate the yearly percentage increase in the number of Lesser White-fronted Geese during the period 1999-2003 assuming that the increase was exponential. (0/2)

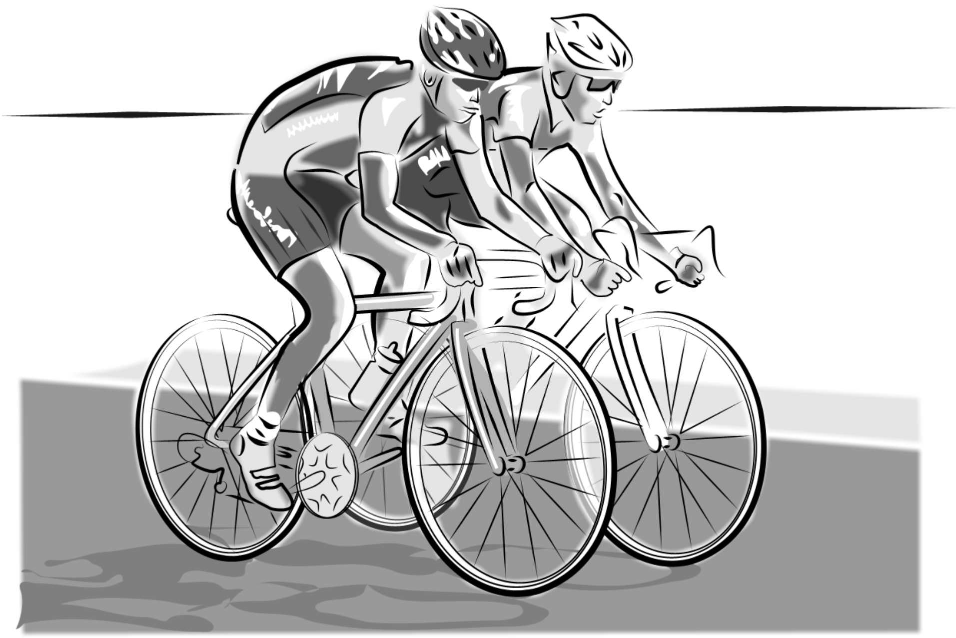
b) In what year will the population again reach 10000 birds if the increase is exponential and at the same rate as during the period 1999-2003? (0/2)

14. Write the expression $\dfrac{k^4-1}{k-1}$, where $k \neq 1$, as a sum with several terms. (0/2)

15. Frida and Gisela participate in the same bicycle race. The race is 90 km. Frida keeps a constant speed throughout the race while Gisela's speed varies. Simplified, the distance (in km) they have cycled can be described by the functions:

$f(t) = 30t$ and $g(t) = t^3 - 6t^2 + 37{,}8t$ where t is the time in hours after the start.

Frida and Gisela start at the same time. Frida finishes after 3 hours and Gisela shortly thereafter.

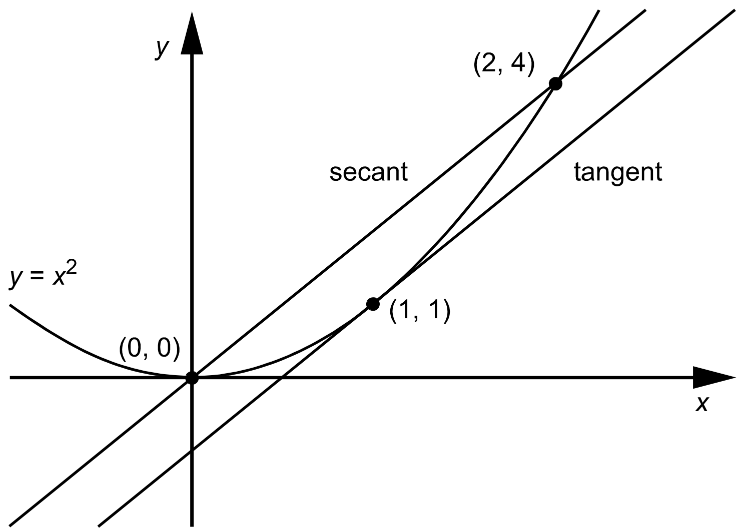


How long after the start is the distance between Frida and Gisela the largest and what is the distance between them at that time? (0/2/¤)

When assessing your work with this problem, the teacher will pay extra attention to:
• How general your solution is
• How well you justify your conclusion
• How well you carry out your calculations
• How well you present your work
• How well you use the mathematical language

16. In this problem you are going to investigate the slope of tangents and secants to quadratic curves that pass through the origin. A secant is a straight line that passes through two points on the curve.

- The figure shows the graph of the function $y = x^2$, a tangent that touches the curve at the point (1, 1) and a secant that passes through the points (0, 0) and (2, 4) on the curve.
  Show that the gradient to the tangent is equal to the gradient of the secant.



The example above shows *one* case where the following three conditions are fulfilled:

- The quadratic curve passes through the origin and can then be written as $y = ax^2 + bx$
- The secant passes through the origin and another point on the curve.
- The *x*-coordinate of the point of tangency is halfway between the two *x*-coordinates of the points of the curve that the secant passes through.

- Now study *all* the cases that fulfil the three conditions above and investigate whether it is *always* true that the gradient of the secant is equal to the gradient of the tangent. (2/4/¤)