NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2005

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators and ”Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 10 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	26 points of which at least 6 ”Pass with distinction” points.
Pass with special distinction:	In addition to the requirements for ”Pass with distinction” you have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show. You must also have at least 12 ”Pass with distinction”-points.

Name: ______________________________ School: __________________________________

Education programme/adult education: ____

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = x^3 - 6x$ *Only answer is required* (1/0)

b) $f(x) = 5e^{4x}$ *Only answer is required* (1/0)

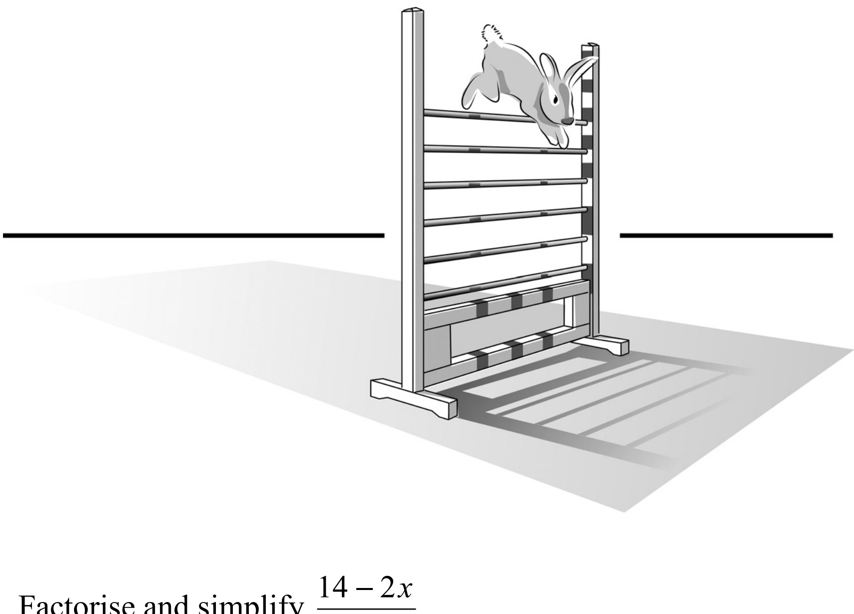
2. The function $y = f(x)$ has a local maximum at $x = 5$
What is the value of $f'(5)$? *Only answer is required* (1/0)

3. In 1997, the rabbit Tösen from Denmark set a world record in the high jump for rabbits. According to one model, Tösen's height during the jump is given by

$h(x) = 4x - 4x^2$

where h is the height in metres above the floor and x is the distance in metres along the floor from the take-off.

Use the derivative to calculate Tösen's maximum height during the jump. (2/0)



4. Factorise and simplify $\dfrac{14 - 2x}{7 - x}$ (1/0)

5. Factorise and simplify $\dfrac{a + 3}{a^2 - 9}$ (1/0)

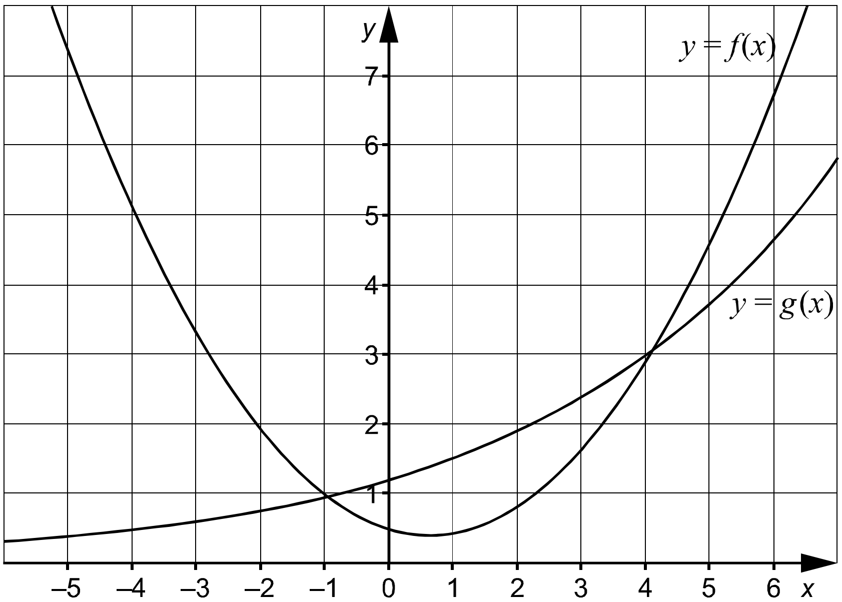
6. One of your friends thinks about what the curve $y = 2x^2 + 3$ looks like and then declares:
"The gradient is always equal to 4, everywhere on the curve."

Is your friend right? Justify your answer. (1/0)

7. Solve the equation $x^3 - x(8x - 16) = 0$ (0/2)

8. Is $\lg 9$ larger or smaller than 1? Justify your answer. (1/1/¤)

9. The figure shows the graphs of $y = f(x)$ and $y = g(x)$



For which value or values of x is it true that $f'(x) = g'(x)$?
Justify your answer with text and/or by drawing a figure. (1/1)

10. Show that $f'(x) \geq 0$ for all x if $f(x) = Ax^5 + Bx^3$ and A and B are positive constants. (0/1/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

11. A certain geometric sum can be calculated by $\dfrac{4000\cdot(1.03^5-1)}{1.03-1}$

a) Write down the terms of the geometric sum that can be calculated by the above expression. (2/0)

b) Formulate a problem about a real-life situation. It should be possible to solve your problem by calculating the expression $\dfrac{4000\cdot(1.03^5-1)}{1.03-1}$ (1/0)

12. The Unit of Applied Field Research at the Swedish University of Agricultural Sciences has investigated how the amount of nitrogen in artificial manure affects the size of the harvest for different kinds of barley.

The barley Baronesse follows the function

$f(x)=0.002x^3-0.81x^2+105.6x+1600 \qquad 0\le x\le 180$

where $f(x)$ is the size of the harvest in kg/hectare and x is the amount of nitrogen added in kg/hectare.

How much nitrogen must be added to maximize the size of the harvest? (2/1)

13. Anders has been given a graphic calculator. Explain to Anders what he should do to solve the equation $x^3-6x^2=1-9x$ on his graphic calculator. (2/0)

14. At the beginning of the year 2000, Karin bought shares worth SEK 3000 in an IT-fund. Five years later the value had decreased to SEK 1712.

Calculate the yearly percentage decrease in value of her shares. (0/2)

15. Carolina Klüft competes in heptathlon and is one of Sweden's strongest medal candidates in the World Championships in Athletics in 2005.

In heptathlon the athletes compete in various events. To be able to sum up the results from these events, the results from each event are converted into a score.

The International Association of Athletics Federation (IAAF) has decided upon the two formulas used for calculating the score.

Formula used for Track Events:

Score = $a \cdot (b - M)^c$

Formula used for Field Events:

Score = $a \cdot (M - b)^c$

Explanation:
M = Measured results (running in seconds, jumps in centimetres, throws in metres)
a, b, c = constants, see the table below (*b* is the worst result that gives a score)

	Constants		
Event	*a*	*b*	*c*
200 m	4.99087	42.5	1.81
800 m	0.11193	254	1.88
100 m Hurdles	9.23076	26.7	1.835
High Jump	1.84523	75	1.348
Long Jump	0.188807	210	1.41
Shot Put	56.0211	1.5	1.05
Javelin	15.9803	3.8	

a) The Swedish record in the women's long jump is 699 cm.
What score will Carolina get for such a long jump in a heptathlon? (1/0)

b) The value of the constant c for the javelin has disappeared from the table.
Determine c if you know that a 48-metre throw gives a score of 822 points. (1/0)

c) At the Athens 2004 Olympic Games Carolina's score was 6047 points before the last event which was 800 m. What time would she have needed to break the European record of 7009 points? (1/1)

d) Why are two different formulas being used? (0/1/¤)

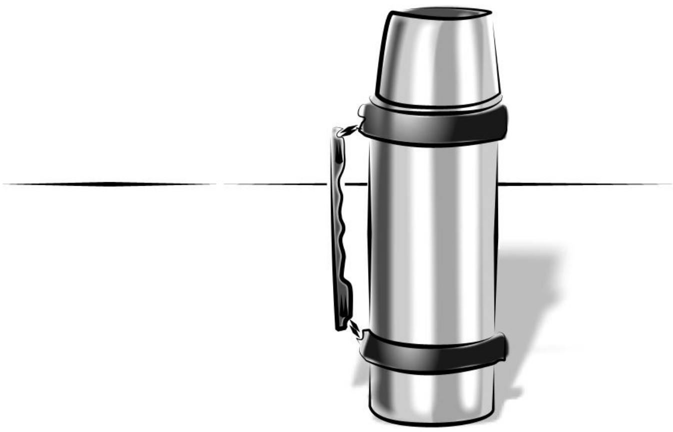
16. In this problem you are going to determine the value of the derivative to $f(x) = x^2 + 3$ at the point on the curve where $x = 4$

a) Solve this problem by using the rules of differentiation. (1/0)

b) Solve this problem by using a suitable rate of change. (1/0)

c) Solve this problem by using the definition of the derivative. (0/1/¤)

17. A thermos is filled with hot coffee and is immediately placed outside where the temperature is around zero degrees. The temperature of the coffee decreases exponentially with time.



After 4 hours, the temperature is 76 °C and at the same time the rate of decrease in temperature is 4.1 °C per hour.

a) What was the temperature of the coffee when it was poured into the thermos? (0/3/¤)

b) The coffee can be regarded as drinkable as long as its temperature does not fall below 55 °C.
How long after the coffee has been poured into the thermos will it still be drinkable? (0/1)

When assessing your work with this problem your teacher will pay extra attention to

- How general your solution is
- How well you justify your conclusions
- How well you carry out your calculations
- How well you present your work
- How well you use the mathematical language

18. Micke and Peter have started a company within Yong Enterprise (YE). They sell T-shirts with the print 'Mmm…Mathematics' and they have customers all over the country. Therefore, they are going to send the sweaters by post. To find out what applies to sending the parcels by post they go to Swedish Posten's homepage where they find the following information:

Length + width + width + height + height may be a maximum of 200 cm

Lenght:	maximum 150 cm
Width:	maximum 70 cm
Height:	maximum 115 cm

They want to send the sweaters in parcels with as large volume as possible, without exceeding the maximum dimensions.

Micke believes that a cubical parcel will give the largest volume. Peter thinks that the volume will be even larger if the parcel is not shaped like a cube, but if the width and height are equal.

- Which of the two types of parcels will give the largest possible volume?
- If the limit of 200 cm is changed, will the same type of parcel still be the largest one? (3/4/¤)