NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2006

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C and D.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators and ”Formulas for the National Test in Mathematics Courses C and D”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 15 problems. **Part I** consists of 7 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 41 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

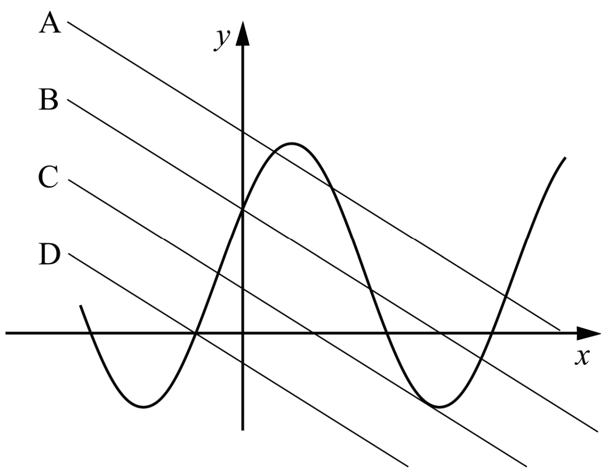
Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	24 points of which at least 6 ”Pass with distinction” points.
Pass with special distinction:	24 points of which at least 13 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Which of the lines A-D is a tangent to the curve? *Only answer is required* (1/0)



2. Differentiate

a) $f(x) = 2x^4 - 7x + 12$ *Only answer is required* (1/0)

b) $f(x) = e^{5x}$ *Only answer is required* (1/0)

3. Study the table below. It shows the sign of the derivative for some different x-values for the cubic function f.

x	-2	-1	0	4	5
$f'(x)$	$+$	0	$-$	0	$+$

What is the value of x at the minimum point of the graph to the function?
Only answer is required (1/0)

4. Solve the equations and give exact answers.

a) $x^5 = 7$ *Only answer is required* (1/0)

b) $10^x = 0.3$ *Only answer is required* (1/0)

c) $4x(3x-7)(7x+3)=0$ *Only answer is required* (0/1)

d) $\ln(3x-1)=0$ *Only answer is required* (0/1)

5. Olle runs a 100-metre run. The distance $s(t)$ metres that he has run is a function of the time t seconds after the start.

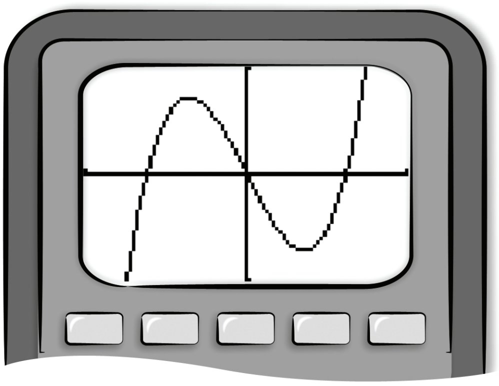
Explain what $s'(6)=8$ means in this context. (1/1)

6. Simplify as far as possible:

a) $\dfrac{x^2-x}{x(x-1)}$ (1/0)

b) $\lg(2x+x)-\lg x$ (0/1)

7. Kajsa is going to solve the equation $x^3-3x=2.1$
She starts by drawing the graph to $y=x^3-3x$. The following picture can then be seen on her graphic calculator:



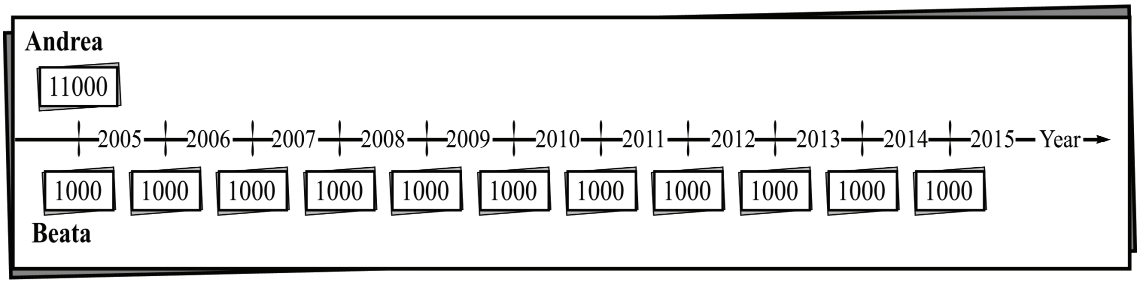
a) Describe how Kajsa should continue to solve the equation $x^3-3x=2.1$ (1/0)

b) How many solutions are there to the equation $x^3-3x=2.1$?
Investigate this and justify your conclusion. (0/2/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

8. Andrea and Beata start saving money at the same time, each in an account where the annual interest is 2%. Andrea deposits a lump sum of SEK 11000. Beata makes a yearly deposit of SEK 1000. Beata makes her last deposit at the turn of the year 2014/2015. See figure.

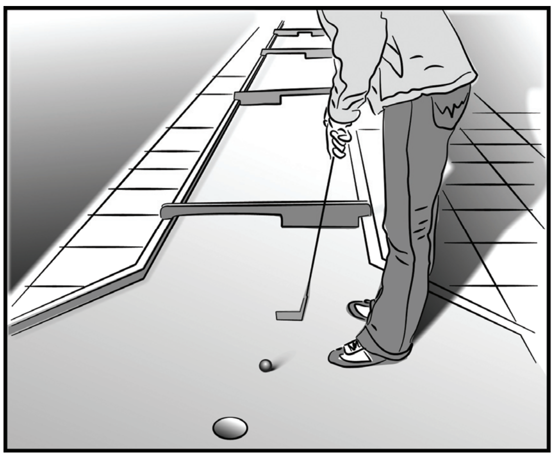


a) How much money does Andrea have in her account immediately after the turn of the year 2014/2015? (1/0)

b) How much money does Beata have in her account immediately after her last deposit? (2/0)

(Disregard any effects of tax.)

9. Bertil and Svea own a miniature golf course. During the last few seasons they have had approximately 200 players each day and each player has brought a profit of SEK 5.



Bertil and Svea are now thinking about raising the fee for playing. For each crown the fee increases, the profit per player will increase by the same amount.

On the other hand, through interviews, they have concluded that each crown in an increase of the fee will lead to a decrease of 20 players per day.

The profit V SEK as a function of the price rise x SEK is then given by
$V(x) = -20x^2 + 100x + 1000$

a) Use the method of differentiation to calculate what price rise will give the largest profit. (3/0)

b) Show that the profit V SEK can be written as $V(x) = -20x^2 + 100x + 1000$ (0/2/¤)

10. Write down a rational expression which is not defined for $x = -5$

Only answer is required (1/0)

11. The White-backed woodpecker is endangered in Sweden. In 2004 only 3 breeding couples were observed. One reason is the forest industry which treats the species which depend on broad-leaf trees, old trees and dead wood unfairly.

The Swedish Society for Nature Conservation and the Swedish Environmental Protection Agency have taken measures to try to preserve the White-backed woodpecker.

The table below shows the number of breeding couples of White-backed woodpecker in two different years.

Year	Number of breeding couples
spring 1984	50
spring1992	20

In 1992, researchers assumed that the decrease between the years 1984-1992 was exponential and that the number of couples would continue to decrease at the same rate.

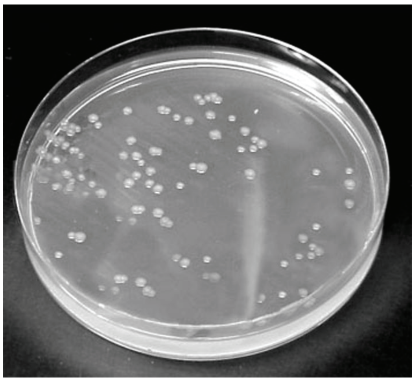
How many couples would there have been in 2004 according to the researchers' assumption? (0/2)

12. For the function $f(x) = \frac{x^4}{4} - k^3 x$ it is true that k is a positive constant.

Determine the largest value of the function within the interval $0 \le x \le 2k$ (0/3)

13. In a laboratory experiment, bacteria are cultivated on an agar plate.
The number of bacteria N as a function of time t minutes after the beginning of the experiment is given by

$$N(t)=\frac{6000}{6.5e^{-0.05\cdot t}+1}$$



The picture shows an agar plate with bacteria.

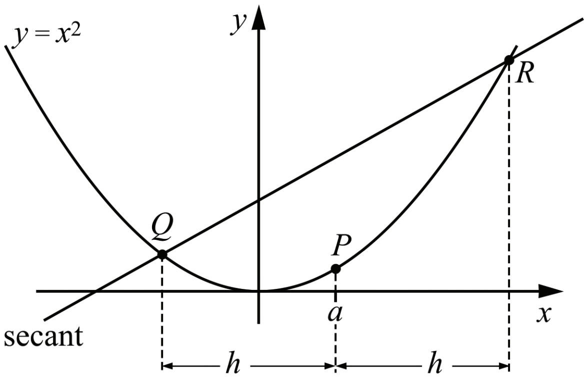
a) How many bacteria are there in the culture at the beginning of the experiment? (1/0)

b) Explain how you can calculate a value for the growth rate when $t=20$ minutes without differentiating $N(t)=\dfrac{6000}{6.5e^{-0.05\cdot t}+1}$ (1/0)

c) The growth rate in the culture is the largest when the number of bacteria is approximately 3000.
After how much time are there 3000 bacteria in the culture? (1/1)

14. The figure shows the graph of the function $y=x^2$ and a straight line (secant) that passes through the points Q and R on the curve. The point P also lies on the curve and has x-coordinate a.

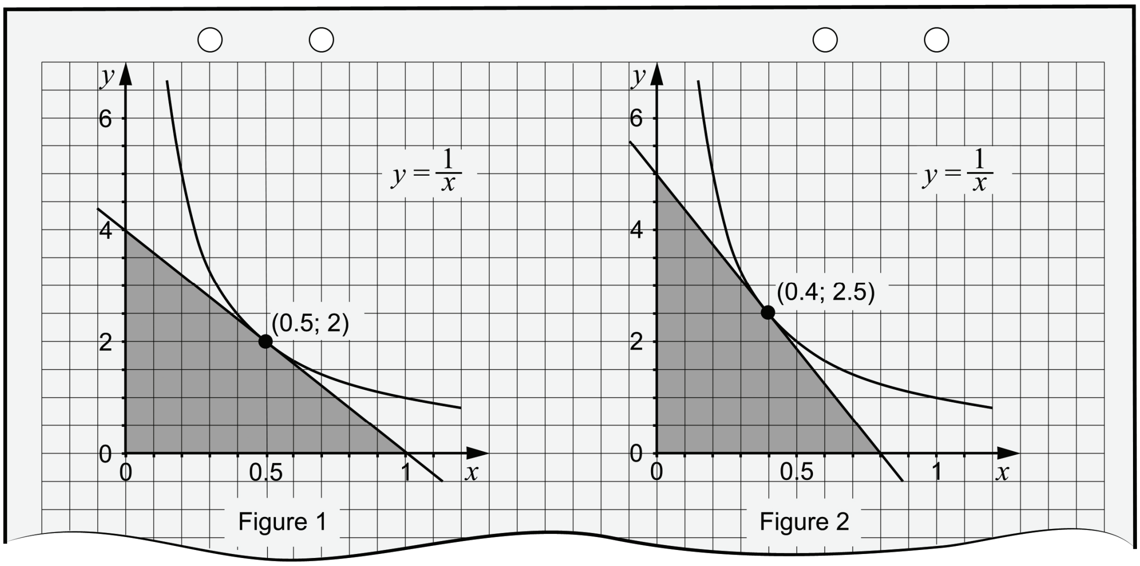
The distance in x between the points Q and P is equal to the distance in x between the points P and R. This distance is denoted h in the figure below.



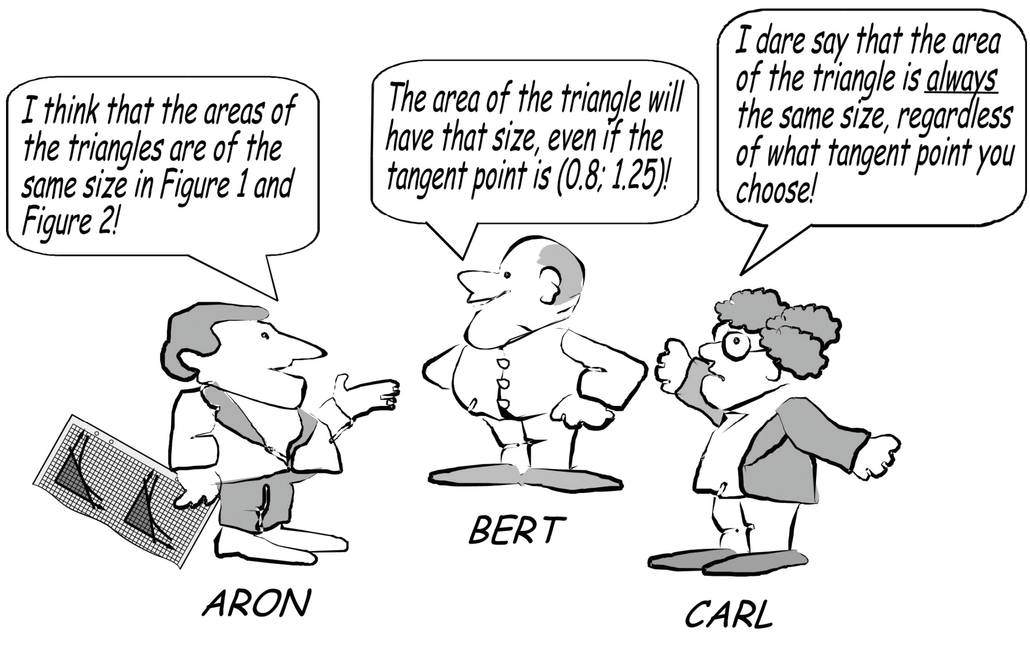
Show that the gradient of the secant is *always* equal to the gradient of the tangent to the curve in point P. (0/2/¤)

When assessing your work, your teacher will pay extra attention to:
• how far you have reached in your investigation
• how general your investigation is
• how well you carry out your calculations
• how well you have presented your work
• how well you have used mathematical language

15. Aron, Bert and Carl study pictures of the graph to the function
$y = \frac{1}{x}$ where $x > 0$.
The pictures show tangents through two different points on the curve. Together with the positive coordinate axes, the tangents form triangles. See Figure 1 and Figure 2.



The three friends discuss the size of the area of the formed triangles and declare the following:



- Investigate and decide who of the three is/are right. (2/4/¤)