Concerning test material in general, the Swedish Board of Education refers to the Official Secrets Act, the regulation about secrecy, 4th chapter 3rd paragraph. For this material, the secrecy is valid until the expiration of June 2014

NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2008

Directions

Test time — 240 minutes for Part I and Part II together. We recommend that you spend no more than 75 minutes on Part I.

Resources — **Part I:** ”Formulas for the National Test in Mathematics Course C.”
Please note that calculators are not allowed in this part.

Part II: Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”

Test material — The test material should be handed in together with your solutions.
Write your name, the name of your education programme/adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 17 problems. **Part I** consists of 9 problems and **Part II** consists of 8 problems.

For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 43 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	24 points of which at least 7 ”Pass with distinction” points.
Pass with special distinction:	24 points of which at least 13 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

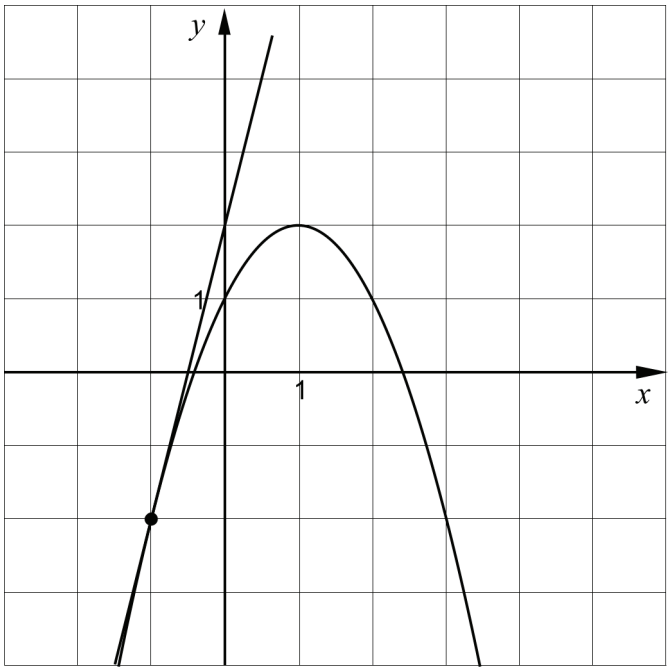
This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = 4x^3 - 2x + 12$ *Only answer is required* (1/0)

b) $g(x) = \mathrm{e}^{3x}$ *Only answer is required* (1/0)

2. The figure shows the graph of $f(x) = -x^2 + 2x + 1$ and a tangent that passes through the point $(-1, -2)$

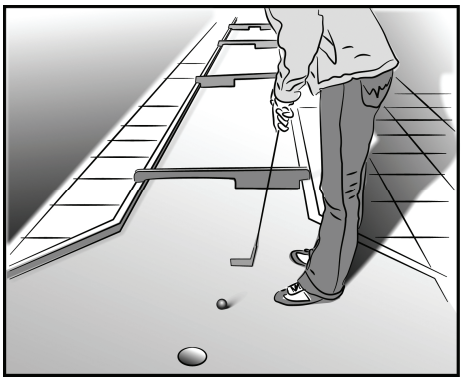


a) Find $f'(-1)$ *Only answer is required* (1/0)

b) Find the equation of the tangent. *Only answer is required* (1/0)

3. Bertil and Svea own a miniature golf course and they are thinking of raising the playing fee to increase profit. They have concluded that the profit SEK V as a function of the fee increase SEK x is given by

$$V(x) = (5 + x)(200 - 20x)$$



Calculate, by using differentiation, what fee increase that will give the greatest profit. (3/0)

4. Simplify the following expressions as far as possible.

a) $(a+b)^3 - (a^3 + b^3)$ (1/0)

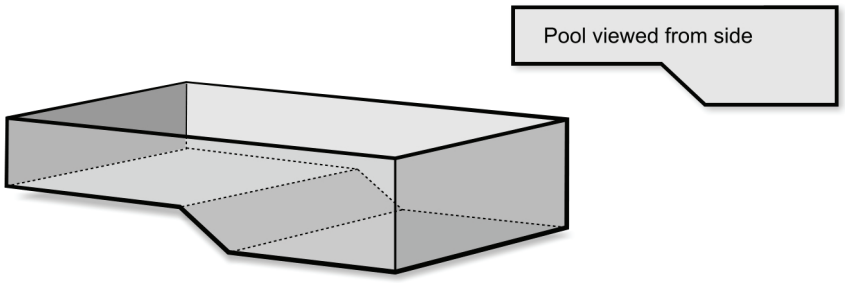
b) $\dfrac{3a^2b + 3ab^2}{3ab}$ (1/0)

c) $\ln a^3 - 3\ln a$ (1/0)

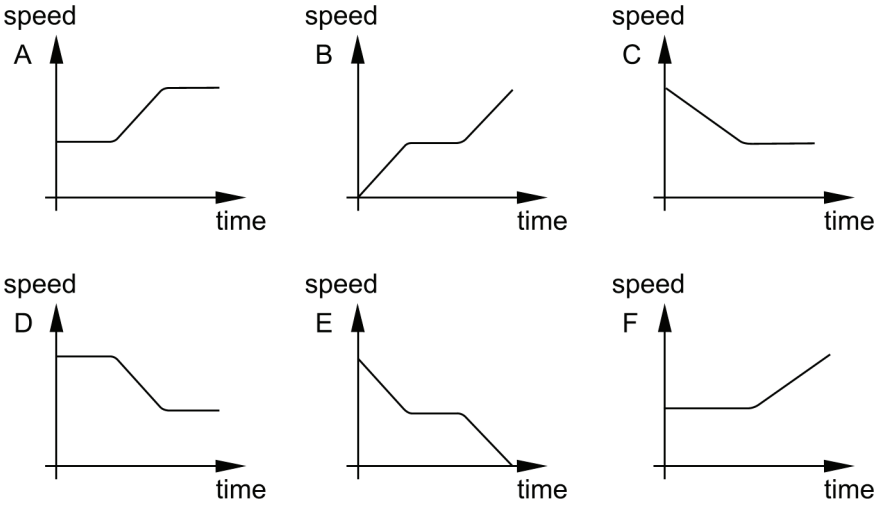
5. For the function f it holds that $f'(x) = 4x - 8$
Kalle says that the graph of the function has a maximum point.

Is Kalle right? Explain. (0/1)

6. The figure below shows an empty pool. As the pool is being filled an equal number of litres of water is added every second.



Which of the graphs A-F *best* describes at what speed the water level in the pool rises? Explain.



(0/2)

7. Factorization is a useful method for solving certain equations.

a) Factorize the expression $x^3(x+3)-8(x+3)$ *Only answer is required* (0/1)

b) Solve the equation $x^3(x+3)-8(x+3)=0$ (0/1)

8. For the function f it holds that $f(x) = \frac{x^2}{A}$ where A is a constant ($A \neq 0$)
Use the definition of the derivative to determine $f'(x)$ (0/2/¤)

9. Which of the six numbers below is smallest? Justify your answer

$\ln e$ $\lg e$ e 1 $\ln 10$ $\lg 10$

(1/1/¤)

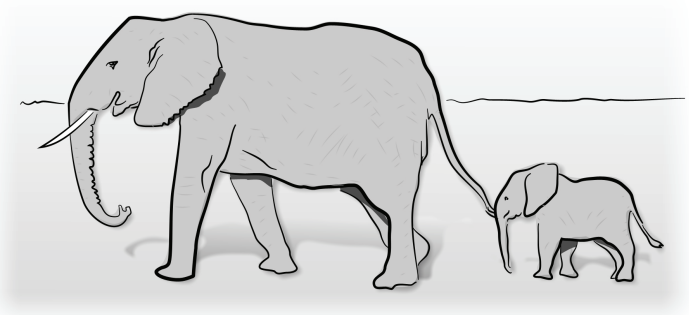
Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

10. The weight of an elephant foetus V kg as a function of time t months is given by the relation

$V(t) = 0.310 \cdot e^{0.271 \cdot t}$ where $1 \le t \le 22$

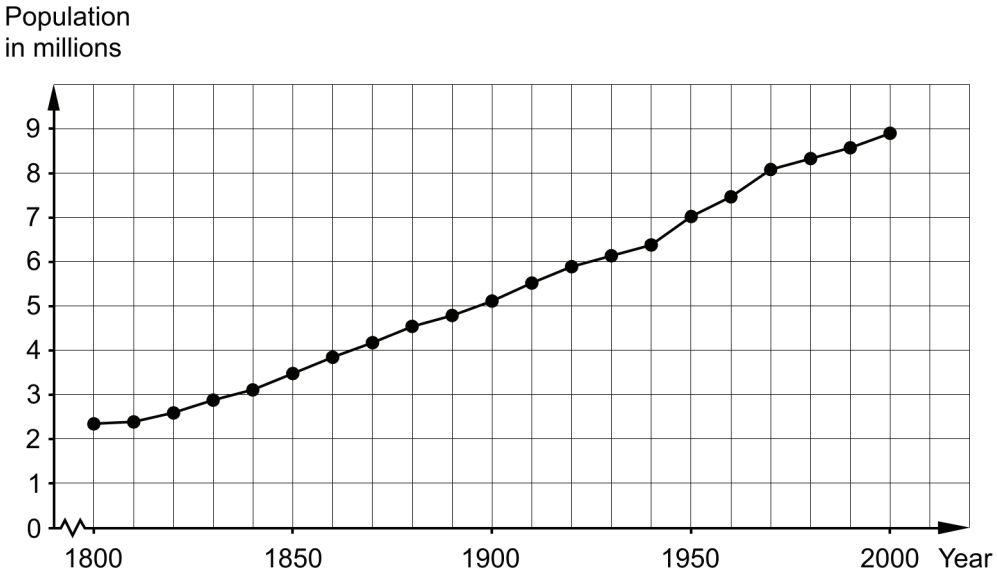
The time is counted from the moment the female is fertilized.



a) An elephant baby is born 22 months after the fertilization. What is the weight of the elephant baby at birth? (1/0)

b) How long after the fertilization is the weight of the elephant foetus 3.5 kg, that is to say the same as a newborn baby? (2/0)

11. The diagram below shows the population growth in Sweden over 200 years.



Estimate the average population growth per year between 1800 and 2000. (2/0)

12. A function f has a continuous graph and the following properties:

- The equation $f(x)=0$ has solutions $x_1=-3, x_2=0$ and $x_3=3$
- The equation $f'(x)=0$ has solutions $x_1=-1.7$ and $x_2=1.7$

Sketch a possible graph of the function f (1/1)

13. The number of sent MMS (multimedia messages via mobile phones) has increased in Sweden.

In 2003 6.7 million MMS were sent.

Three years later, in 2006, 70.3 million MMS were sent.

(Source: The National Post and Telecom Agency)

The increase is assumed to be exponential.
Calculate the yearly percentage increase in the number of MMS sent. (0/2)

14. Alva has planned to save up SEK 100 000 by depositing the same sum of money into a savings account at the end of each year. She has planned to make 15 deposits and she counts on a yearly interest rate of 2 %.

How much does she have to deposit each year if there is going to be SEK 100 000 in her account immediately after the last deposit? (1/2)

15. The figure shows Russian nesting dolls, so called ’Matryoshka’. The dolls are made of wood and can be nested inside of each other.

The largest Russian nesting doll ever made is 100 cm tall. Assume that the second largest doll is 5 % shorter than the largest doll and that the percentage decrease is the same throughout the whole series of dolls.

How many dolls does the series contain if the smallest doll is 2.6 cm tall? (0/3)

16. Does the curve $y = -3x^2 + 6x$ have any tangent that passes through the point $(0, -1)$? (0/2/¤)

When assessing your work with the following problem your teacher will take into consideration:

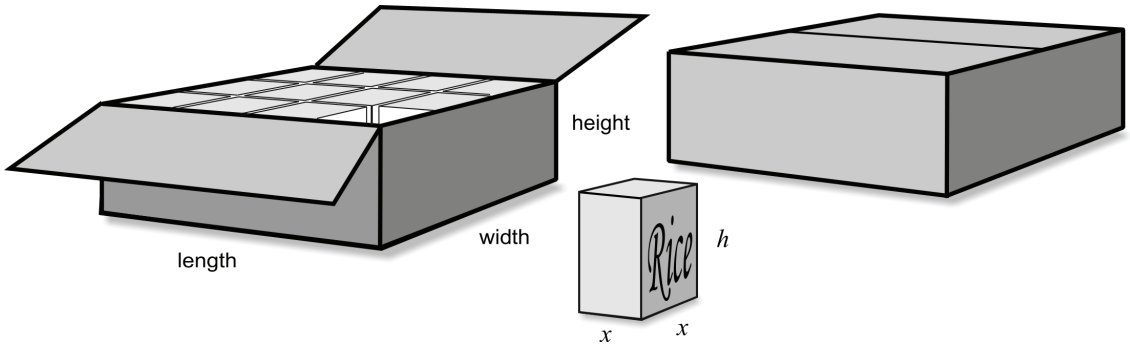
- How well you carry out your calculations
- How close to a solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

17. In this problem you are going to investigate how different ways of placing rice packages affect the surface area of the box. The surface area is the total area of the lid, bottom and sides. For cost reasons, it is desirable to use as little material as possible for the box.

The following requirements should be satisfied (for aesthetical and practical reasons):

- Each rice package should hold 0.5 dm^3 of rice.
- The rice packages should have a square base.
- The height of the rice packages h must be at least 0.5 dm and not more than 2 dm.
- The box should hold 12 rice packages.

The box can have different appearances. If the 12 rice packages are placed standing, with 4 rice packages lengthwise and 3 rice packages widthwise, the box is given the appearance shown in the figure below.



The relation $A(x) = 24x^2 + \frac{7}{x}$ describes how the surface area of this box A dm^2 varies with the dimensions of the rice packages base x dm.

- Calculate the surface area of the box when the dimensions of the rice package´s base x are 1 dm.
- Calculate the value of x that gives the smallest surface area of the box when the rice packages are placed according to the figure above.
- Choose **one** other way to place the rice packages. This entails that the box where the rice packages are placed must have another shape. Calculate the smallest surface area of this new box.
- Which of the two boxes should be produced, if you only take the cost into consideration? (3/4/¤)