NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2010

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 90 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 9 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 12 points Pass with distinction: 24 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: 24 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Find $f'(x)$ when

a) $f(x) = x^{11} + 11x$ *Only answer is required* (1/0)

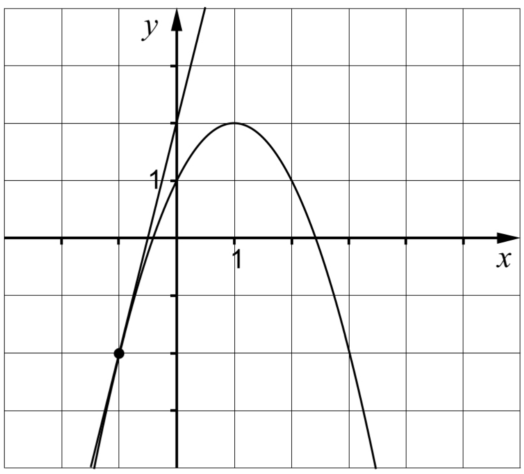
b) $f(x) = \dfrac{x}{3}$ *Only answer is required* (1/0)

2. Solve the equations. Give exact answers.

a) $\lg x = 3.2$ *Only answer is required* (1/0)

b) $6^x = 13$ *Only answer is required* (1/0)

3. The figure shows the graph to $f(x) = -x^2 + 2x + 1$ and a tangent that passes through the point $(-1, -2)$

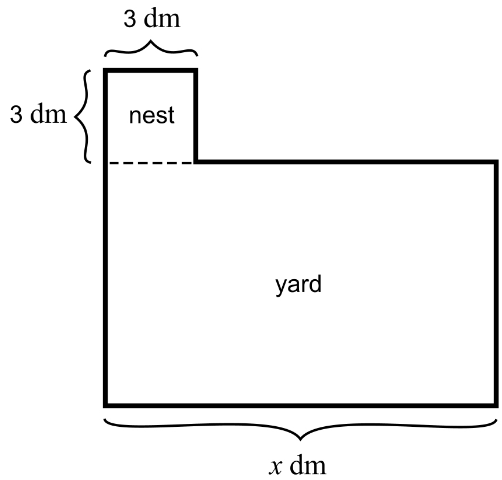


a) Find $f'(-1)$ *Only answer is required* (1/0)

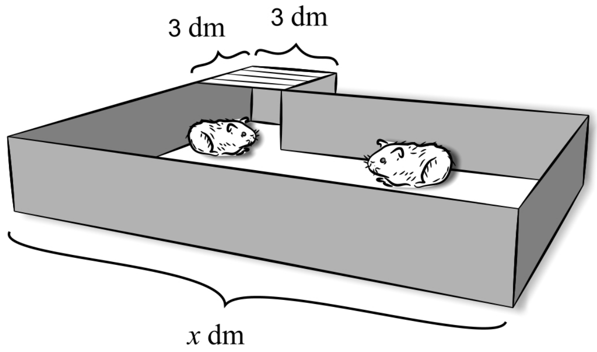
b) For what value of x is it true that $f'(x) = -4$? *Only answer is required* (0/1)

4. Solve the equation $4x^3 - 20x = 0$ (2/0)

5. Sigrid is making an enclosure for her guinea pigs. It will consist of two parts. One open rectangular part where the guinea pigs can hop around (a yard) and a quadratic part with a plastic roof (a nest). In the nest, the guinea pig will be protected from wind and weather.



Enclosure seen from above



Enclosure seen at an angle

The nest will have sides with a length of 3 dm. The opening will be placed according to the figure. Sigrid has 50 dm of fencing that she will use both for the yard and for the nest. She finds out that the area of the enclosure (the yard and the nest) is determined by

$A(x) = 22x - x^2 + 9$ where x dm is the breadth of the yard and $10 \le x \le 20$

a) Sigrid wants the enclosure to have the largest possible area. Use the derivative and calculate x so that the enclosure has the largest possible area. (3/0)

b) Show that the area of the enclosure is determined by
$A(x) = 22x - x^2 + 9$ where x dm is the breadth of the yard. (0/2)

6. Simplify the following expressions as far as possible.

a) $(a+b)^3-(a^3+b^3)$ (1/0)

b) $\lg 3a - \lg a$ (0/1)

c) $\dfrac{a}{a-3}+\dfrac{3}{3-a}$ (0/1)

7. It is true for the function f that $f(x)=x^2+x$

a) Calculate $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}$ (1/1/¤)

b) From what you have calculated in task a), explain what it can tell you about the graph of the function *f*. (0/1)

8. Which of the six numbers below is smallest? Justify your answer.

ln e lg e e 1 ln 10 lg 10 (1/1/¤)

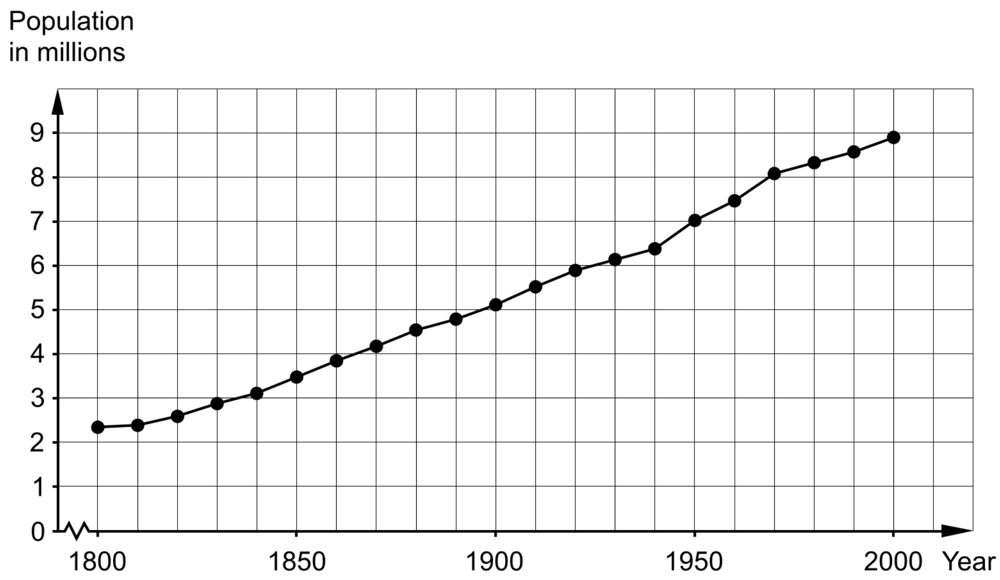
9. It is true for the quadratic function f that $f(x)=k(x-a)(x-b)$ where $k\neq 0$
Show algebraically that $f'(a)+f'(b)=0$ (1/2/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. The graph of the function $f(x) = 10x^4$ and the graph of the function $g(x) = \mathrm{e}^{2x}$ have different gradients when $x = 0.5$
Which graph has the larger gradient when $x = 0.5$? (2/0)

11. The diagram below shows the population growth in Sweden over 200 years.



Estimate the average population growth per year between 1800 and 2000. (2/0)

12. Ali's mother regularly saves money for Ali. She deposits SEK 3000 into a bank account every birthday, starting the year Ali turns 1 and finishing the year he turns 18.

What amount can Ali withdraw from his bank account on his 18^{th} birthday, right after the final deposit assuming that the yearly interest rate is 1.5 %? (2/0)

13. For the function f it holds that $f'(x) = 4x - 8$
Kalle says that the graph of the function has a maximum point.

Is Kalle right? Explain. (0/1)

14. Below is an extract from an article published in ST 2008-02-18 (the online version of Sundsvalls Tidning)

> **Sweden has 2 600 bears**
>
> DELA
>
> SUNDSVALL (ST) 2008-02-18 03:00
>
> There were 2600 bears in Sweden according to measurements from 2006. The population lives mainly from Värmland, Dalarna and Gästrikland and futher north.
>
> The bear was endangered in the 1930s. At that time there were only 130 bears in Sweden.

Depending on how the information in the article is interpreted there will be different results when calculating the yearly percentage increase.

Assume that the increase in the number of bears is exponential during the period from the 1930s to 2006 and that the number of bears is estimated at the same time every year. Determine the largest yearly percentage increase that can be calculated from the figures in the article. (0/2)

15. A certain medicine starts to work immediately after it is taken and the amount of medicine in the blood decreases exponentially. At 10.00 am, one patient is given a dose that gives 160 mg of medicine in the blood. Two hours later the amount of medicine is measured again to 127 mg.

a) The medicine will not have the intended effect when the amount of medicine in the blood is less than 40 mg. When must the patient have the next dose in order to avoid the amount of medicine in the blood getting too low? (0/2)

b) Calculate the speed at which the amount of medicine decreases at 12.00 pm. (0/2)

16. It is true for the point P that $P = (0,\ a)$

Show for **what** values of a the curve $y = -3x^2 + 6x$ has a tangent that passes through point P. (0/2/¤)

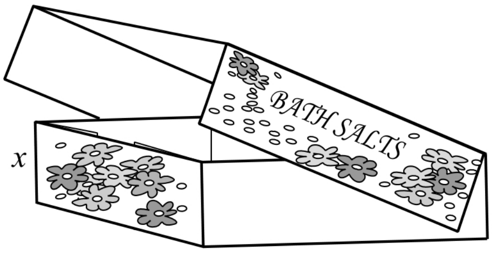
When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

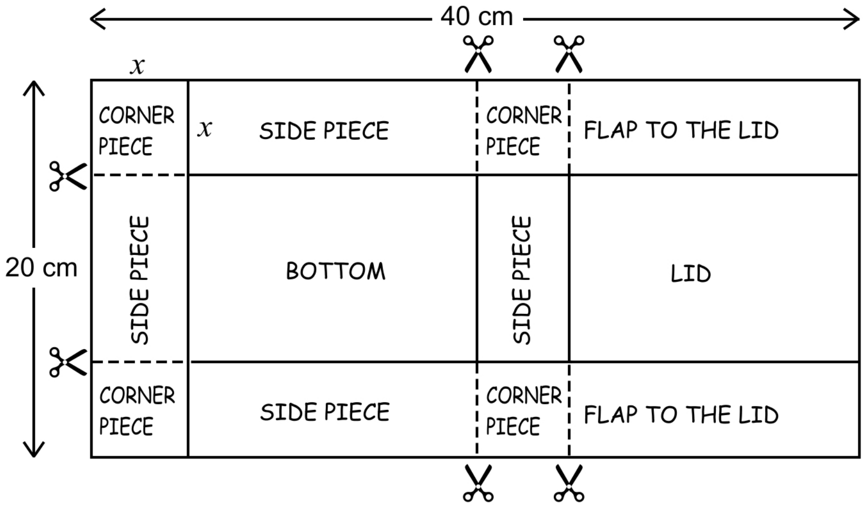
17. Agnes has started a company within ”Junior Achievement” (JA). She has received a large amount of bath salts from an official receiver and is planning to sell the bath salts in nice boxes that she will make herself. Agnes wants to know how the design of the box affects the economic results. She thinks for a while and then decides on the following:

MY BUSINESS IDEA:

Sale of bath salts in boxes that I will make myself.



Sketch:

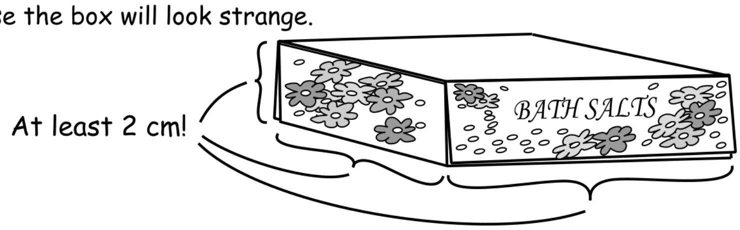


THE BOXES HAVE TO SATISFY:

SHAPE: The box should consist of a bottom, four side pieces that are turned up, four corner pieces that are folded in and a lid. The lid has two flaps that should be turned down over the box.

MATERIAL: I will use a sheet of paper measuring 20 cm × 40 cm.

DEMAND: Each dimension of the box has to be at least 2 cm. Otherwise the box will look strange.



ECONOMY:

INCOME: I can sell the bath salts for 0.03 SEK/cm^3.

COST: I have to pay 12 SEK for each sheet of paper.

AGNES SP 3A

Agnes first constructs a box where x is 6 cm, see figure.

- What is the volume of this box?

The cost of one box is constant, SEK 12, but the income depends on the design of the box. Agnes writes down the cost C and the income I as a function of x:

$I(x) = 12x - 1.8x^2 + 0.06x^3$ and $C(x) = 12$

- Show how Agnes came to the conclusion that the income per box can be written
  $I(x) = 12x - 1.8x^2 + 0.06x^3$

The economic result is the difference between the income and the cost.

- Help Agnes to examine, as fully as possible, between which values the economic result per box may vary. (3/3/¤)