NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2011

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 90 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 8 problems and **Part II** consists of 9 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 8 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 46 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 12 points. Pass with distinction: 25 points of which at least 7 ”Pass with distinction” points. Pass with special distinction: 25 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Differentiate

a) $f(x) = 2x^3 - 5x$ *Only answer is required* (1/0)

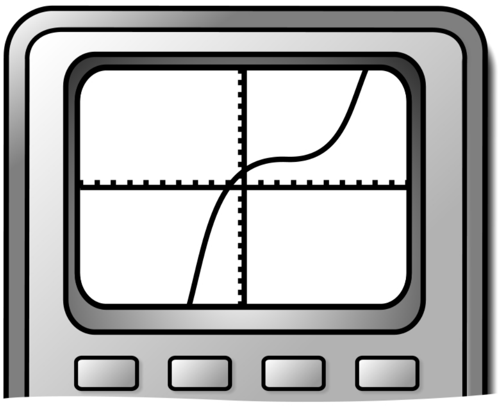
b) $g(x) = e^{2x} + 7$ *Only answer is required* (1/0)

2. Solve the equations and give exact answers.

a) $6x^5 = 24$ *Only answer is required* (1/0)

b) $6^x = 24$ *Only answer is required* (1/0)

3. Kalle has been given the task of finding out what the graph to a certain cubic function looks like. He plots the graph on his calculator, see figure.



He says: ”It looks like the graph has a saddle point!”

Can Kalle, based on the image he sees on his calculator, be sure the graph has a saddle point? Justify your answer. (1/0)

4. Solve the equations

a) $4x^3 - x^5 = 0$ (2/0)

b) $\lg x + \lg 2 = 3$ (0/2)

5. It holds for the function f that $f(x) = \mathrm{e}^x$
Which of the following statements A-E is correct? *Only answer is required* (1/0)

A. f has the property that $f'(x) = f(x)$ for all x

B. f is an exponential function with base e where $\mathrm{e} \approx 1.718$

C. f has a graph that passes through the point $(1, 0)$

D. f is decreasing for $x < 0$ and increasing for $x > 0$

E. f has the property that $f'(1) = 0$

6. Simplify the following expressions as far as possible.

a) $\dfrac{(17+x)^3}{(x+17)^2}$ (1/0)

b) $\dfrac{(8-2x)^3}{(4-x)^4}$ (0/1)

7. Bertil deposits SEK *B* into an account with a yearly interest rate of *r* %. The interest rate remains unchanged during the time the money is in the account. The balance of the account is SEK *K*.

Write down a function that indicates how the balance *K* depends on *B* and *r* if the money is in the account for three years. *Only answer is required* (0/2)

When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

8. In this problem you will investigate a property of the extreme points of the functions given by $f(x)=3x^2-\dfrac{kx^3}{3}$ where k is a constant.

The table shows the coordinates of the extreme points of the function f for some different values of k.

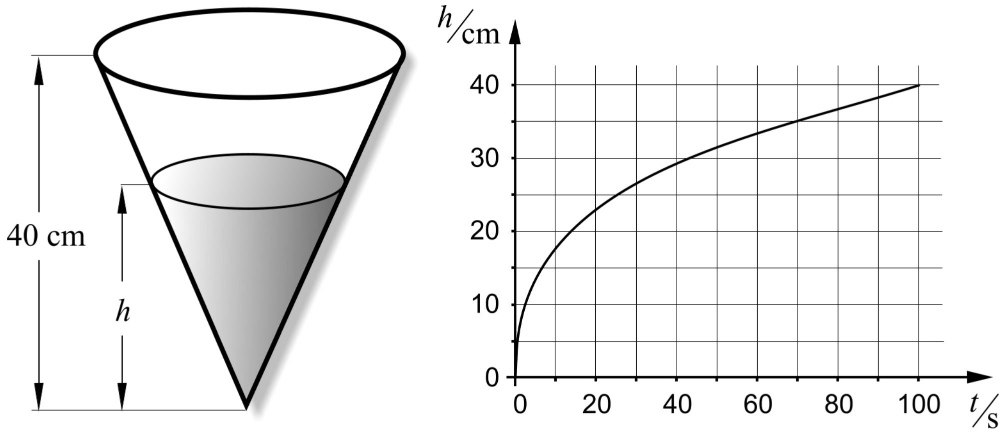
k	Extreme point/s
-2	$(0,0)$ and $(-3,9)$
-1	$(0,0)$ and $(-6,36)$
0	$(0,0)$
1	$(0,0)$ and $(6,36)$
2	$(0,0)$ and $(3,9)$
3	

- Complete the table by calculating the coordinates of the extreme points when $k=3$, that is when the function is given by $f(x)=3x^2-x^3$
- Study the extreme points in the table. They lie on the graph of another function that we call g. Write down the function $g(x)$ that you find likely and justify your answer.
- Investigate as completely and in as much detail as possible whether it always holds that the extreme points to $f(x)=3x^2-\dfrac{kx^3}{3}$ lie on the graph of the function g. (4/3/¤)

Part II

This part consists of 9 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

9. A conical container is filled with water. The diagram shows how the height *h* in centimetres of the water level depends on the time *t* seconds.



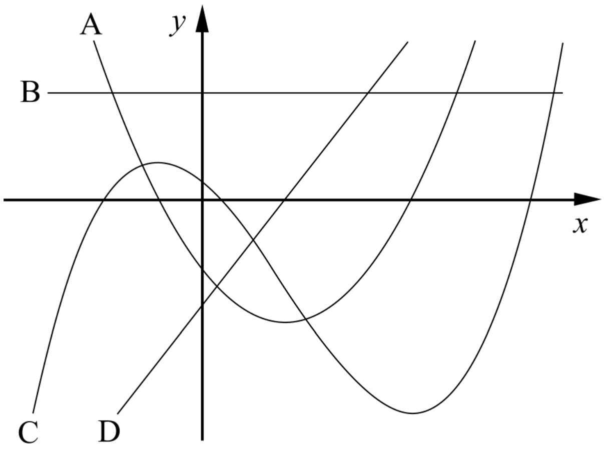
a) It takes 100 seconds to fill the container. At what average velocity does the height *h* of the water level increase during the time period $10 \le t \le 100$? (2/0)

b) Interpret what $h'(50) = 0.20$ means in this context, that is as the cone is filled with water. (0/1)

10. At the end of each year, Lisa's parents deposit SEK 1000 into a bank account. The yearly interest rate is 3 %. Her parents make the first deposit the year Lisa turns 2 years old and they then deposit money every year until the year she turns 30. How much money will there be on the bank account immediately after the last deposit? (2/0)

11. Give an example of a rational expression that is not defined for $x = 3$ and which has the value 2 when $x = 0$ *Only answer is required* (0/1)

12. The figure shows the graphs of the four functions *p*, *q*, *r* and *s*.



The function *p* is a polynomial function of the third degree. The other functions are formed by a repeated differentiation of *p*, that is:

$q(x) = p'(x)$

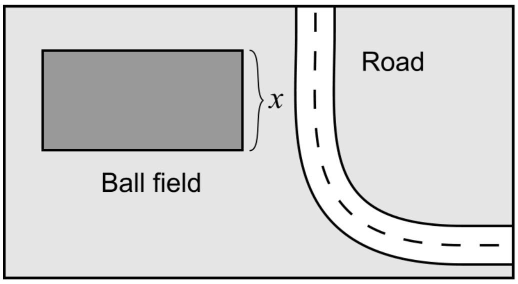
$r(x) = q'(x)$

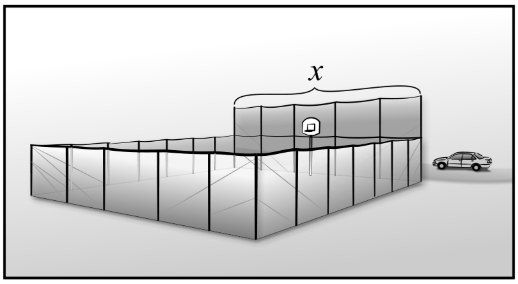
$s(x) = r'(x)$

Match the functions *p*, *q*, *r* and *s* with the corresponding graphs A, B, C and D.

Only answer is required (0/1)

13. Garfesta municipality is going to build a ball field. It will be rectangular with a surrounding fence. To prevent the balls from ending up on the road they decide to build a higher fence on the side closest to the road, see figure.





The municipality has decided that the fence may cost a maximum of SEK 6600. The cost of the lower fence is SEK 75 per metre and the higher SEK 225 per metre. The cost for poles and gates is included in the price of the fence.

If the municipality use SEK 6600 for the fence, the area A m^2 of the ball field can be calculated with the relation below:

$A(x) = 44x - 2x^2$ where x m is the length of the side of the ball field that is closest to the road.

a) Use differentiation to determine the value of x that gives the maximum area of the ball field. (3/0)

b) Show that the area of the ball field A m^2 can be written $A(x) = 44x - 2x^2$ (0/2/¤)

14. Earthquakes are more common in Sweden than one may think, but most of the time they are minor ones that are hardly felt. By using the Richter magnitude scale, the size of an earthquake is indicated with magnitude M .

The magnitude M is given by the relation $M = \frac{2}{3}(\lg E - 4.84)$

where E is the released energy measured in the unit joules, J.

a) On 16 December 2008, Skåne was hit by an earthquake, large by Swedish standards. The energy $2.75 \cdot 10^{11}$ J were released.
What magnitude does this correspond to on the Richter magnitude scale? (1/0)

b) The largest measured earthquake in Sweden is called Kosteröskalvet, the Kosterö quake, and it occurred on 23 October 1904. The magnitude was 5.4 on the Richter magnitude scale. How much energy was released during the Kosterö quake? (2/0)

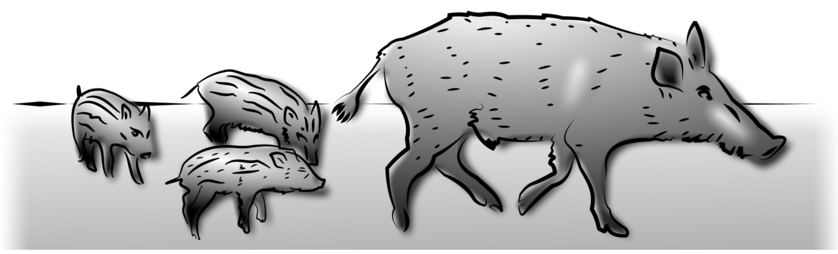
c) Assume that there are two different earthquakes, one with a magnitude of 5 and another with a magnitude of 7. How many times larger is the amount of released energy during the larger earthquake compared to the amount of released energy during the smaller one? (0/1)

d) Assume once again that there are two different earthquakes, one with a magnitude that is two units larger than the other. Investigate generally how many times larger the released energy is during the larger earthquake compared to the released energy during the smaller one. (0/1/¤)

15. The number of boar in Sweden is increasing heavily. The following quotation is from a report:

> In the year 2007 the number of boar was estimated to be approximately 60000 from Skåne and up to the Dalälv, which yet is the northern limit for their spread.
> ---
> From 1990 to 2007 the population of boar has increased so strongly that the number of boar in Sweden has doubled every five-six years.
>
> Source: Svensk Naturförvaltning AB (2008), Rapport 04, *Vildsvin, jakt och förvaltning*

Assume that the number of boar is estimated at the same time of year every year.



Based on the quotation different prognoses can be made on the number of boar in Sweden in the future.

How many boar can there be in Sweden 2011, *at most*, if the increase continues at the same rate as described above? (0/3)

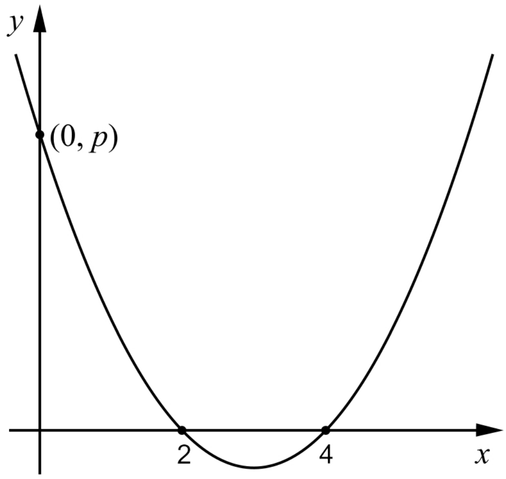
16. The value of the derivative of a function *f* at a given point *P* is given below.

$$\lim_{h\to 0}\frac{((2+h)^5+3)-(2^5+3)}{h}=80$$

a) Give the function f *Only answer is required* (0/1)

b) A tangent is drawn at point *P*. Determine the equation of the tangent. (0/2)

17. Below is the graph of a quadratic function with zeros $x_1 = 2$ and $x_2 = 4$, see figure. The graph intersects the *y*-axis at the point $(0, p)$.



Assume that we draw a tangent to the graph at point $(0, p)$. Calculate the gradient of this tangent expressed in *p*. (0/2/¤)