NATIONAL TEST IN MATHEMATICS COURSE C
SPRING 2012

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 10 problems and **Part II** consists of 6 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

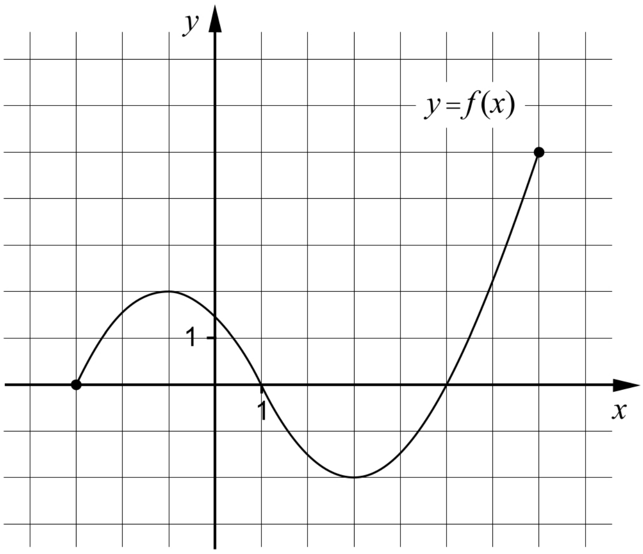
Lower limit for the mark on the test

Pass:	12 points.
Pass with distinction:	25 points of which at least 7 ”Pass with distinction” points.
Pass with special distinction:	25 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. The figure shows the graph of the function f. The function is defined on the interval $-3 \le x \le 7$



Use the graph to answer the following questions:

a) For what values of x is $f'(x) = 0$? *Only answer is required* (1/0)

b) What is the largest value of the function? *Only answer is required* (1/0)

c) What is the smallest value of the function? *Only answer is required* (1/0)

d) For what value of x is it true that both $f(x) \le 0$ and $f'(x) \ge 0$?
Only answer is required (0/1)

2. Simplify the expression $\dfrac{5x^2 - 20}{x - 2}$ as far as possible. (2/0)

3. It holds for the function f that $f(x) = x^6 + 3x^2 + 6x + 6$

a) Determine $f'(x)$ *Only answer is required* (1/0)

b) Write down another function g with the same derivative as function f
Only answer is required (1/0)

4. Solve the equations. Give exact answers.

a) $\lg x = 5$ *Only answer is required* (1/0)

b) $\ln \mathrm{e}^x = 5$ *Only answer is required* (1/0)

5. a) Simplify the expression $x^2(x-9)+9(x^2-x)$ (1/0)

b) Solve the equation $x^2(x-9)+9(x^2-x)=0$ (2/0)

6. Investigate and specify the largest of the following numbers.

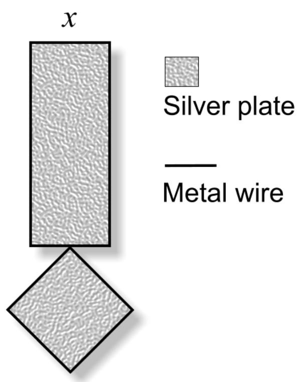
$\lg 1000$ $\quad$ e $\quad$ π $\quad$ $\ln \mathrm{e}^3$ $\quad$ $\sqrt{8}$ (2/0)

7. For a function f it holds that $f'(x) = x^2 + 2x - 5$
Calculate the values of x for which the graph of the function f has a tangent with gradient 3 (0/2)

8. For the function p it holds that $p(r) = \dfrac{r^2+1}{a}$, where a is a constant ($a \neq 0$).
Determine $p'(a)$ (0/2)

9. Sofie has a company within ”Junior Achievement” (JA) and she is going to make jewellery from metal wire and silver plate. She has 150 metal wires where the length of each wire is 28 cm.

From each wire, she wants to make jewellery in the shape of a rectangle and a square. Sofie plans to cover the jewellery with silver plate, see figure.



Sofie decides that the length of the rectangle should be three times the width.

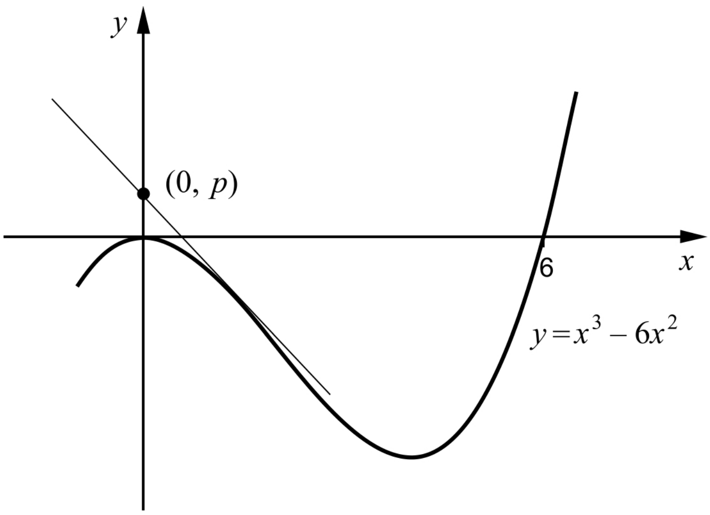
Out of a 28-cm metal thread, the area of the jewellery A cm^2 can be written as $A(x) = 7x^2 - 28x + 49$ where x is the width of the rectangle in cm.

a) Since silver plate is expensive, Sofie wants the area of the jewellery to be as small as possible. Use the method of differentiation and determine the width x of the rectangle so that the area A becomes as small as possible. (3/0)

b) Explain why the domain of definition of A is $0 < x < \frac{7}{2}$ when the length of the rectangle is three times the width. (0/1/¤)

c) Show that the area A as a function of the width x can be written $A(x) = 7x^2 - 28x + 49$ where the area is measured in cm^2 and the width in cm. (0/1/¤)

10. The figure below shows the graph of $y = x^3 - 6x^2$



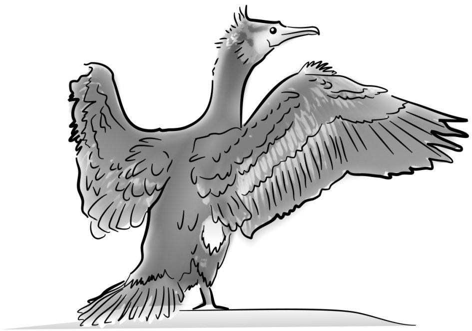
On the interval $0 \leq x \leq 6$, the curve has an infinite number of tangents that intersect the y-axis at the point $(0, p)$. What are the possible values of p? (0/3/¤)

Part II

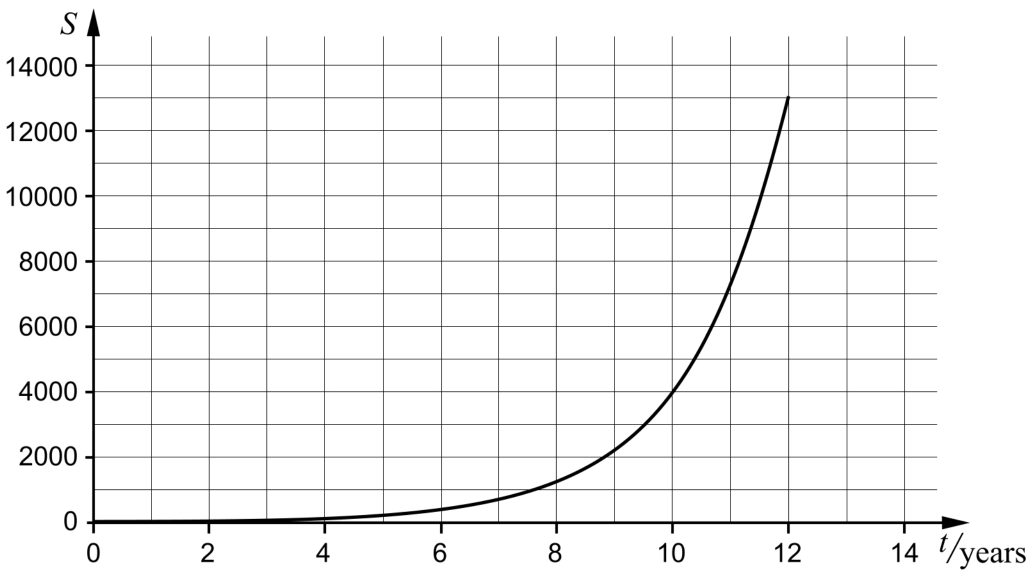
This part consists of 6 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11. $a_1 + 10 + a_3 + a_4$ is a geometric sum. Give an example of possible values of the terms a_1, a_3 and a_4 (1/0)

12. The Great Black Cormorant is a handsome seabird that started to nest in the Finnish archipelago in 1996.



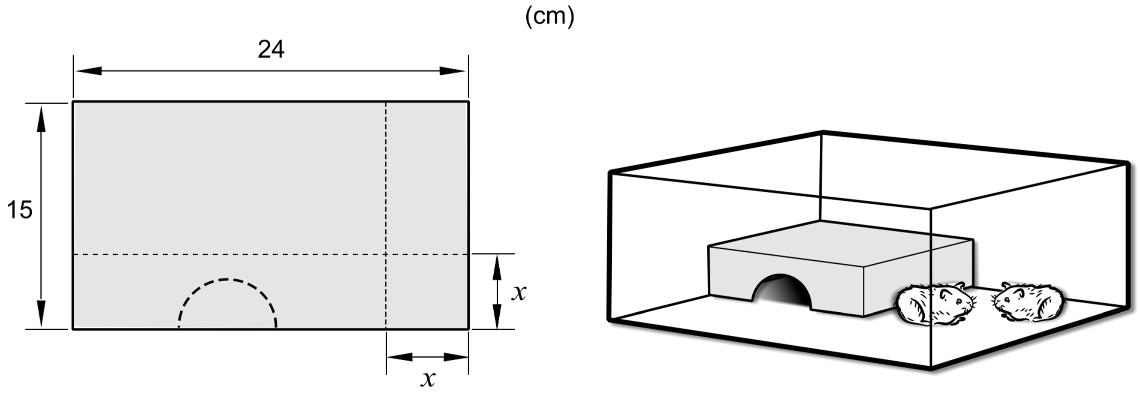
The diagram below shows the number of nesting couples S during the time t years where $t = 0$ corresponds to year 1996.



a) Use the graph and determine an approximate value of $S'(10)$ (1/0)

b) Give an interpretation of what $S'(10)$ means to the number of nesting couples in this context. (0/1)

13. Elias has a rectangular tin sheet which he will use to build a den for his dwarf hamsters. The den will be placed in a cage and to save material he plans to use the walls of the cage for two of the walls of the den. Elias is going to cut out a quadratic piece from one of the corners of the tin sheet and then fold the tin sheet into a den, see figure.



What measures should the den have to get the largest possible volume? (0/3)

14. The most expensive stamp in the world is the Treskilling Yellow. There is only one copy of the stamp and it is valuable because it was given a yellow colour instead of a green one in the printing process.

In 1885 the stamp was sold to a stamp collector for SEK 7.

According to Statistics Sweden, consumer prices have had a yearly average increase of 3.3 % between years 1885 and 2010.

a) How much would the stamp have been worth in 2010 if its value had followed the consumer prices? (1/0)

In 1928 the stamp was sold for SEK 36340 and in 1998 it was sold once again, now for SEK 15 million. In 2010 the stamp was sold for a secret sum.

b) How much should the stamp have cost in 2010 if its value had followed the same yearly per cent development as during the period from 1928 to 1998? (0/3)

15. The function f is a polynomial function of degree four. f', the derivative of the function, has only two zeros. The table below shows the value of the derivative $f'(x)$ for some different values of x.

x	-2	-1	0	1	2	3	4
$f'(x)$	-32	0	8	4	0	8	40

a) For what value of x does the graph of f have a minimum?
Only answer is required (1/0)

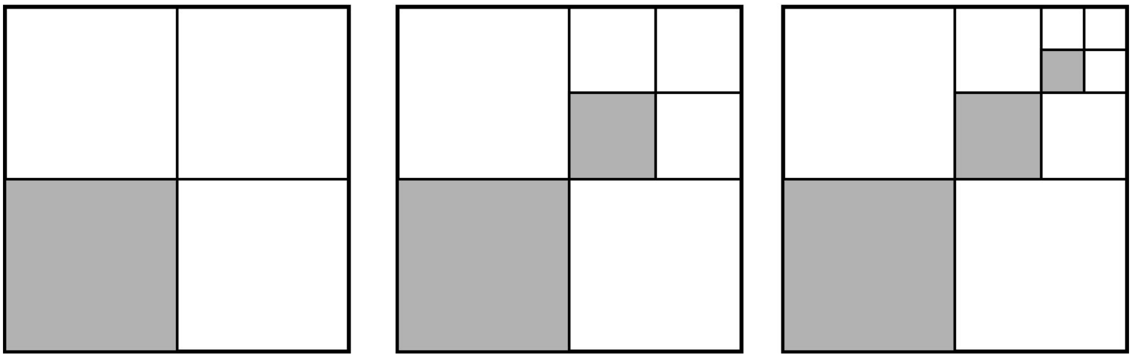
b) There are many functions that satisfy the above conditions. Investigate how many zeros the graph of the function f can have. (0/2/¤)

When assessing your work with this problem, the teacher will take into consideration:
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

16. The figure below shows three squares A, B and C which each consists of a number of smaller squares. Some of the squares have been shaded.

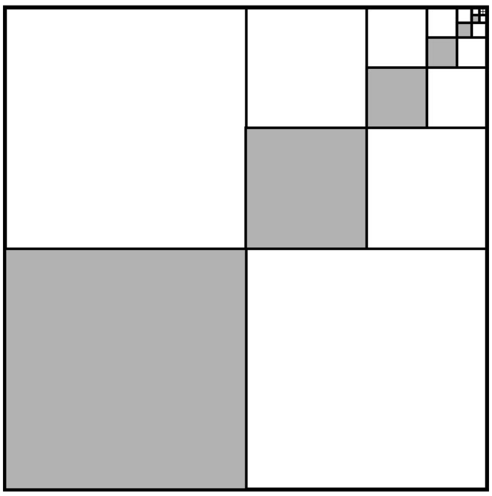
- Determine the *proportion* of the shaded area of each of the squares A, B and C.

A B C



Now take a look at square D. Assume that we introduce more and more squares of decreasing size into this square and that we shade some of them according to the same principle as above.

D



- Investigate and describe what then happens to the *proportion* of the shaded area in square D. Use your knowledge of geometric sums. (2/3/¤)