# NATIONAL TEST IN MATHEMATICS COURSE C
# SPRING 2013

## Directions

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| Test time | 240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.** |
| Resources | **Part I:** ”Formulas for the National Test in Mathematics Course C.” *Please note that calculators are not allowed in this part.*<br>**Part II:** Calculators, also symbolic calculators and ”Formulas for the National Test in Mathematics Course C.” |
| Test material | The test material should be handed in together with your solutions.<br>Write your name, the name of your education programme/adult education on all sheets of paper you hand in.<br>*Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.* |
| The test | The test consists of a total of 18 problems. **Part I** consists of 10 problems and **Part II** consists of 8 problems.<br>For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.<br>Problem 10 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.<br>Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions. |
| Score and mark levels | The maximum score is 44 points.<br>The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. |

Lower limit for the mark on the test

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|---|---|
| Pass: | 12 points |
| Pass with distinction: | 25 points of which at least 7 ”Pass with distinction” points. |
| Pass with special distinction: | 25 points of which at least 14 ”Pass with distinction” points. You also have to show most of the ”Pass with special distinction” qualities that the ¤-problems give the opportunity to show. |

# Part I

**This part consists of 10 problems that should be solved without the aid of a calculator.** Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

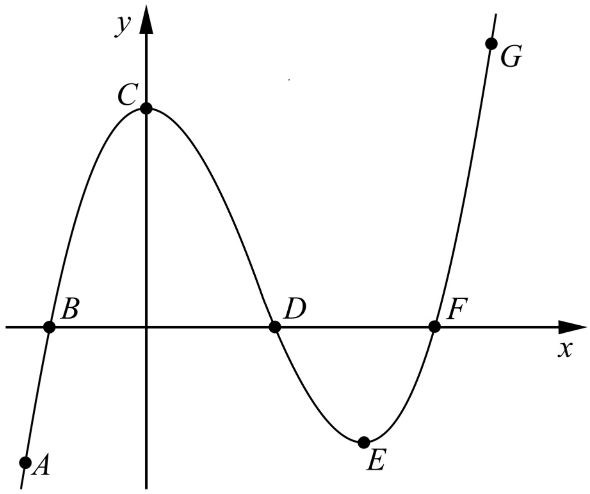
**1.** Differentiate

a) $f(x) = 2x^4 + 3x^2$ *Only answer is required* (1/0)

b) $f(x) = e^{-5x}$ *Only answer is required* (1/0)

c) $f(x) = \frac{x}{7}$ *Only answer is required* (1/0)

**2.** The figure below shows the main features of the graph $y = f(x)$




a) Specify a point on the graph where $f'(x) < 0$ *Only answer is required* (1/0)

b) How many solutions are there to the equation $f(x) = 0$? *Only answer is required* (1/0)

**3.** Solve the equation $(x-1)(x+2)(x-3) = 0$ *Only answer is required* (1/0)

**4.** Simplify $(x+1)^3 - (x+1)^2$ as far as possible. (2/0)

**5.** Calculate

a) $\lg 10000 - \lg 100$ (1/0)

b) $10^{\lg 3} + 10^3$ (1/0)

**6.** The graph of $f(x) = x^3 - 3x$ has a maximum point. Calculate the coordinates of this maximum point. (3/0)

**7.** The equation $a \cdot e^x = b$ contains two constants, *a* and *b*. Suggest a value *a* and a value *b* so that $x = \ln 3$ is a solution to $a \cdot e^x = b$ (0/2)

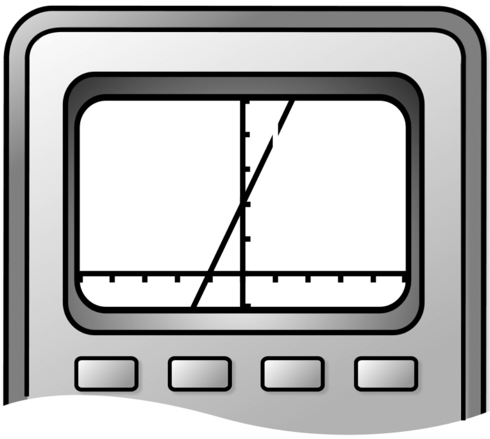
**8.** In the geometric sum $2 - 2 \cdot 0.1 + 2 \cdot 0.1^2 - 2 \cdot 0.1^3 + 2 \cdot 0.1^4 - \ ... \ + 2 \cdot 0.1^{48}$ the terms are alternately positive and negative.

a) How many terms are there in this geometric sum?
*Only answer is required* (1/0)

b) Investigate whether the value of the geometric sum is larger or smaller than 2 (0/1/¤)

**9.** Ebba has drawn the graph of $f(x) = \dfrac{2x^2 - 2}{x - 1}$ with her graphic calculator, see figure.

She can neither understand why the graph is linear, nor why there is a break.



a) Explain why the graph is linear. (0/2)

b) Explain why there is a break in the graph. (0/1)

**When assessing your work with the following problem, the teacher will take into consideration:**

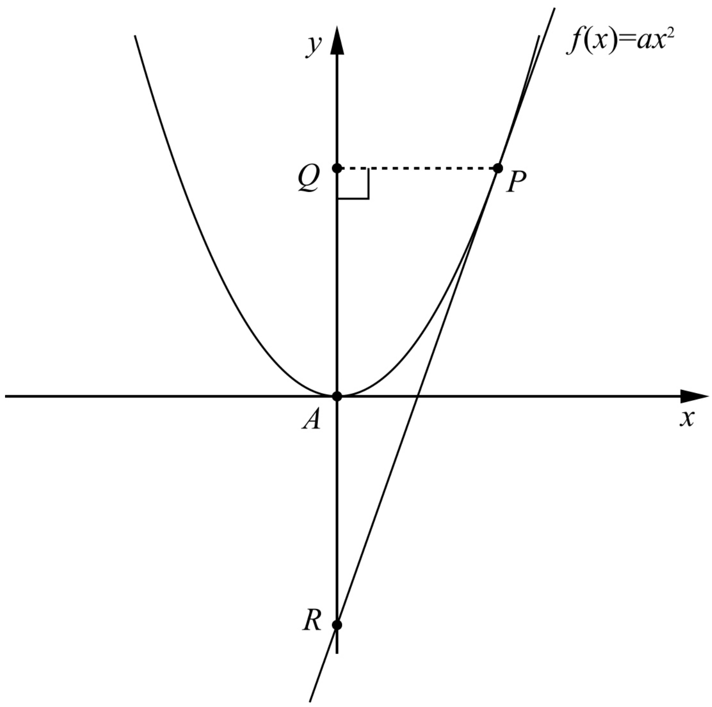
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

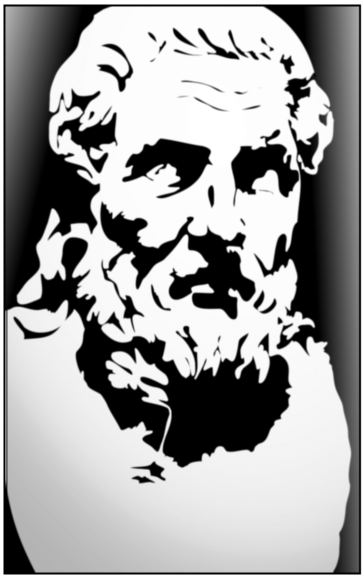
**10.** Apollonius developed a method for constructing the tangent line to the parabola $f(x) = ax^2$ ($a > 0$) at a point *P*. Apollonius' method can be described as follows:

1. The point *Q* is marked on the *y*-axis so that *PQ* is perpendicular to the *y*-axis.

2. The point *R* is marked on the *y*-axis so that the distance *AR* is equal to the distance *AQ*, where point *A* is the origin.

3. The straight line that passes through points *R* and *P* is now a tangent to the parabola at point *P*.



Apollonius was a Greek mathematician who lived 262 BC-190 BC. He produced many mathematical writings and was known as 'The Great Geometer'.

Show that Apollonius' method for constructing tangents works

- for the function $f(x) = x^2$ where the point of tangency is $P =$ (2, 4)
- for all types of quadratic functions of the form $f(x) = ax^2$ where the point *P* does not coincide with point *A*. (2/3/¤)

# Part II

**This part consists of 8 problems and you may use a calculator when solving them.**
Please note that you may begin working on Part II without a calculator.

**11.** A baked potato is placed in an oven. The oven has been heated to 200 °C. The temperature of the potato $T$ °C increases according to the function

$$T(t) = 200 - 179 \cdot 0.990^t$$

where $t$ is the time in minutes from when the potato is placed in the oven. The potato is considered cooked when it has a temperature of 100 °C.

a) What is the temperature of the potato when it is placed in the oven? (1/0)

b) How long does it take for the potato to be cooked? (2/0)

**12.** Let $f(x) = 2\sqrt{x-2}$ where $x \geq 2$

a) Write down a suitable difference quotient and use it to determine an approximate value of $f'(3)$ (1/0)

b) The derivative to the above function is $f'(x) = \dfrac{1}{\sqrt{x-2}}$

Calculate an exact value of $f'(3)$ (1/0)

c) Draw a figure and use it to explain in words what you have calculated in problems a) and b) respectively. (0/2)

**13.** Olle is planning to save up SEK 50 000 by depositing the same amount of money into a savings account at the end of each year. He is going to make 10 deposits. Olle has found out that the bank applies an annual interest rate of 3 %.

How much does he have to deposit each year, if he wants the balance to be SEK 50 000 immediately after the last deposit? (1/2)

**14.** The picture shows the share price development (in SEK) for a Beijer share over the course of one year. (Source: OMX)


Assume that the percentage decrease has been the same each month. Calculate the percentage decrease in value of the share per month during the period April 1 2006 to August 1 2006. (0/2)

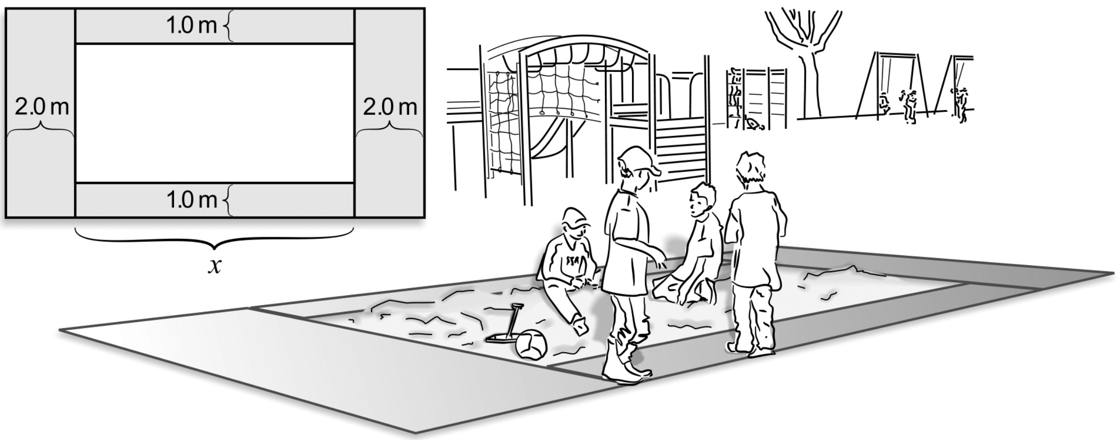
**15.** Give an example of a function *f* that has the property $f'(0) = 1$

*Endast svar fordras* (0/1)

**16.** For the function $f$ it holds that the derivative $f'(x) = 2x$

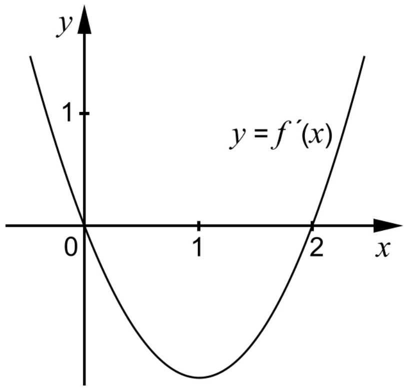
Calculate the value $\lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$ (0/1)

**17.** The Larch House Association are going to build a new rectangular sandpit. They have enough sand for a sandpit with an area of 30 $m^2$. They want to have rubberized asphalt around the sandpit. The rubberized asphalt should be 1.0 metre wide along the long sides, and 2.0 metres wide along the short sides, so there are enough room to put some benches on the short sides.




The rubberized asphalt is expensive to buy and spread out. Help The Larch House Association to determine the value of $x$ that gives the smallest area covered by rubberized asphalt. (0/3/¤)

**18.** The figure shows the graph of the derivative $y = f'(x)$. The derivative is a quadratic function.




There are several functions with derivatives whose graphs have the same appearance as the one in the figure.

Sketch the graphs of some of these functions in the same coordinate system. Justify why your graphs have this appearance. (0/2/¤)