

Directions

Test period	3 May - 15 May 1996.
Test time	180 minutes without a break.
Resources	Non-programmable calculator and formula tables. A formula sheet is attached to the examination paper.
Test material	Examination material should be handed in together with the solutions. Write your name, gymnasium programme/adult education and date of birth on the papers you hand in.
Test	The test is made up of 13 questions. The problems which are short answer problems (problems which can earn 1 point) require, for the most part, only an answer. Most of the problems are long answer problems (problems which can earn 2 or more points). For these problems it is not enough with just a short answer. They also require • that you write down what you do and explain your train of thought, • that you draw figures when necessary and • that you write down all your computations Try all of the problems. It can be relatively easy, even at the end of the test, to earn some points for a partial solution or presentation.
The grading levels	The teacher responsible will explain the grade levels which are required for "Passed" and "Passed with Distinction". The test can earn a maximum of 42 points.

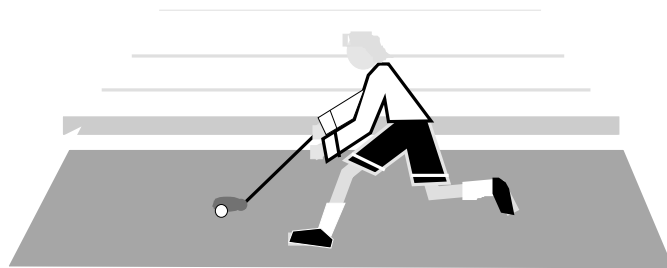
1. a) Differentiate $f(x) = 3x^4 - 4x + 3$ (1p)
- b) Evaluate $f'(2)$ (1p)

2. a) Differentiate $g(x) = \frac{x^2}{4}$ (1p)
- b) Differentiate $h(x) = 2e^{3x}$ (1p)
- c) Solve the equation $\ln x = 2$ (1p)
- d) Calculate the geometric sum
 $2000 + 2000 \cdot 1.07 + 2000 \cdot 1.07^2 + \dots + 2000 \cdot 1.07^{49}$ (1p)

3. Some employees of a company which sells floor ball sticks, have examined the company's costs in relation to purchase and storage.

If the company buys a large number of sticks every order, then there will be few orders. This leads to, amongst other things, lower administrative costs but higher storage costs.

If instead the company buys less sticks with every order, then the orders will occur more frequently. Then the company will have higher administrative costs, but lower storage costs.



They concluded that the total annual cost $K(x)$ can be expressed by the formula

$$K(x) = 0.05x^2 - 15x + 4125 \quad 100 \leq x \leq 300$$

where $K(x)$ is the total annual cost in crowns and x is the number of floor ball sticks the company requests per order.

Use differentiation to decide how many floor ball sticks the company should purchase per order so that the annual cost will be as small as possible.

(3p)

4. A person buys shares for 20 000 kr. Five years later she sells them for 56 720 kr.

What is the corresponding yearly percent increase in her capital?

(3p)

- 5 A sports club has 500 members. The committee plans to build a club house. As the issue is of such importance to the club, the committee will only build the club house if a majority of the club's assembled members can be expected to support the plans.

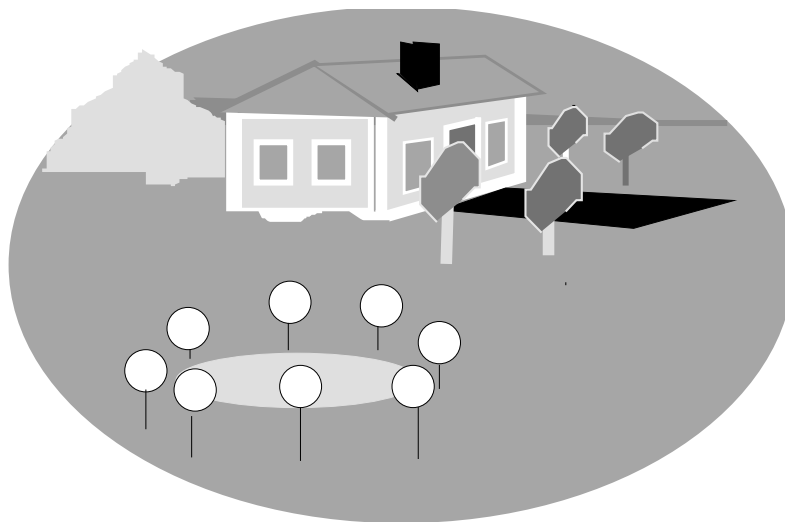
Therefore a meeting was arranged for the members. Unfortunately only 185 members attended. Of these members, 125 wanted the club house to be built and the rest did not.

As so many members did not participate in the meeting, the committee did a supplementary survey. They rang 75 randomly chosen members who had not attended the meeting. Of these, 26 answered yes and 49 no.

Do you think that the committee should decide to build the club house?

Take into consideration the result of both the meeting and the supplementary survey.

(3p)

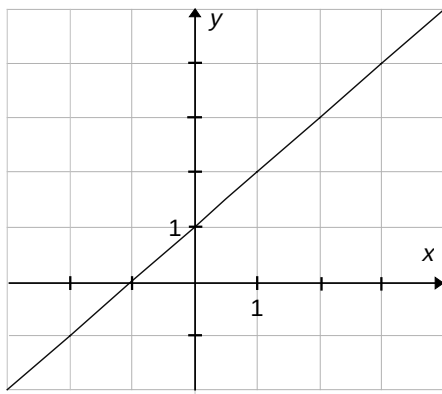


6. The functions f , g and h have the following properties:

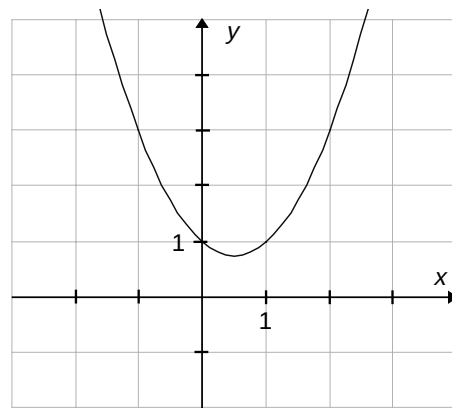
- $f(2) = 3$ and $f'(0) = 1$
- g is increasing for all values of x and $g(0) = -1$
- $h'(2) = 0$ and $h(0) = 1$

The graphs of each of the functions f , g , and h are amongst the graphs below. Decide which graph corresponds to each function. *Only an answer is required.* (3p)

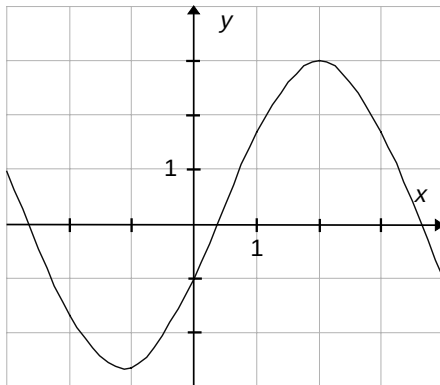
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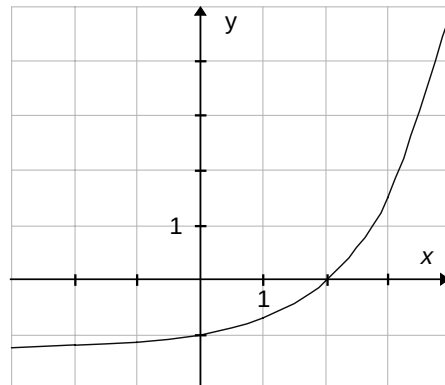
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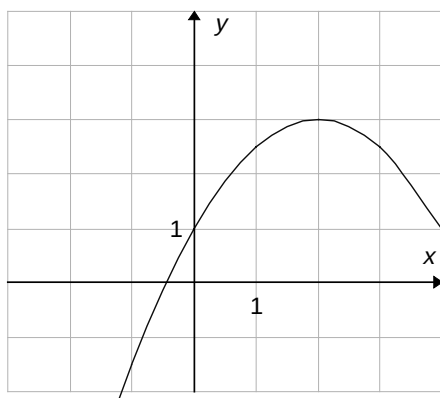
C



D



E



7. Many people interested in films think that the price of cinema tickets has increased a lot over the last few years.

Using the table below, compare the price of a cinema ticket with the change in the Consumer Price Index during the period 1992-1995. You should consider each year during the period under question, and your comparison should include both calculations and comments. (3p)

Year	1992	1993	1994	1995
Average price of cinema ticket (crowns each)	60.00	61.80	63.80	63.80
CPI (Base year 1980)	234.9	244.3	250.4	256.0

(The information in the table was provided by the Statistics Central Office.
CPI = consumer price index.)

8. In the equation $160\,000 \cdot 0.95^x = 50\,000$, x denotes the time in years.

- a) Formulate a problem that can be solved using this equation. (2p)
- b) Solve the equation and give an answer to the problem you have formulated. (2p)

9. A student wants to find the gradient of the curve $y = \sqrt{3x}$ at $x = 2$. As he can not differentiate $y = \sqrt{3x}$ he can not solve the problem by calculating the value of the derivative at $x = 2$.

Therefore he determines an approximate value of the derivative by calculating the rate of change quotient

$$\frac{\sqrt{3 \cdot 2.1} - \sqrt{3 \cdot 1.9}}{2.1 - 1.9}$$

Write down a new rate of change quotient which should give a better approximation to the derivative.

An answer consisting of just the new rate of change quotient is required..

(2p)

- 10.** The health clinic at a school with 700 students would like to know to what extent the pupils take part in sporting activities. Therefore they carried out a survey and reported the results in the following form.

Extent of pupils' participation in sporting activities

Aim: The aim of this survey is to determine to what extent pupils in the school take part in sporting activities.

Method: We asked 100 of the school's pupils to fill in the following questionnaire.

1. Are you a girl or a boy? Girl ☐ Boy ☐

2. How many hours a week do you take part in sporting activities?

None at all ☐ Less than ☐ At least ☐
4 h/week 4 h/week

Results: Not everyone filled in the questionnaire, but the results for those that did respond are given by the table below:

Extent of sporting activities	Boys	Girls
At least 4 h/week	34 %	33 %
Less than 4 h/week	53 %	55 %
Non at all	13 %	12%

Conclusion: 13% of the school's boys and 12% of the school's girls do not participate in any sporting activities at all.

Lisa is writing a special project about to what extent school pupils take part in sporting activities and wants to use the school health clinic's conclusion. Therefore she wants to find out if the survey was carried out properly but she realises that the report does not contain sufficient information for her to be able to do it. Therefore she decides that she will not be able to make a good evaluation of the quality of the survey.

Which parts of the research does Lisa need to know more about in order to be able to evaluate the quality of the research?

Why are these things important?

(4p)

11. A friend of yours, who is studying the same mathematics course as you, comes to you and says, "I don't understand a thing about this differentiation."

Help your friend by explaining what a derivative is.

Explain in as much detail as you can and in as many ways as you can.

You should neither derive nor write down the differentiation rules.

(4p)

12. For a given medicine to have the desired effect, a patient must have 15 mg of the medicine in his body.

If the total amount of the medicine is given in one dose, there is a risk of serious side effects. Therefore the patient is given small doses at hourly intervals. After 10 such equally sized doses the medication is stopped and the patient should then have 15 mg of the medicine in his body.

How large should the dose be if it is known that the medicine starts to work immediately and that 16% of it is broken down by the body per hour?

(4p)

13. The following is known about the function f :

- $f(7) = 3$ and
- $0.8 \leq f'(x) \leq 1.2$ for $7 \leq x \leq 9$

Determine the largest possible value of $f(9)$.

(3p)