

This material is confidential until the end of November 1999.

Directions

Test period	May 15 – June 3 1998.
Test time	180 minutes without a break.
Resources	Calculator and formula sheet. The formula sheet is attached to the test.
Test material	The test material should be handed in with your solutions. Write your name, gymnasium programme/adult education and date of birth on the papers you hand in.
The test	The test is made up of 14 problems. Most of the problems are of the <i>long-answer type</i>, where a short answer is not sufficient, but it is required • that you write down what you do • that you explain your train of thought • that you draw figures when necessary • that you show how you have used your resources when you have solved problems numerically/graphically. Some of the problems (where it is stated <i>Only an answer is required</i>) need only an answer. Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation
The score levels	The teacher responsible will inform you about the scores required for "Passed" and "Passed with Distinction". The maximum score is 41 points

1. Let $f(x) = 3x^2 - 6x + 7$
 - a) Differentiate $f(x)$ *Only an answer is required* (1p)
 - b) By using the derivative, find the minimum value of the function. (2p)

2.
 - a) Differentiate $g(x) = e^{4x}$ *Only an answer is required* (1p)
 - b) Calculate $g'(2)$ *Only an answer is required* (1p)

3.
 - a) Solve the equation $x^{5.5} = 20$ *Only an answer is required* (1p)
 - b) Solve the equation $5 \cdot 10^x = 8$ *Only an answer is required* (1p)

4. Markus has won on the pools and is planning to save parts of the winning. He catches sight of an advertisement about bonds in a bank-window (see figure).



Markus wants to be able to compare the interest of this bond to the interest of other ways of saving.

Calculate the yearly interest rate Markus receives if he buys a bond according to the above advertisement. (2p)

5. Explain, by giving an example, the notion *decline* in a statistical survey. (2p)

6. Karin is 25 years old and she decides to take out a retirement annuity. From one insurance company she finds out that the lowest guaranteed yearly interest rate is 4%.

She decides to save 3000 SEK in the beginning of each year. She is planning to do the first deposit the year she turns 26 and the last one the year she turns 65.

How much is her retirement annuity worth immediately after the last deposit, if the interest rate is 4%?

No tax is paid on her savings until the pension is paid. (3p)

7. A company that manufactures Thermos flasks has developed a new Thermos flask. By means of measurements the ability of keeping the temperature for different beverages has been examined. For coffee, it has been found that under certain circumstances the following formula holds:

$$f(t) = 85 \cdot e^{-0.038t}$$

where $f(t)$ is the temperature in °C and t is the time in hours after the coffee has been poured into the Thermos.

- Calculate the temperature of the coffee after 24 hours.
Only an answer is required (1p)
- Formulate a question that can be answered by the solution to the equation $f(t) = 50$ (1p)
- Solve the equation and answer your question. (2p)
- What does the value $f'(5)$ tell you about the coffee? (1p)
- State one condition that must be fulfilled for the formula to be valid. (1p)



8. A function f has the following properties

$$f(0) = 2$$

$$f'(0) = 1$$

$$f'(2) = 0$$

Sketch the graph to a function that has these properties.

(2p)

9. In later years, saving in unit trusts has become an increasingly popular way of saving. In one of the tables, you can see the value of a share in the World Wildlife Fund at some different points of time. In the other table you can see the development of the consumer price index

Take the consumer price index into consideration and compare the appreciation of the fund from

December 31 1991 to December 31 1994 with the appreciation from

December 31 1994 to December 31 1997.

	Value of share (SEK)						
	Dec 31 1991	Dec 31 1992	Dec 31 1993	Dec 31 1994	Dec 31 1995	Dec 31 1996	Dec 31 1997
The World Wildlife Fund	15.41	15.32	23.34	35.39	30.44	42.58	52.10

Tab.236. Konsumentprisindex: månadstal (1980 = 100), totalindex

Consumer price index: monthly, total

År Year	Jan	Feb	Mars March	April	Maj May	Juni June	Juli July	Aug	Sept	Okt Oct	Nov	Dec	Årsmedeltal Annual average
1985	149,6	151,0	152,1	152,7	154,5	153,9	153,8	153,8	154,5	155,5	156,5	157,1	153,8
1986	158,9	159,0	158,7	159,7	159,7	159,7	160,1	159,9	161,3	161,9	161,9	162,3	160,3
1987	164,4	164,4	164,7	165,1	165,2	164,9	166,9	167,8	169,4	170,1	170,7	170,7	167,0
1988	171,6	172,9	173,7	175,2	175,8	176,3	177,1	177,5	178,8	180,2	180,5	180,9	176,7
1989	183,0	184,0	184,7	186,5	187,3	187,9	187,9	188,7	190,2	191,8	192,2	192,8	188,1
1990	199,0	199,9	205,4	205,2	206,4	206,2	208,2	209,6	212,0	213,4	214,1	213,9	207,6
1991	218,9	225,0	225,8	227,1	227,3	227,0	227,1	226,7	229,2	230,1	231,1	230,8	227,2
1992	230,2	230,3	231,3	231,9	232,0	231,5	231,2	231,3	234,6	235,1	234,0	234,9	232,3
1993	241,0	241,6	242,7	243,7	243,1	242,3	241,9	242,3	244,5	245,2	245,3	244,3	243,2
1994	245,1	245,9	246,8	247,8	248,3	248,4	248,4	248,5	250,7	251,0	250,8	250,4	248,5
1995	251,3	252,3	253,3	255,0	255,3	255,1	254,8	254,5	256,2	256,9	256,8	256,0	254,8
1996	255,6	255,8	257,0	257,6	257,3	256,3	255,7	254,5	256,0	255,9	255,3	254,9	256,0
1997	254,6	254,2	255,2	257,0	257,0	257,4	257,3	257,4	259,8	259,6	259,2	259,1	257,3

Se Tabellänmärkingar.

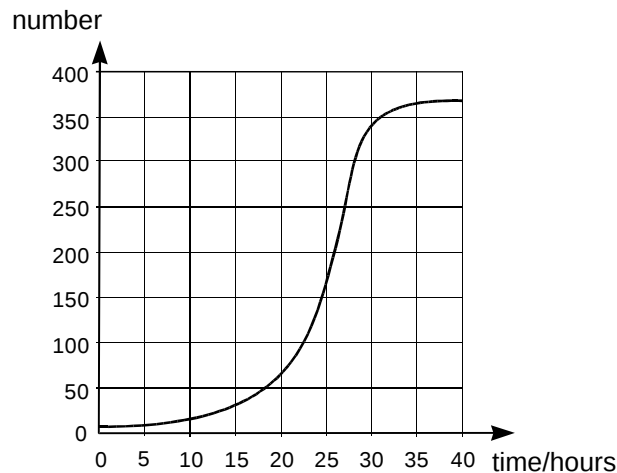
Källa: SCB Statistiska meddelanden, serie P 14

(3p)

10. The graph below shows how the number of bacteria in a certain culture of bacteria changes with time. To describe the properties of the culture of bacteria, a researcher wants to find the rate of change of the number of bacteria in the culture at certain points of time.

Find the rate of change, and the unit, for the number of bacteria 25 hours after the experiment has started.

(2p)



11. In an upper secondary school with 52 classes, there are 936 girls and 506 boys. The pupil's council planned to carry through a day of equality on Sunday the 8th of March and they wanted to investigate the pupil's attitudes to this. The first girl and the first boy on the list of each class had to answer the question: "Will you participate if we arrange a day of equality?"

36 girls and 15 boys answered "Yes", the rest of them answered "No". Some pupils from the pupil's council did the following calculation:

$$\frac{36 + 15}{2 \cdot 52} \approx 0.49$$

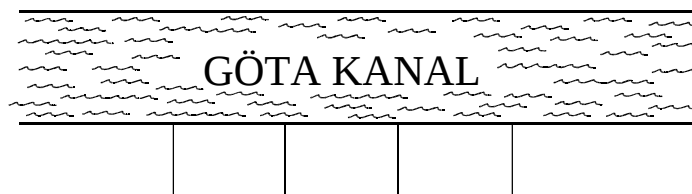
where $2 \cdot 52$ is the number of pupils asked and $36 + 15$ is the share of pupils who answered "Yes". The pupil's council concluded that 49 % of the pupils would participate on the actual day.

- a) Mention one mistake the pupils from the pupil's council made. (2p)
- b) Calculate the percentage of the pupils that according to the survey will participate in the equality day. (2p)

12. Lasse at Viggeby has the intention of building enclosed pastures to his horses on the meadow-land that borders on Göta kanal. He has 360 m of fencing at his disposal. To avoid having too many horses in the same pasture, he intends to divide the area into 3 identical rectangular pastures, see figure below. No fencing is needed along the canal

What measures should the enclosed pasture have so that the total area becomes as large as possible?

(4p)



13. When the weed-killer Meklorprop is used in nature, it is gradually decomposed. When the temperature of the ground is at a constant level, it is true that the remaining amount of weed-killer is decreasing exponentially with time. The time it takes until half of the weed-killer is left (half-life) depends on the temperature of the ground, according to the table below.

Temperature of ground (°C)	Half-life (days and nights)
5	20
10	12
20	3

Source: Miljøforskning, Nyhedsbrev nr 10, 1994

On one occasion, a field was sprayed with 8 kg of Meklorprop. The temperature of the ground was 5 °C at the time of the spraying, and is assumed to be constant during the following weeks.

What is the percentage of the initial amount of weed-killer left in the ground after 10 days and nights?

(3p)

14. The derivative of the function f is $f'(x) = x(x-a)^2$, where a is a positive constant.

Describe what the graph to the function f might look like.

(3p)