NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2001

Directions

Test time — 240 minutes without a break for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources — **Part I:** "Formulas for the National Test in Mathematics Courses C, D and E." Please note calculators are not allowed in this part.

Part II: Calculators, and "Formulas for the National Test in Mathematics Courses C, D and E".

Test material — The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 16 problems. **Part I** consists of 6 problems and **Part II** consists of 10 problems.

To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 45 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	25 points of which at least 5 "Pass with distinction points".

Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: _____ School: _____

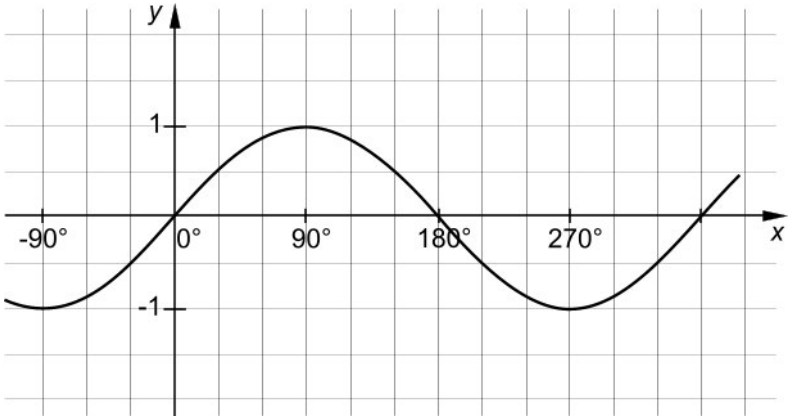
Education programme/adult education: _____

Part I

This part consists of 6 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

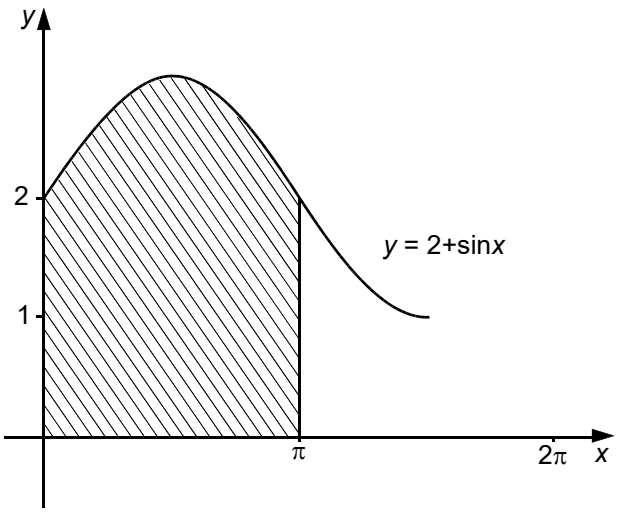
1. Calculate $\int_0^2 (x^2 + 3)\mathrm{d}x$ (2/0)

2. In the figure below the curve $y = \sin x$ is drawn.
On your paper, draw the curve $y = 0.5\sin(x + 60^\circ)$, using the same gradations on the axes as in the figure.



(2/0)

3. Calculate the exact value of the area of the shaded region in the figure below.



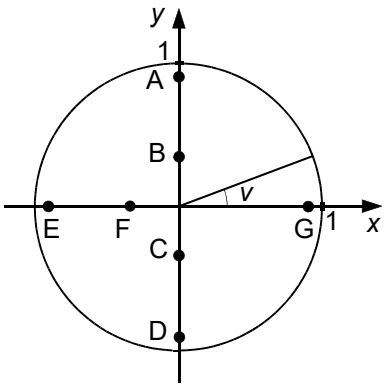
(2/0)

4. The function f is defined by $f(x) = x \cdot e^x$

Solve the equation $f'(x) = 0$ (1/1)

5. The figure below shows a circle of unit radius where an angle v has been pointed out.
In which of the points A – G can you read $\sin(180° - v)$?

Only answer is required



(1/0)

6. You know that $\sin u = \frac{3}{5}$ and that the angle u is between 0º and 90º.
Find the exact value of $\sin(u + 60°)$ (0/3)

Part II

This part consists of 10 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

7. Write down all the primitive functions F to $f(x) = 2x + 5$

Only answer is required (2/0)

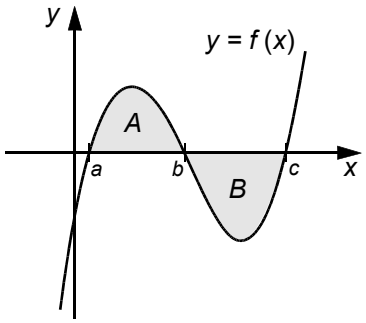
8. As the crow flies it is 105 km between Jönköping and Gothenburg. As the crow flies it is 182 km between Jönköping and Karlstad.
The angle between these two directions is $78°$.

What is the distance as the crow flies between Karlstad and Gothenburg? (2/0)

9. Which solutions to the equation $\sin 2x = 0.47$ are within the interval $0^\circ \le x \le 180^\circ$? (2/0)

10.

Together with the x-axis, the graph of the function $y = f(x)$ bounds two regions with areas A and B area units as indicated. The graph intersects the x-axis at points a, b and c.



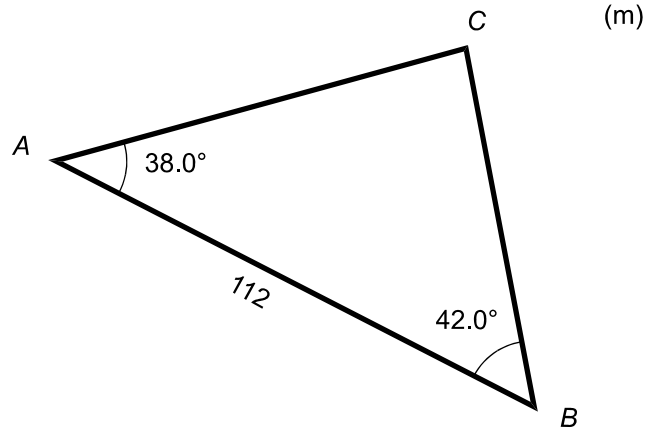
Using integrals, write down an expression for

a) A *Only answer is required* (1/0)

b) $B - A$ *Only answer is required* (0/1)

11. The triangle ABC is given according to the figure below.
Calculate the area of the triangle.

(Calculations based on measurements are not accepted)

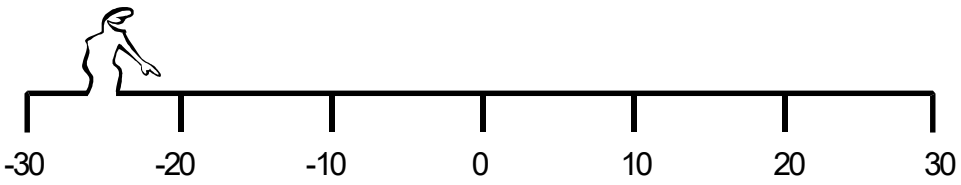


(3/0)

12. The function $y = f(x)$ has a primitive function $F(x) = Ax^2 + Bx$ where A and B are constants.

Determine A and B if $\int_0^1 f(x)\mathrm{d}x = 2$ and $\int_0^2 f(x)\mathrm{d}x = 0$ (0/3)

13. Linus is moving along a straight line according to the figure below. To be able to describe Linus' position on the line, it has been graded from −30 m to 30 m as can be seen from the figure.



Linus starts at time $t = 0$. His position $x(t)$ metres on the line is given by the function $x(t) = (t-2)^2(6-t)$

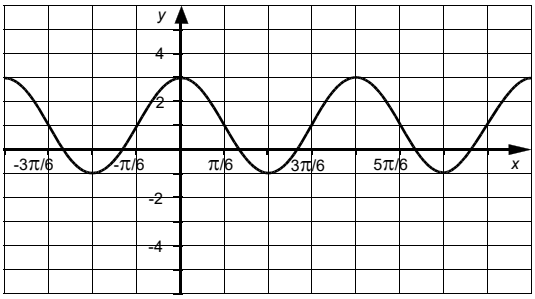
a) What is Linus' position on the line at the time $t = 0$?
Only answer is required (1/0)

b) Write down an expression for Linus' velocity at the time t. (0/2)

c) When Linus changes directions his velocity is zero. When does this happen? (1/0)

14. The graph to the function $y = f(x)$ is reproduced in the figure below.

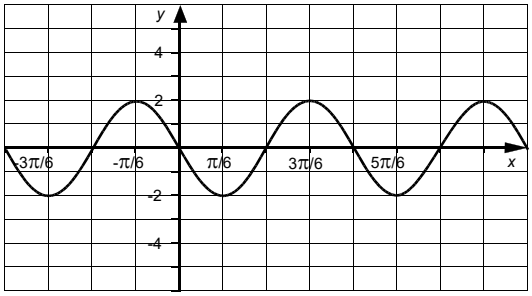
$y = f(x)$



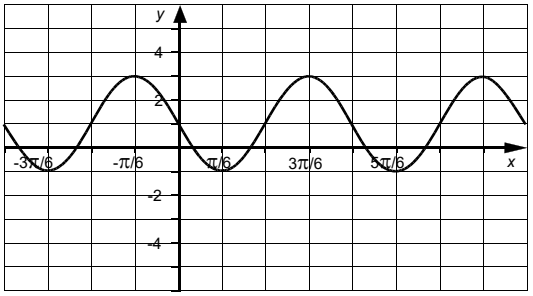
a) Which of the graphs in figures A-D is the best representation of the derivative of the function $y = f(x)$? *Only answer is required* (1/0)

b) Motivate your answer. (0/2/¤)

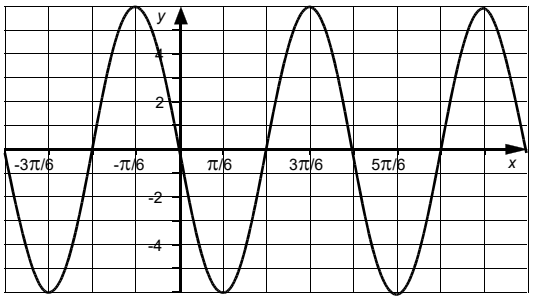
A



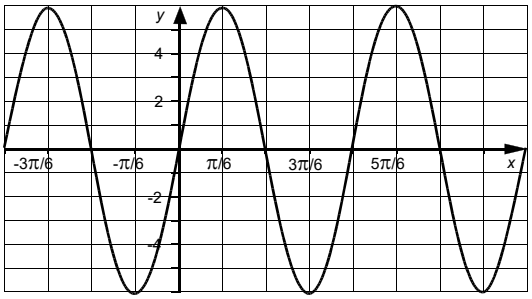
B



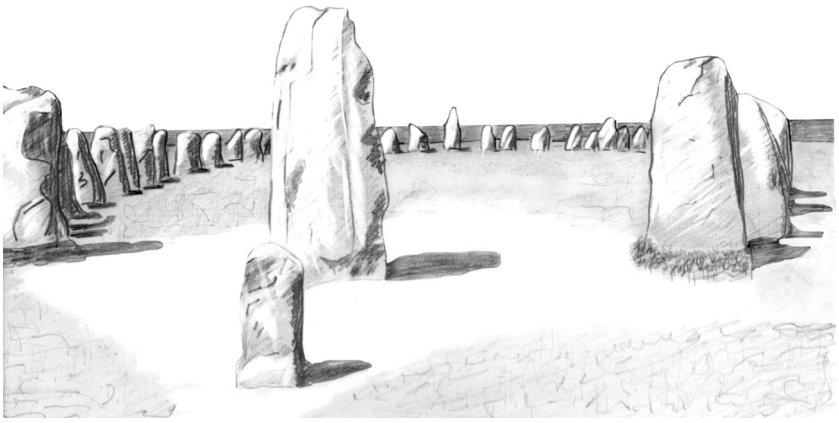
C



D



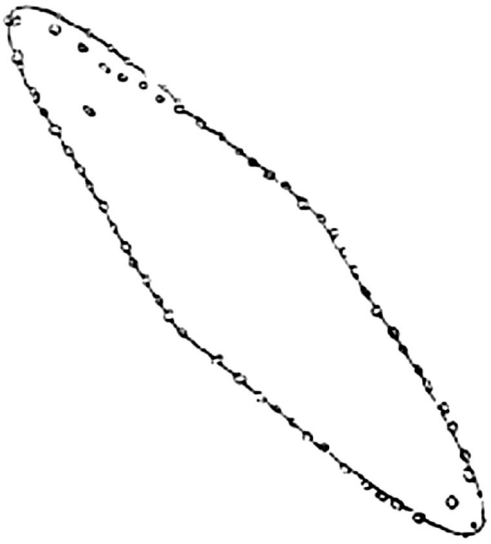
15. Some twenty kilometres east of Ystad, on the 42 m high ridge of Kåseberga is a place called the Stones of Ale. The stone circle is 70 m long, 18 m wide and consists of 59 stones. The shape of the stone circle has for a long time made people believe that it was a stone ship from the Viking Age. More recent research indicates that it might be a cult centre from the Bronze Age.



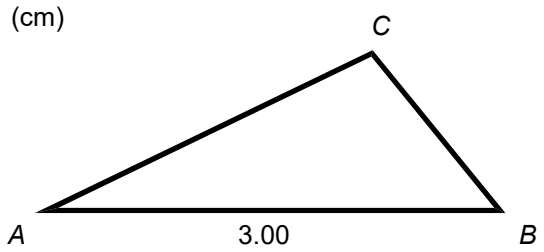
The placing of the stones (see the figure below) can be assumed to follow two opposite parabolas (= graphs to quadratic functions). Your task is to

a) find a suitable function to one of the parabolas. (0/3/¤)

b) calculate the area enclosed by the stones. (0/2)



16. In this problem, your task is to investigate how large the area of the triangle A*BC* can be. The first two points in this problem can be used as support for the investigation. You can choose whether you want to do the general investigation (the third point) at once or if you want to solve the problem step by step using all three points in sequence.



In the triangle *ABC* side *AB* is 3.00 cm long and side *AC* is twice as long as side *BC*.

- Choose a value of the length of side *BC* and calculate the area of the triangle *ABC* by first calculating the angle *C*.
- Find a value for the length of side *BC* that gives an area of the triangle larger than the one you calculated in the previous point.
- Investigate how large the area of the triangle *ABC* can be.

(3/4/¤)

When assessing your work your teacher will consider the following:

- how close to a general solution you are
- how well you present your work
- how well you justify your conclusions