NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2002

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note calculators are not allowed in this part.* **Part II:** Calculators, and “Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 7 problems and **Part II** consists of 10 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 43 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 “Pass”-points and 1 “Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 11 points Pass with distinction: 23 points of which at least 7 “Pass with distinction points”. Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: ______________________________ School: ________________________

Education programme/adult education: ____

Part I

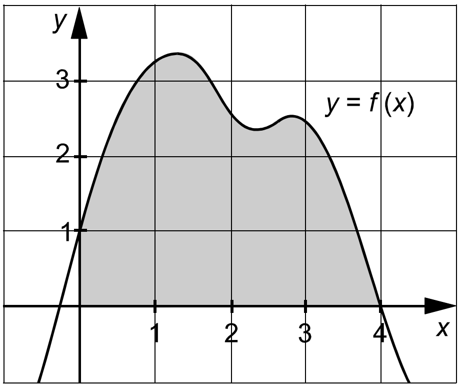
This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Evaluate the integral $\int_0^2 (2x^3 - x)\,dx$ (2/0)

2. Find the antiderivative F to $f(x) = e^x$ so that $F(0) = 2$ (2/0)

3. Solve the equation $\tan 3x = 1$
Write down all the solutions to the equation. (2/0)

4. Using an integral, write down an expression for the shaded area below the curve $y = f(x)$
Only answer is required

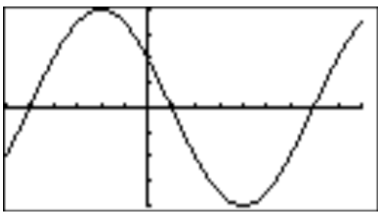


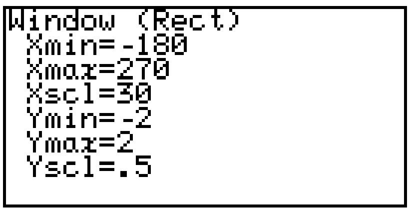
(1/0)

5. Which of the functions below corresponds to the plotted curve?
Only answer is required (1/0)

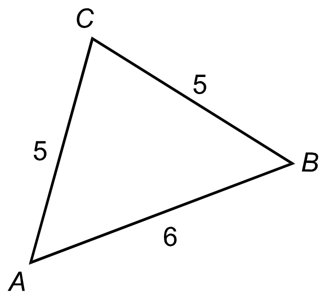
A $y = 2\sin(x - 30^\circ)$ B $y = 2\cos(x - 30^\circ)$ C $y = 2\sin(x + 60^\circ)$

D $y = -2\sin(x + 30^\circ)$ E $y = -\sin(x - 30^\circ)$ F $y = 2\cos(x + 60^\circ)$



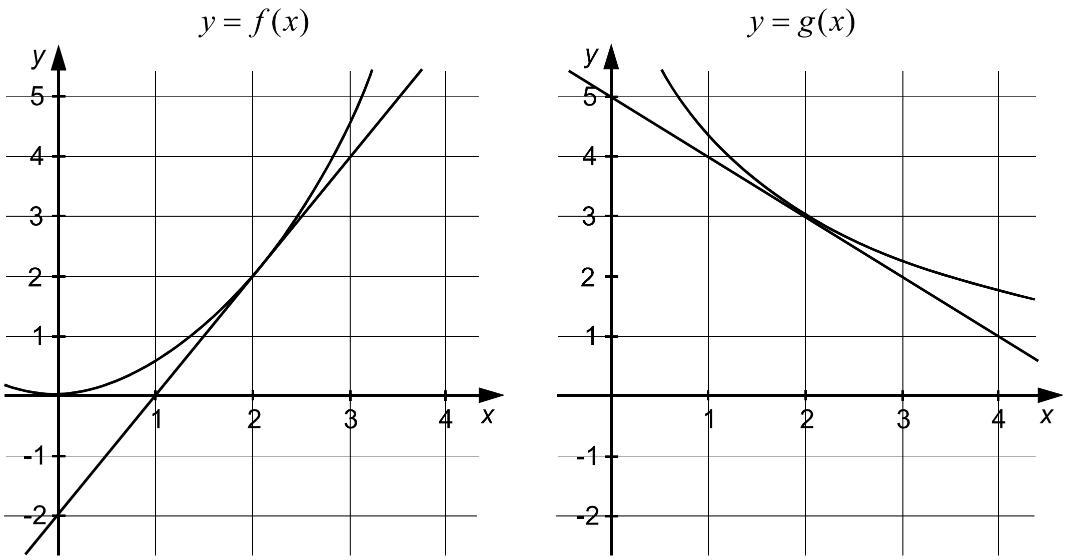


6. The figure shows an equilateral triangle ABC.
Calculate $\sin A$ and $\sin C$.



(0/3)

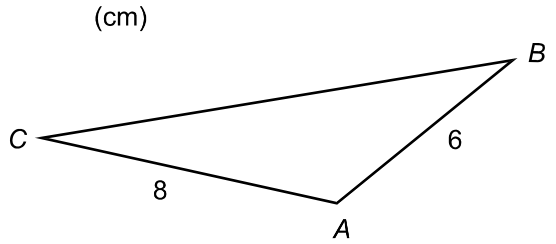
7. The figures below show the curves $y = f(x)$ and $y = g(x)$ and also the tangents at $x = 2$
The function h is defined by $h(x) = f(x) \cdot g(x)$
Use the figures to determine $h'(2)$. (0/3)



Part II

This part consists of 10 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

8.



In the above triangle *ABC* the angle *CAB* is obtuse. The area of the triangle is $12\ \text{cm}^2$.
Calculate the angle *CAB*. (2/0)

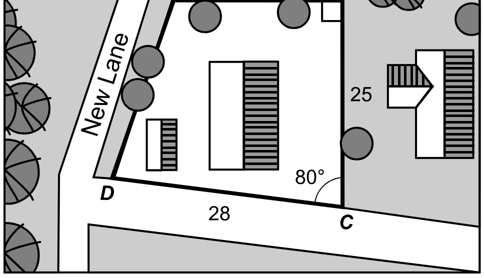
9. Write down a trigonometric equation that has solutions 38° and 142°
Only answer is required (1/0)

10. Differentiate the following functions

a) $f(x) = 3\sin 3x$ *Only answer is required* (1/0)
b) $g(x) = x \cdot \cos x$ *Only answer is required* (0/1)
c) $h(x) = \frac{1}{2}\mathrm{e}^{-x^2}$ *Only answer is required* (0/1)

11. Using your calculator, estimate the value of the integral $\int_2^3 \frac{1}{\ln x}\,\mathrm{d}x$
Give your answer correct to at least three decimal places.
Only answer is required (1/0)

12. A garden has the shape of a quadrilateral *ABCD*. *AB* is 20 m, *BC* is 25 m and *CD* is 28 m. Angle *B* is a right angle and angle *C* is 80°. Calculate the area of the garden. (3/0)



13.

The number of starlings in Sweden has halved since 1979 (in 23 years). Starlings depend on open grasslands, preferably pastures, and these have decreased. Mathematically, the situation can be described by the differential equation

$$\frac{dy}{dt} = ky$$

The differential equation has the solution $y = C \cdot e^{kt}$ where C is the number of starlings in 1979, k is a constant and y is the number of starlings at the time t, where t is the number of years after 1979.

a) In your own words, explain the meaning of the differential equation. (0/1)

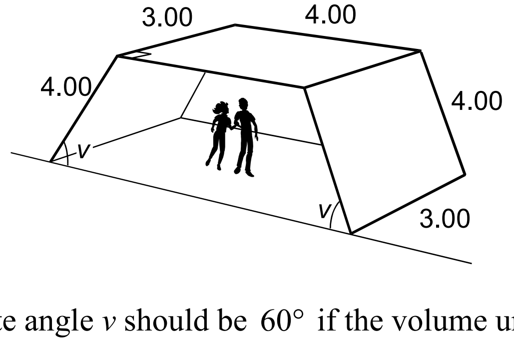
b) How long does it take until the number of starlings has decreased to 30 percent of the level in 1979 if the negative trend continues? (0/2)

14. Using your calculator, find the least value of the integer A for which the inequality $\lg x \cdot e^{4x-x^2} < A$ holds for all $x > 0$ (0/2)

15. A water container, empty at the beginning, is filled at a rate of $8e^{-0,2t}$ litres/minute, where t is the time in minutes after the empty container has started to fill. A leak causes water to pour out at the same time at a rate of $5e^{-0,1t}$ litres/minute.

How long does it take before the container is empty again? (0/3)

16. A travelling theatre company playing *Hamlet* has a stage with a steel tubing frame that can be assembled according to the figure. A canvas is then put up over the steel tubing frame. The frame can be assembled with greater or smaller slope of the sides.



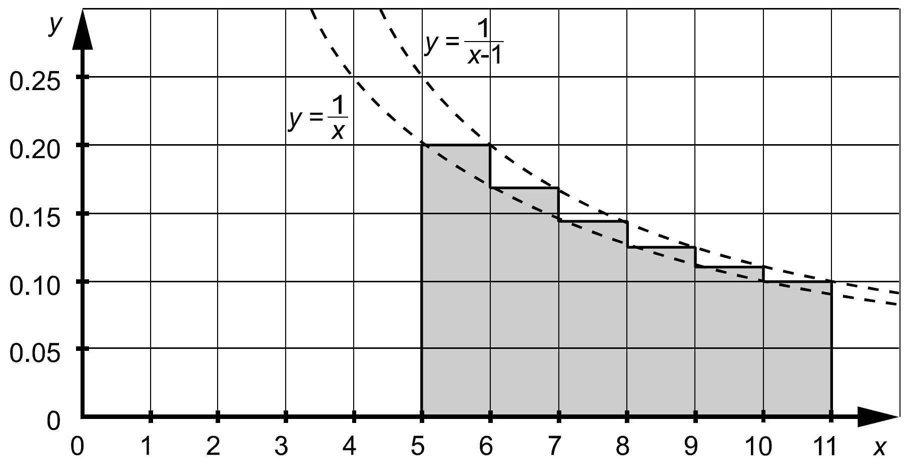
Show that the acute angle v should be $60°$ if the volume under the canvas is to be as large as possible. (0/4/¤)

When assessing your work with problem 17, your teacher will take into consideration:

- How well you present your work
- If your calculations are correct
- How close to a general solution you are
- How well you motivate your conclusion
- How well you use the mathematical language

17. It can be difficult to calculate a sum with a large number of terms. A formula can often be found, but when this cannot be done it is sometimes possible to estimate the sum by using integrals. In this problem you are going to estimate the value of some different sums by forming suitable integrals.

The sum $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$ is illustrated by the area of the shaded region in the figure below. You can also see the graphs to the functions $y=\frac{1}{x}$ and $y=\frac{1}{x-1}$



You can see that $\int_5^{11}\frac{1}{x}\,dx < \frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10} < \int_5^{11}\frac{1}{x-1}\,dx$

Thus, the sum is captured between the values of the integrals.

- Use the integrals and decide between which values the sum $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$ is.

- Use the same functions as above to capture the sum $\frac{1}{60}+\frac{1}{61}+\frac{1}{62}+\cdots+\frac{1}{118}+\frac{1}{119}+\frac{1}{120}$ between two values. Calculate these values.

- The sums above are on the form $\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{2n}$, where n is a positive number.
  In a similar way, examine these sums for large values of n.
  What is your conclusion? (3/4/¤)