NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2004

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators, and ”Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 8 problems and **Part II** consists of 8 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 46 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction”-points. Pass with special distinction: In addition to the requirements for ”Pass with distinction” you have to show *“Pass with special distinction” qualities in at least two* of the ¤-problems. You must also have at least 14 ”Pass with distinction”-points.

Name: ______________________________ School: ______________________________

Education programme/adult education: ____

Part I

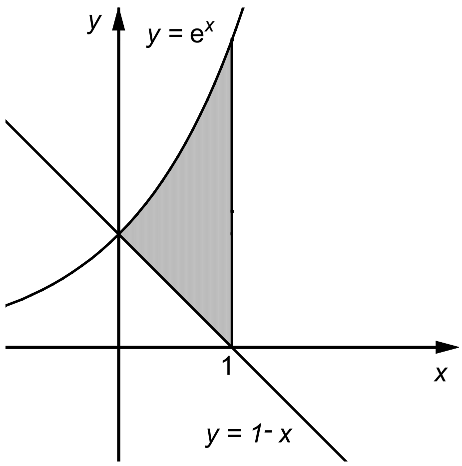
This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

1. Differentiate

a) $f(x) = \sin 3x$ *Only answer is required* (1/0)

b) $g(x) = (x+1)^{11}$ *Only answer is required* (1/0)

2. Calculate the area of the shaded region in the figure below.



(3/0)

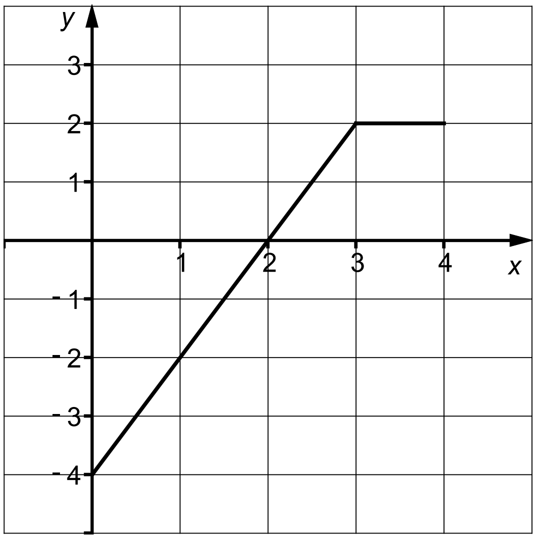
3. v is an angle between $0°$ och $90°$ such that $\tan v = \frac{1}{5}$

a) Draw a right-angled triangle where one angle is v. (1/0)

b) Determine $\tan(90° - v)$ (1/0)

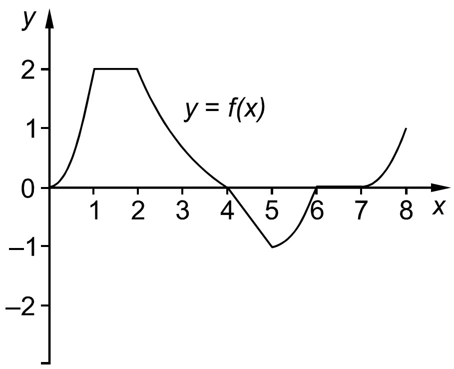
4. Find the two smallest positive solutions to the equation $\cos 2x = 0$ (1/1)

5. The figure below shows the graph of the function $y = f(x)$. Determine $\int_0^4 f(x)dx$ (1/1)



6. For which value of x, $0 \leq x \leq 2\pi$, does the function
$f(x) = x + 2\cos x$ have a local minimum value? (2/1)

7. The function $y = f(x)$ is given by its graph in the figure below. We form the function $A(x) = \int_0^x f(t)\,\mathrm{d}t$; $0 \leq x \leq 8$



a) The function A is constant throughout one interval.
Give this interval and justify your answer. (0/2)

b) Determine for which value of x the function A assumes its maximum value? (0/2/¤)

8. A triangle has sides 3 cm, 5 cm and 6 cm.
Show that the triangle is obtuse. (0/1/¤)

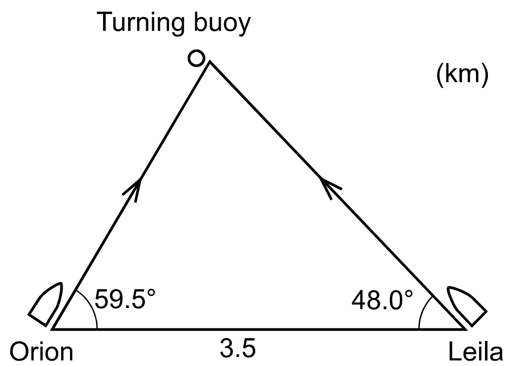
Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

9. Find the antiderivative F of $f(x) = e^{3x}$
that satisfies the condition $F(0) = 1$ (2/0)

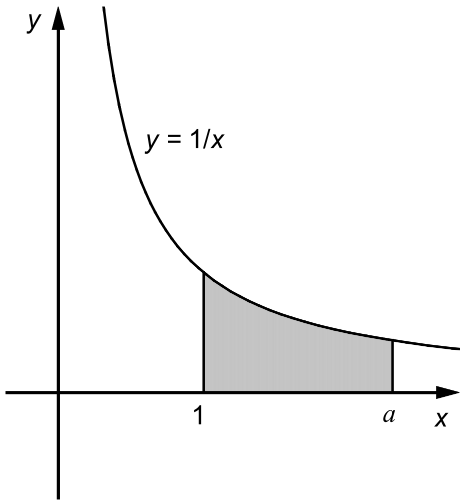
10. Show that $y = x \cdot \sin x$ is a solution to the differential equation
$x \cdot y' - y = x^2 \cdot \cos x$ (2/0)

11. The two sailing-boats 'Orion' and 'Leila' are competing in a race. Due to the different choice of routes, they are 3.5 km apart at a certain point in time. A little further away the boats are going to pass a turning buoy, see figure.



Which of the sailing-boats will be first to the turning buoy if Leila's velocity is 4.0 knots and Orion's 3.2 knots?
(1 knot = 1.852 km/h) (3/0)

12. Determine the number *a*, so that the area of the shaded region is exactly 1 square unit.
(See figure below).



(1/1)

13. $\sin^4 15° - \cos^4 15°$ can be determined exactly in the following way:

$$\sin^4 15° - \cos^4 15° \overset{1}{=} (\sin^2 15° - \cos^2 15°)(\sin^2 15° + \cos^2 15°) \overset{2}{=} \sin^2 15° - \cos^2 15° =$$

$$-(\cos^2 15° - \sin^2 15°) \underset{3}{=} -\cos 30° = -\frac{\sqrt{3}}{2}$$

Justify the three equalities that are pointed out. (1/2)

14. Determine *a* so that

$$\int_a^{2a} (3x^2 + 2x)\,dx = 14$$

Answer to four significant figures. (0/2)

15. In the mid 19th century a famous Swedish physicist, Anders Ångström, made comparative studies of the temperature of the air and the temperature of the earth.

The diagram shows the results of Ångström's experiment.

From the diagram you can read that the difference between the highest and lowest value in the air is approximately $24\ °C$. At a depth of 10 feet the difference between the highest and lowest earth temperature is only approximately $6\ °C$. Furthermore, the points when the maximum- and minimum- temperatures occur are different for the air and earth.

If a sine function is adjusted to the measurements, the air temperature, y °C, can be described by

$$y = 5.0 + 12\sin(0.52x - 2.09)$$

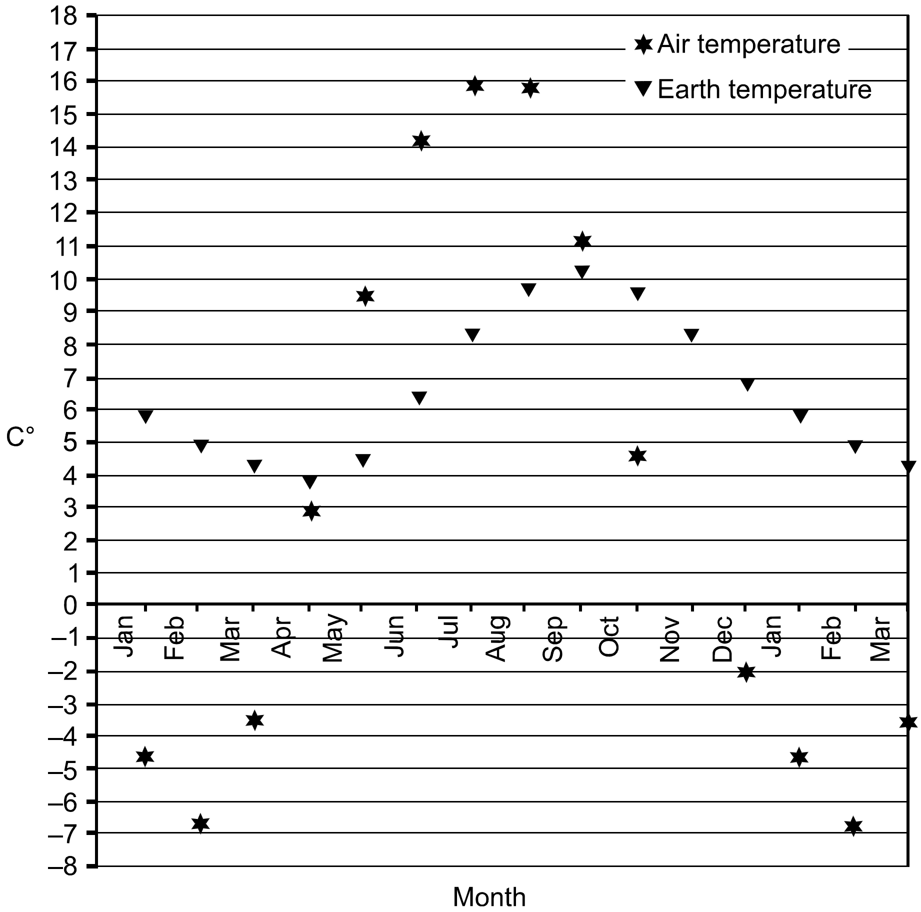
where x is the number of months from the beginning of the year.

a) Determine $y'(x)$ *Only answer is required* (1/0)

b) Determine $y'(6)$ and describe in your own words what $y'(6)$ implies. (1/1)

c) Use the diagram to write down a realtionship of the form $y = a + b\sin(kx + d)$ between the earth's temperature y °C and the time x months. (0/3)

In 1855 Ångström published a thorough investigation of the earth's temperature in *Nova Acta Regiae Societatis Scientiarium Upsaliensis*. The Royal Swedish Academy of Science had earlier paid for a number of earth thermometers. In 1837 measurements at different depths begun. Originally there were nine thermometers. They all reached the surface of the earth but went down to different depths, the longest one went down to 15 feet. This thermometer was however broken after only a couple of months because of the pressure of the earth. More thermometers were broken but still a number of connected measurements at different depths were received over eight years in total.

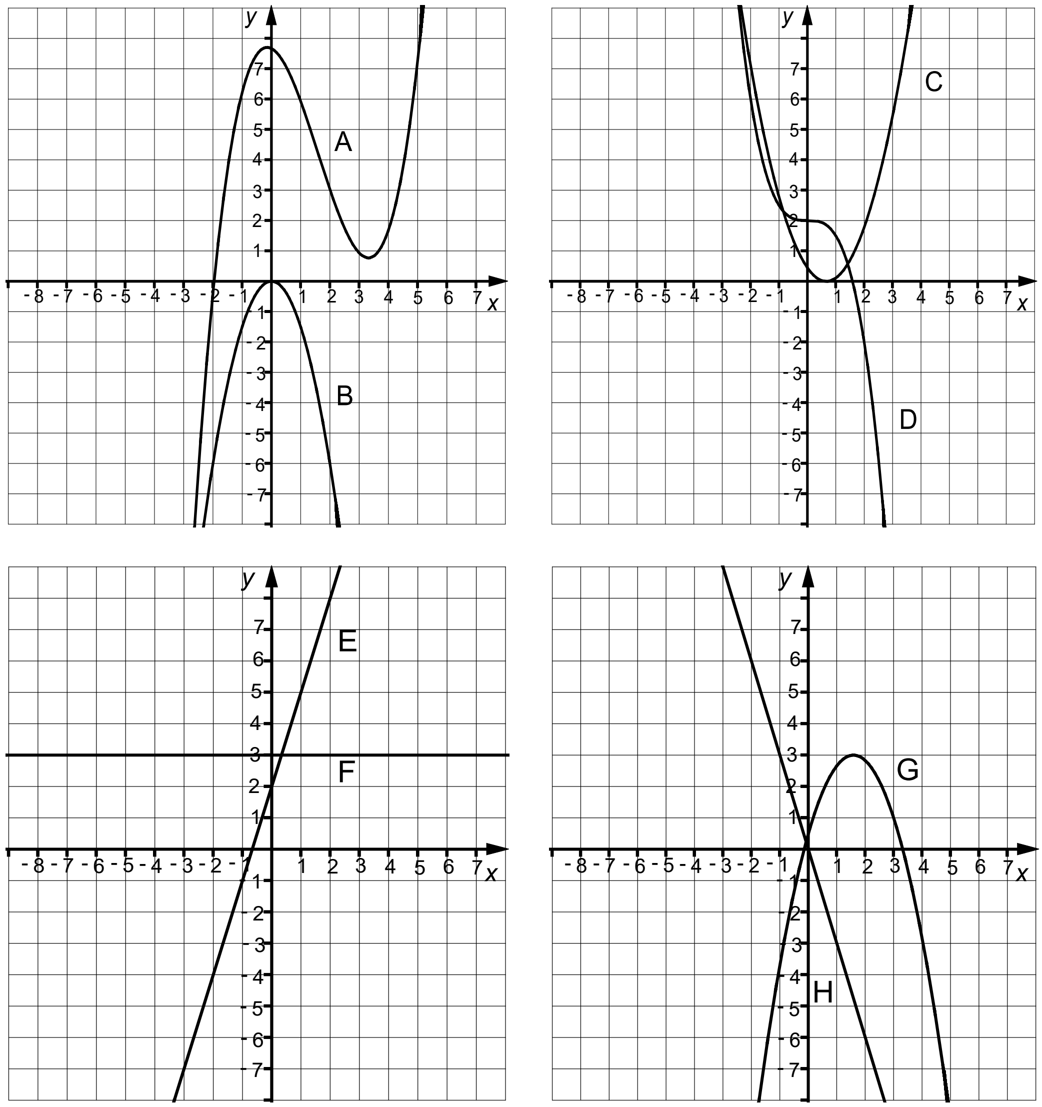


Monthly averages of the air temperature (✶) and the earth temperature (▼) at a depth of 10 feet during the years 1838-1845.

When assessing your solution your teacher will take into consideration:

- how well you justify your conclusions
- how well you present your work
- how well you use the mathematical language
- how well your sketch reflects properties of the grapf to the derivate

16. The graphs below belong to polynomial functions with a degree of no more than three.

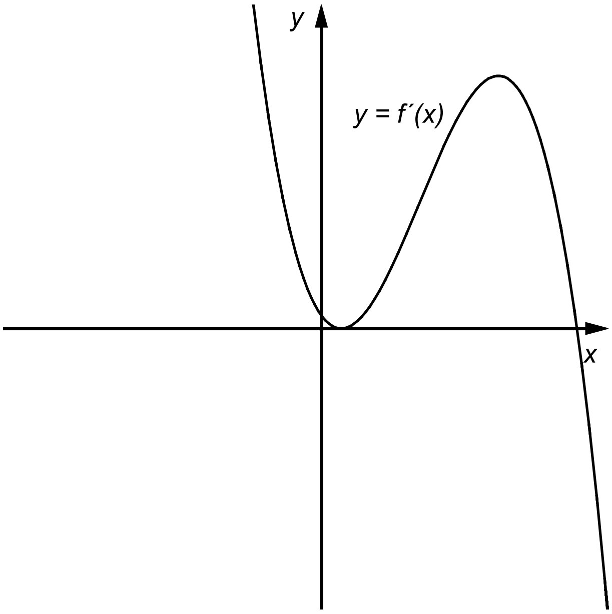


- Find two graphs among the figures above where one graph is the derivative of the other. Point out which graph is the function and the derivative respectively, and explain how you can see the connection.
  Graphs that belong together do *not* have to be in the same coordinate system.

- Repeat the above task and find all pairs of graphs that can be found.

- Among the graphs above, there is a triplet consisting of function, derivative and second derivative. Find these three graphs and point out which is the function, the derivative and the second derivative respectively.

- The figure below shows the graph of the derivative of a funcion. Sketch how the corresponding graph of the function might look. Explain how you arrived at the look of the graph.



- Could your graph have looked any different? Justify your answer. (2/5/¤)