NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2005

Directions

Test time: 240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources: **Part I:** ”Formulas for the National Test in Mathematics Courses C and D.” *Please note that calculators are not allowed in this part.*

Part II: Calculators and ”Formulas for the National Test in Mathematics Courses C and D”.

Test material: The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test: The test consists of a total of 18 problems. **Part I** consists of 9 problems and **Part II** consists of 9 problems.

For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels: The maximum score is 43 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	25 points of which at least 7 ”Pass with distinction”-points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”-points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

1. Find an antiderivative F for $f(x) = 8x^3 - 2x$ *Only answer is required* (1/0)

2. Evaluate $\sin\frac{7\pi}{2}$ *Only answer is required* (1/0)

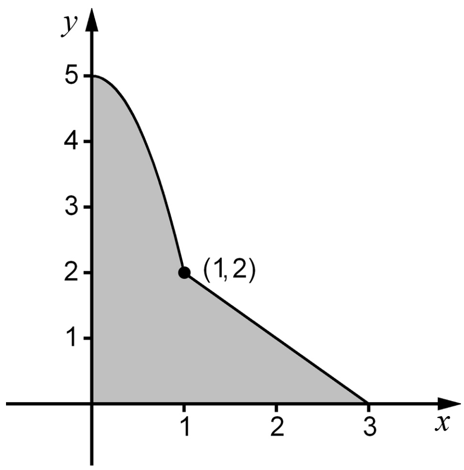
3. Differentiate

a) $f(x) = 4\sqrt{x}$ *Only answer is required* (1/0)

b) $g(x) = (2x+1)^5$ *Only answer is required* (1/0)

4. Evaluate $f'\left(\frac{\pi}{2}\right)$ when $f(x) = x - 2\cos x$ (2/0)

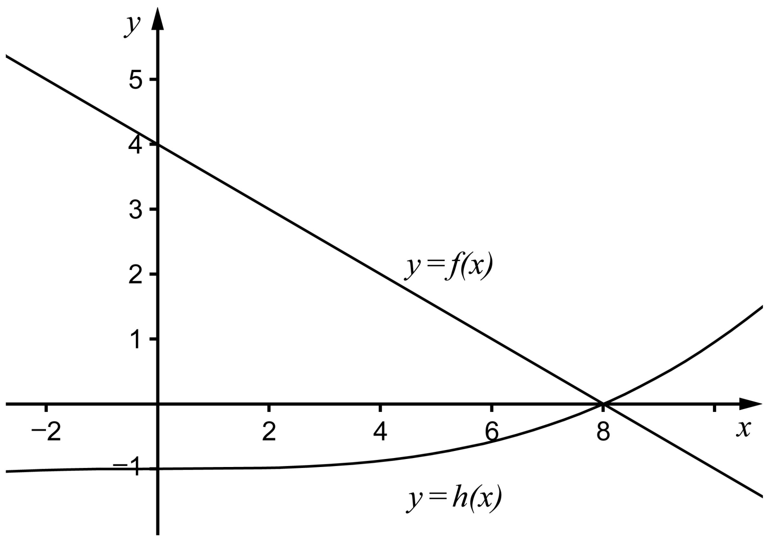
5. The figure below shows a region bounded by $y = 5 - 3x^2$, $y = 3 - x$ and the positive coordinate axes. Calculate an exact area of that region. (2/0)



6. Evaluate $f(x)$ if $f'(x) = 2x + \frac{1}{x}$, $x > 0$ and $f(1) = 2$ (2/0)

7. Evaluate $\cos v$ if $\sin v = \frac{\sqrt{45}}{7}$ and $90^\circ < v < 180^\circ$ (0/2)

8. The region bounded by the line $y = f(x)$, the curve $y = h(x)$ and the y-axis has the area $\frac{65}{3}$ square units.

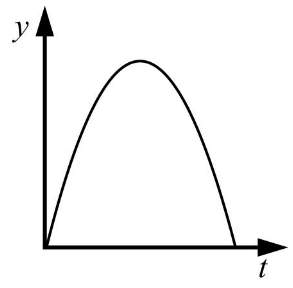


Determine the value of the integrals:

a) $\int_0^8 f(x)\mathrm{d}x$ (1/0)

b) $\int_0^8 h(x)\mathrm{d}x$ (0/2)

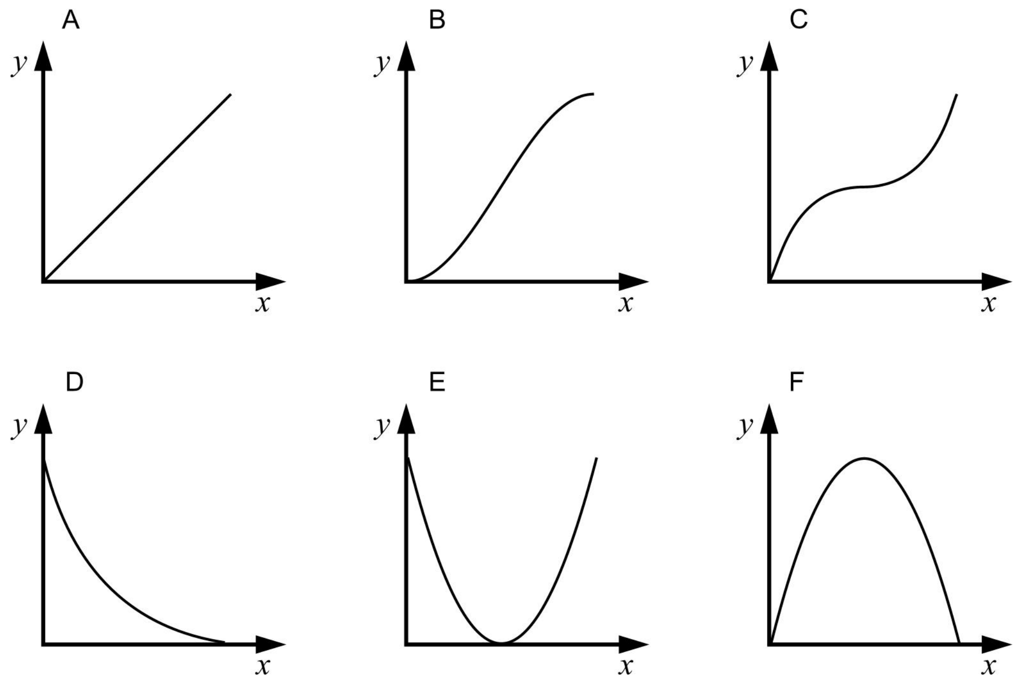
9. The figure below shows the graph of the function f.



a) Which of the graphs A – F below shows the function $g(x)=\int_0^x f(t)\,\mathrm{d}t$?

Only answer is required (0/1)

b) Motivate your answer. (0/1/¤)

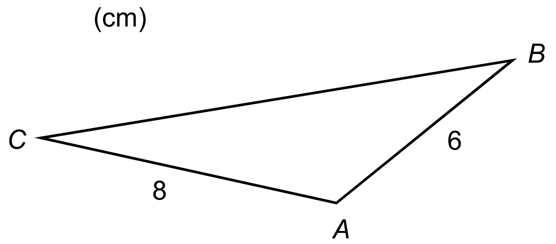


Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

10. A trigonometric curve has amplitude 3 and period π.
Find an equation for such a curve. *Only answer is required* (2/0)

11.



In the triangle *ABC* above, the angle *CAB* is obtuse. The area of the triangle is $12\,\text{cm}^2$, $AB = 6\,\text{cm}$, and $AC = 8$ cm. Find angle *CAB*. (2/0)

12. Find all solutions to the equation $\cos 2x = 0.78$ (2/1)

13. A region is bounded by the curves $y = e^{0.2x}$ and $y = 6x - x^2 + 1$

Write down an integral expression for the area of this region and determine an approximative value of this area, correct to at least 3 significant digits. (1/2)

14.

The number of bees in a colony of bees after *t* weeks is denoted $n(t)$. The number of bees in the beginning is 5000. The bee-keeper writes down the expression

$$5000 + \int_0^{15} n'(t)\,dt$$

a) What does the second term in the expression represent? (0/1)

b) What is the meaning of the whole expression? (0/1)

15. A hot tub contains 3000 litres of water. A leak causes water to pour out at a rate of y litres/min. This rate is given by $y = 22e^{-0.011 \cdot t}$, where t is the time in minutes after the emergence of the leak.

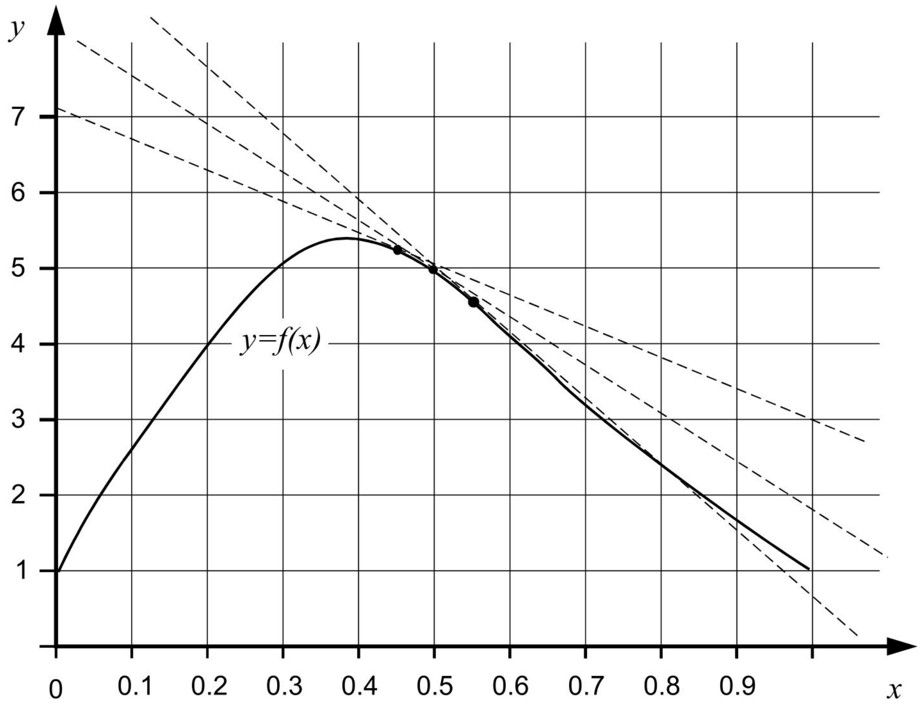
After how long is there 2000 litres left in the hot tub?

(0/3)

16. Use derivatives to determine the constant a so that $f(x) = x^2 \cdot e^{ax}$ has a local maximum when $x = 2$ (0/3)

17. The figure below shows the graph to the function $y = f(x)$. Tangents are drawn at the points where $x = 0.45$, $x = 0.50$ and $x = 0.55$. The equations to the tangents are given in the table below.

Determine the best possible approximate value to $f''(0.5)$ (0/2/¤)



x	Equation of the tangent
0.45	$y = -4.1x + 7.1$
0.50	$y = -6.3x + 8.2$
0.55	$y = -8.8x + 9.4$

When assessing your work with this problem, your teacher will take into consideration:

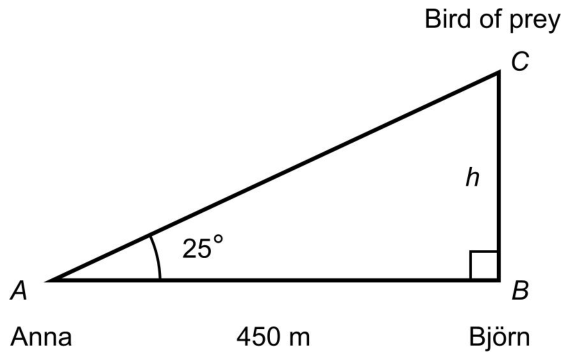
- How well you carry out your calculations
- How well you justify your conclusions
- How well you present your work
- How well you use the mathematical language

18. A large number of birds die every year due to collision with power lines and other obstacles. In the local press, there is discussion about whether a planned wind power station poses a threat to migratory birds.

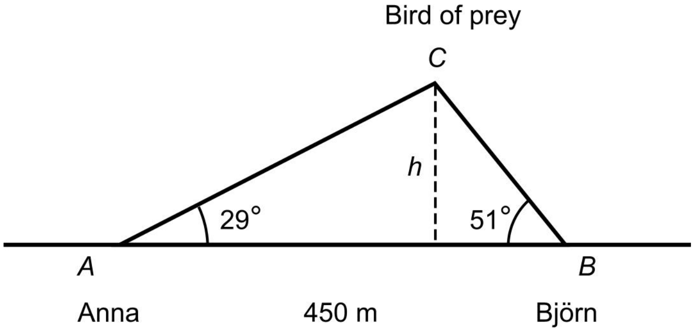
Two students, Anna and Björn, who live 450 m away from each other, decide to try to determine at what height migrating birds of prey fly.

- On one occasion Björn calls Anna when he sees a bird approaching. When the bird is right above Björn, Anna observes the bird under the vertical angle 25°. At what height h does this bird of prey fly?

Anna uses a clinometer to measure the angle.



- Another bird of prey passes between Anna's and Björn's homes. When they see that the bird is right above an imaginary line between their homes, they both measure the angle under which they observe the bird. See figure below. The angles they measure are $A = 29°$ and $B = 51°$. At what height does the bird fly?



- They decide to dedicate their project work to the problem with the migratory birds. They want to find a formula that gives you the height if you insert the measured values of the angles A and B. Derive a suitable formula in a simplified form.

- Is the formula valid even if the bird flies to the right of Björn so that the angle B is obtuse? Motivate your answer.

(3/3/¤)