NATIONAL TEST IN MATHEMATICS COURSE D

AUTUMN 2008

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 120 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.*
	Part II: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions.
	Write your name, the name of your education programme/adult education on all sheets of paper you hand in.
	S*olutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 9 problems and **Part II** consists of 7 problems.
	For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.
	Problem 9 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.
	Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points.
	The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test:

Pass:	13 points
Pass with distinction:	25 points of which at least 6 ”Pass with distinction”- points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Determine an antiderivative for $f(x) = 8x^3 + 4x + 1$

Only answer is required (1/0)

2. a) Express 3π in degrees. *Only answer is required* (1/0)

b) Express 20° in radians. *Only answer is required* (1/0)

3. Differentiate

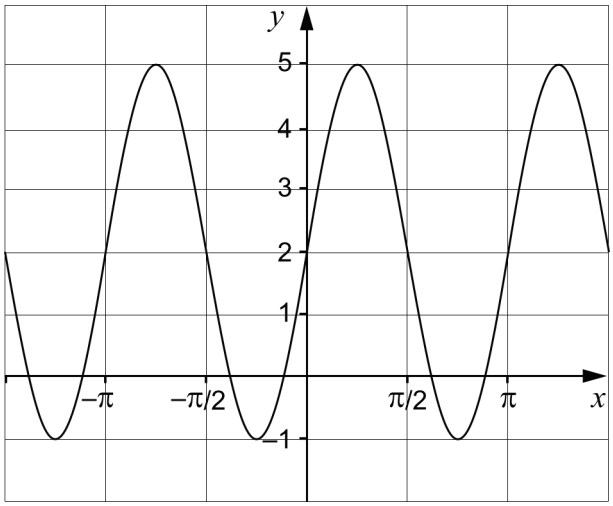
a) $f(x) = 4\sin 3x$ *Only answer is required* (1/0)

b) $f(x) = x^2 \cdot \mathrm{e}^x$ *Only answer is required* (1/0)

c) $f(x) = \ln(2x + 1)$ *Only answer is required* (0/1)

4. Evaluate the integral $\int_0^2 (x^2 - x)\,\mathrm{d}x$ (2/0)

5. The equation for the curve below can be written as $y = A\sin kx + b$



a) Determine the values of the constants A and b. *Only answer is required* (2/0)

b) Determine the value of constant k. (0/1)

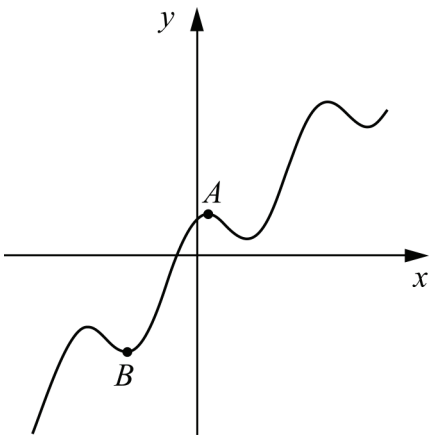
6. Draw a graph of a function f where $y = f(x)$ which satisfies the condition $\int_1^3 f(x)\mathrm{d}x = 1$ (0/2)

7. Decide for each of the following statements whether it is true or false. Remember to justify your answer.

a) $\cos\frac{8\pi}{3} = \frac{1}{2}$ (0/1)

b) $\tan\frac{4\pi}{3} = \sqrt{3}$ (0/1)

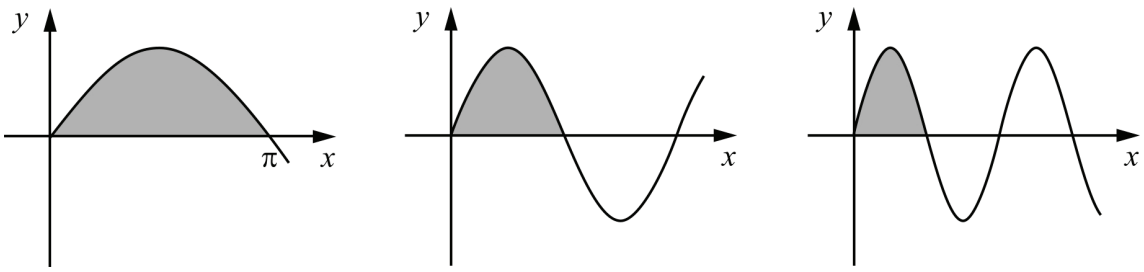
8. The figure shows the graph of the function $f(x) = x + \cos 2x$
Find the x-coordinate of the local maximum point at A and the local minimum point at B. (1/2)



When assessing your work with the following problem your teacher will take into consideration:

- How well you carry out your calculations
- How close you come to a general solution
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

9. In the figures below parts of the graphs of curves of the type $y = \sin kx$ are drawn for $k = 1$ $k = 2$ and $k = 3$. In each figure there is a shaded part. Your task is to investigate if there is a relationship between the area of the shaded part and the value of the constant *k*.



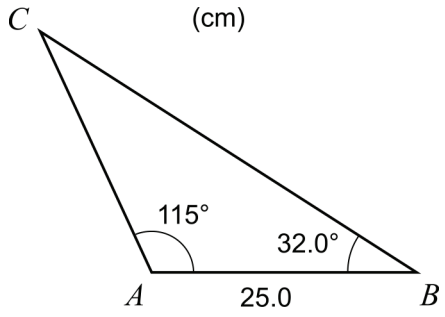
- Calculate the area of the shaded part in the three cases above.
- Use your results to formulate a statement about how the area of the shaded part depends on *k*.
- Show that your statement is true where $k > 0$

(2/4/¤)

Part II

This part consists of 7 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

10. The triangle ABC is shown in the figure below.



a) Determine the length of the side BC. (2/0)

b) Calculate the area of the triangle. (1/0)

11. It is calculated that the population of a region increases at a rate of
$650\,e^{0.4t}$ persons/year,
where t is the time in years counted from the beginning of the year 2008.

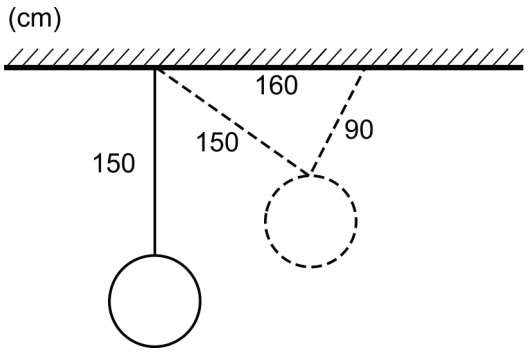
a) Evaluate the integral $\int_0^8 650\,e^{0.4t}\,dt$ *Only answer is required* (1/0)

b) Interpret what $\int_0^8 650\,e^{0.4t}\,dt$ means in this context. (0/1)

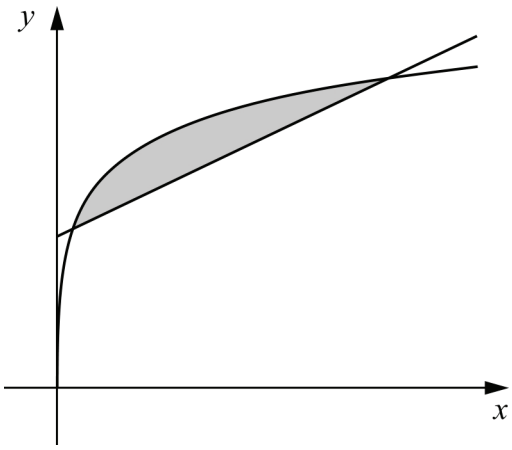
12. Stina and Nisse have a lamp above the coffee table. Sometimes they want to raise the lamp so that it doesn't block their view. The lamp hangs from the ceiling by a cord that is 150 cm long. They have a hook 160 cm from the ceiling fixture and from there they attach a 90 cm long cord to the lamp.

How much higher will the lamp hang when they attach it this way? (3/0)

(Calculations based on measurements are not accepted)



13. An area is enclosed by the curve $y = 5 + \ln x$ and the line $y = x + 3$



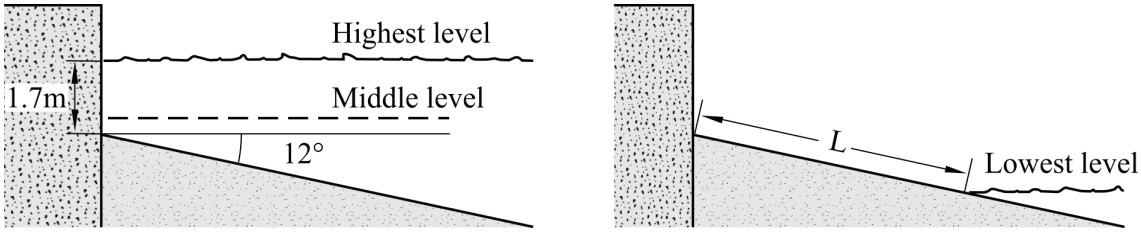
a) Determine the x-coordinates for the curve's points of intersection correct to three decimal places. (1/0)

b) Determine the area of the shaded part correct to two decimal places. (0/2)

14. The tide causes the water level in the harbour vary around an average level according to

$y = 1.5\sin\frac{\pi}{6}t$ where y is the water level in metres and t is the time in hours from 6.00 am.

During parts of a 24-hour period the water does not reach all the way to the dock in the harbour, a part of the sea bed is exposed. The sea bed's slant is even and creates a $12°$ angle with the surface of the water. The water's maximum depth at the dock is 1.7 metres.



a) How great a distance L, from the dock, along the sea bed is exposed when the water is at its lowest level? (1/1)

b) When, during a 24-hour period, is the water at its lowest level? (1/1)

15. The figure below shows a sign chart for a function f where the derivative has two zeros and where the second derivative has two zeros. The second derivative's signs between the zeros are marked.

x		a		b		c	
$f'(x)$		0				0	
$f''(x)$	+	+	+	0	−	0	−

Specify for each of the following statements whether it is true or false. Remember to justify your answer.

a) The function f has a maximum for $x = a$ (0/1)

b) The derivative f' has a maximum for $x = b$ (0/1)

c) The function f has a maximum for $x = c$ (0/1/¤)

16. Examine the integral $\int_a^b (x^2 - 1)\,\mathrm{d}x$ when $a < b$

Determine which values the integral can have. (0/2/¤)