NATIONAL TEST IN MATHEMATICS COURSE D

AUTUMN 2009

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.*
	Part II: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions.
	Write your name, the name of your education programme/adult education on all sheets of paper you hand in.
	S*olutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 10 problems and **Part II** consists of 7 problems.
	For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.
	Problem 10 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.
	Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points.
	The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test:

Pass:	12 points
Pass with distinction:	25 points of which at least 6 ”Pass with distinction”- points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Determine the antiderivative F of $f(x) = 8x^3 + 4x$
that satisfies $F(0) = 1$ (2/0)

2. Calculate $\int_1^2 (3x^2 + 1)\mathrm{d}x$ (2/0)

3. Differentiate

a) $f(x) = 2\sin 3x$ *Only answer is required* (1/0)

b) $g(x) = x^3 \cdot \mathrm{e}^x$ *Only answer is required* (1/0)

c) $h(x) = (2x+1)^6$ *Only answer is required* (1/0)

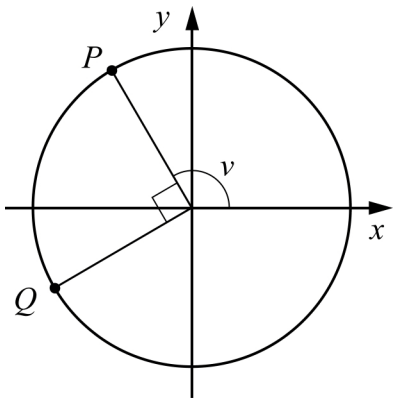
4. Calculate $\dfrac{\pi}{\cos\pi} + \dfrac{2\pi}{\cos 2\pi} + \dfrac{3\pi}{\cos 3\pi}$ *Only answer is required* (1/0)

5. Let $f(x) = 3\sin x + b$
For which values of b is $f(x) > 0$ for all x? *Only answer is required* (1/0)

6. It holds that $\sin 44° \approx 0.69$
Use this to determine all solutions to the equation $\sin 4x = 0.69$ (1/2)

7. The points P and Q lie on the unit circle.

Which of the following alternatives A - E gives the coordinates of the point Q?



A $(\sin v, \cos v)$

B $(-\sin v, -\cos v)$

C $(-\cos v, -\sin v)$

D $(-\sin v, \cos v)$

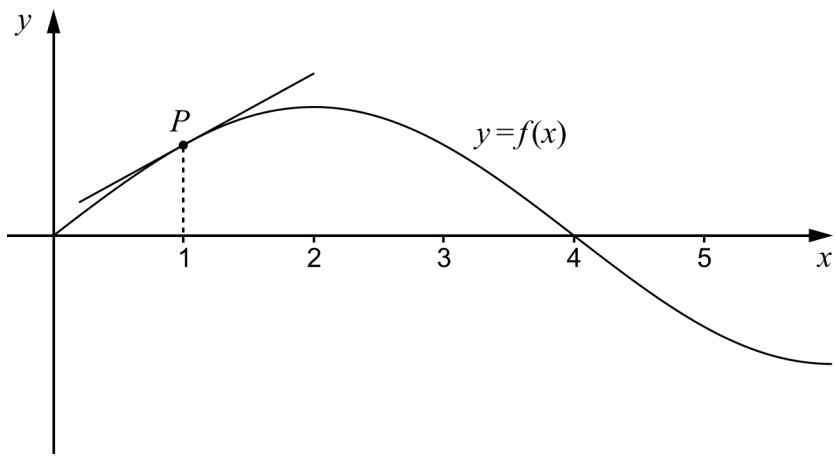
E $(\cos v, -\sin v)$

Only answer is required (0/1)

8. The figure shows the graph of $y = f(x)$ where $f(x) = A\sin bx$

The tangent to the graph at point P has slope $\frac{1}{2}$

Use the information from the figure to determine the constants A and b. (0/3)



9. Show that $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = \dfrac{2}{\sin x}$

for all x where the expressions on both sides are defined. (0/2/¤)

When assessing your work with the following problem the teacher will take into consideration

- How well you carry out your calculations
- How well you justify your conclusions
- How well you present your work
- How general your solution is
- How well you use mathematical language

10. In this problem you will be investigating areas of regions bounded by the *x*-axis, a vertical line and the graph of the function $f(x) = x^p$ where $x \geq 0$

- Determine the area *A* of the marked region in Figure 1, and then determine *b* so that the area of the marked region in Figure 2 is twice as great as in Figure 1.

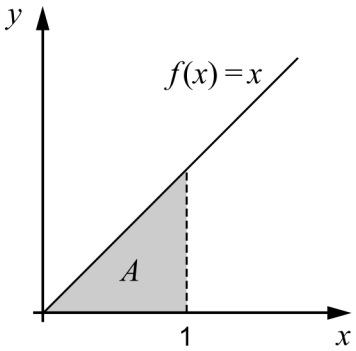


Figure 1

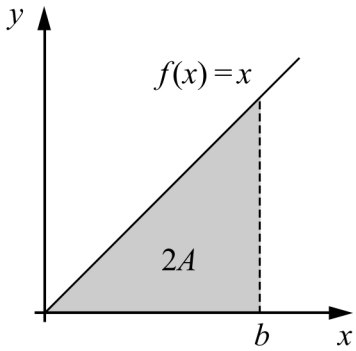


Figure 2

- Determine *b* such that the area in Figure 4 is twice as great as in Figure 3.

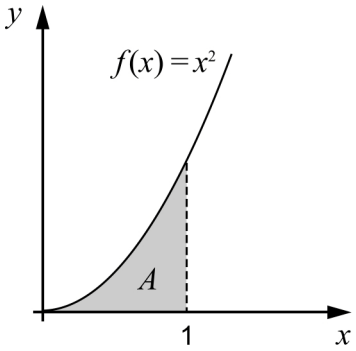


Figure 3

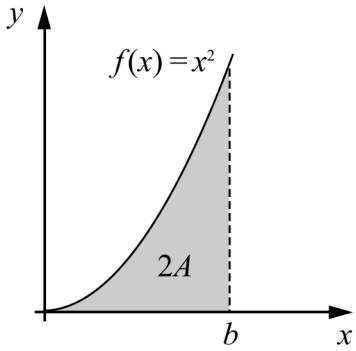


Figure 4

The figures below show the graphs of the function $f(x) = x^p$, $x \geq 0$, $p \geq 1$

Figure 5 shows a region with area *A*, bounded by the *x*-axis, the line $x = a$ and the graph of the function $f(x)$.

Figure 6 shows a region with an area is twice the size of the area in Figure 5, that is 2*A*. The region is bounded by the *x*-axis, the line $x = b$ and the graph of the function.

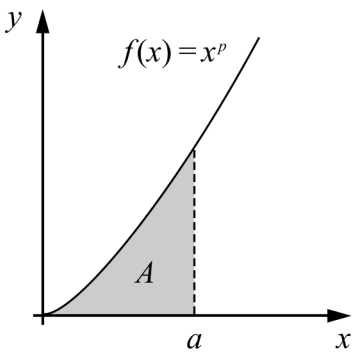


Figure 5

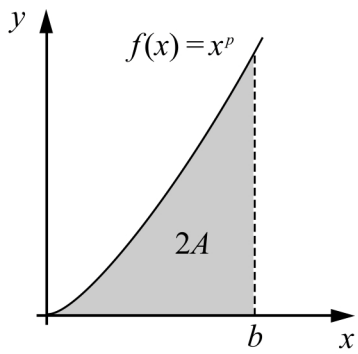


Figure 6

- Start from Figures 5 and 6 and let $a = 1$ and $p = 3$. Then determine *b* so that the area in Figure 6 is twice as great as the area in Figure 5.

- Copy the table below and complete it by calculating the quotient of $\frac{b}{a}$ for the different values of *p* indicated.

p	1	2	3
a	1	1	1
$\frac{b}{a}$			

- Investigate the quotient of $\frac{b}{a}$ and formulate a hypothesis for how the value of $\frac{b}{a}$ depends on the value of *p*.

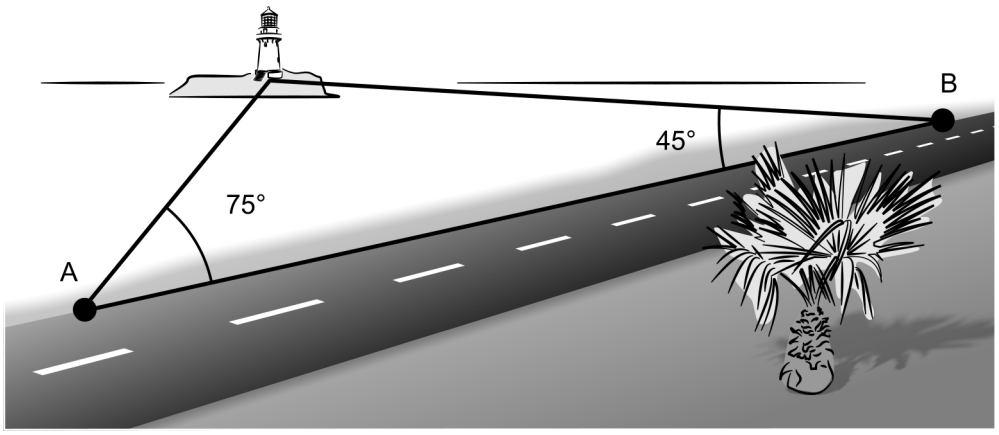
- Show that your hypothesis is true regardless of which value of *a* you choose. (3/4/¤)

Part II

This part consists of 7 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11. A triangle has sides of length 4.0 cm, 6.0 cm and 7.0 cm. Determine the area of the triangle. (3/0)

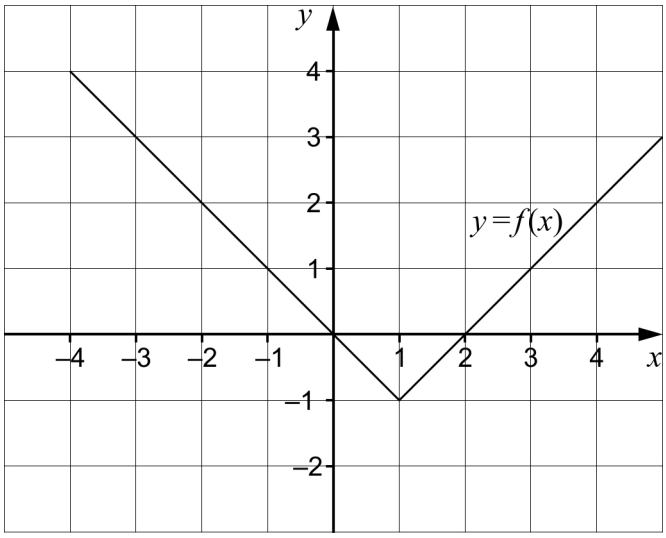
12.



A straight road runs 4.0 km along the shoreline from point A to point B. When Amir stands at point A he sees a lighthouse. His line of sight towards the lighthouse makes an angle of 75 degrees with the road. When standing at point B, Amir's line of sight back towards the lighthouse makes an angle of 45 degrees with the road.

Calculate the perpendicular distance from the road to the lighthouse. (2/0)

13. Determine $\int_{-2}^{4} f(x)\,\mathrm{d}x$ using the figure. (1/1)



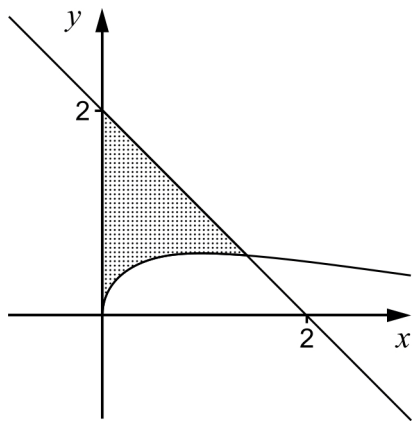
14. During an experiment in a laboratory, running over 7 days, the temperature in a sample is measured regulary. It has been established that the relation between time and temperature can be described by

$y(x) = 122 + 20\sin\frac{\pi(x+1.5)}{7}$, where $y(x)$ is the temperature in °C and x is the time measured in days from the start of the experiment.

a) What is the temperature of the sample when the experiment is initiated? (1/0)

b) At what rate is the temperature changing at the moment when the experiment has been running for 5.0 days? (1/1)

15.



A region in the first quadrant is bounded by the curve $y = \sqrt{x} \cdot e^{-x}$, the line $y = 2 - x$ and the y-axis.

Write down an expression for the area of the region using integrals and determine the area correct to two decimal places. (1/2)

16. The following is known about the polynomial function *f*:

- $f'(0) = 1$
- $f'(3) = -3$
- $f''(x) < 0$ for $x > 0$

What conclusions can be drawn regarding extreme points of *f* within the interval $x > 0$? (0/2/¤)

17. The function $y = f(x)$ is defined in the interval $0 \le x \le 6$ and is an increasing function. The following values are known:

x	$f(x)$
0	1.0
1	1.2
2	1.7
3	3.0
4	3.5
5	3.6
6	5.0

a) Make use of all the values in the above table to calculate an approximation of the value of $\int_0^6 f(x)\mathrm{d}x$ (2/0)

b) Bind the value of $\int_0^6 f(x)\mathrm{d}x$ with as narrow an interval as possible, that is determine *A* as large as possible and *B* as small as possible so that $A \le \int_0^6 f(x)\mathrm{d}x \le B$ for all functions $y = f(x)$ that are increasing and have the values specified in the table. (0/2/¤)