NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2010

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 135 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 11 problems and **Part II** consists of 6 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 11 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test

Pass:	13 points.
Pass with distinction:	25 points of which at least 6 ”Pass with distinction”- points.
Pass with special distinction:	25 points of which at least 13 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 11 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Evaluate $\int_1^2 2x^3\,dx$ (2/0)

2. a) Express $210°$ in radians. *Only answer is required* (1/0)

b) Express -3π in degrees. *Only answer is required* (1/0)

3. Differentiate

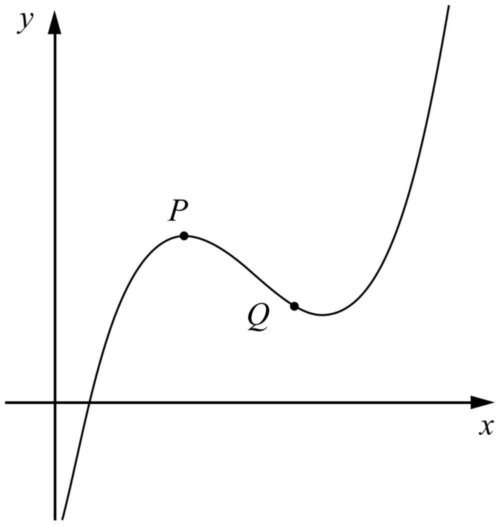
a) $g(x) = 4\sin 5x$ *Only answer is required* (1/0)

b) $h(x) = \dfrac{3x}{x^2+1}$ *Only answer is required* (0/1)

4. Determine the antiderivative F of $f(x) = e^{3x}$ for which $F(0) = \dfrac{4}{3}$ (2/0)

5. For which angles v in the interval $0° \le v \le 360°$ is it true that $\cos v < \cos 160°$? (1/1)

6. The figure below shows the cubic curve $y = f(x)$. The points P and Q lie on the curve. P is a local maximum.
The table below shows different alternatives A-H of the signs of $f'(x)$ and $f''(x)$.

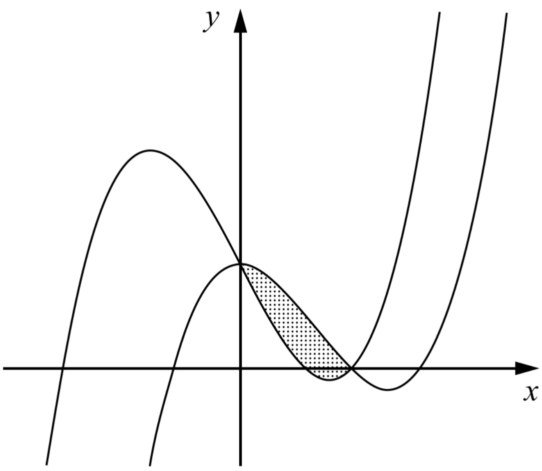


	$f'(x)$	$f''(x)$
A	+	+
B	+	0
C	+	–
D	0	+
E	0	–
F	–	+
G	–	0
H	–	–

a) Which of the alternatives A-H is applicable to point P?
Only answer is required (1/0)

b) Which of the alternatives A-H is applicable to point Q?
Only answer is required (0/1)

7. The figure shows a region bounded by the curves $y = x^3 - 2x + 1$ and $y = x^3 - 2x^2 + 1$



Write down an integral expression for the area of the region and evaluate this integral. (1/2)

8. Find all solutions to the equation $f'(x) = 0$ in the interval $0 \le x \le \pi$ when $f(x) = 2x + \cos 4x$ (1/2)

9. Show that $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$ for all x where expressions on both sides are defined. (0/2/¤)

10. Eva and Kerstin discuss how to solve the following problem:

Show that $F_1(x) = \dfrac{1}{1 - \sin x}$ and $F_2(x) = \dfrac{2\sin x - 1}{1 - \sin x}$ are antiderivatives of the same function.

Eva says that she will investigate the derivative of the given functions. Kerstin says that it is also possible to solve the problem by investigating the difference of the antiderivatives.

a) Solve the problem by using one of the methods. (0/2)

b) Explain why Kerstin's method works. (0/0/¤)

When assessing your work with this problem, the teacher will take into consideration:

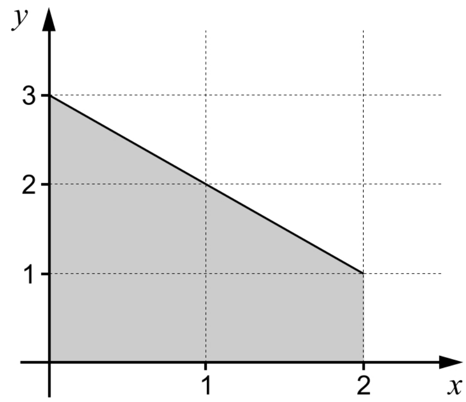
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

11. In this problem you are going to investigate the relation between area and integral.

The figure shows $y = g(x)$ in the interval $0 \le x \le 2$

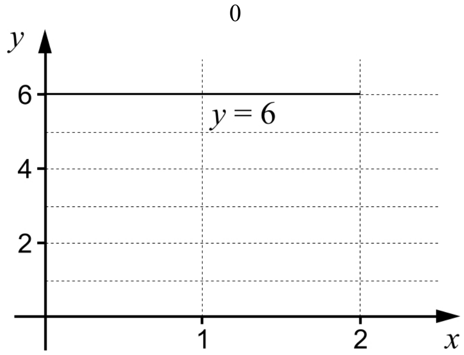
The area of the shaded region is 4 area units.
Therefore, it holds that

$$\int_0^2 g(x)\mathrm{d}x = 4$$

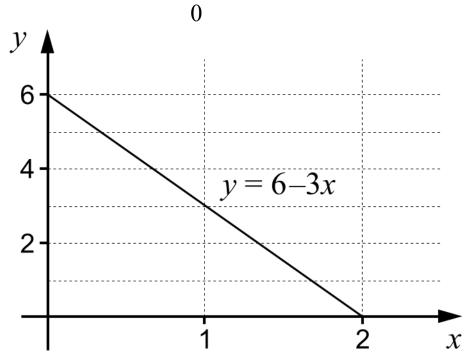


- Evaluate the integrals A and B, for example by interpreting them as areas.

$$A = \int_0^2 6\mathrm{d}x$$



$$B = \int_0^2 (6-3x)\mathrm{d}x$$

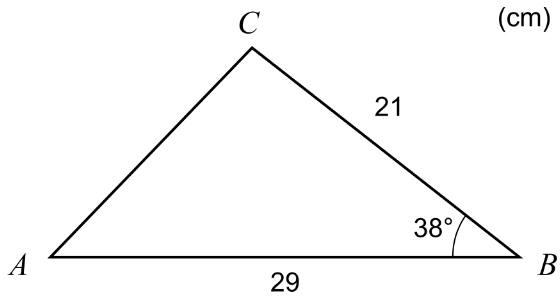


- Give two examples of a linear function of the form $y = kx + m$ that has the feature $\int_0^2 (kx+m)\mathrm{d}x = 1$
- Find a general relationship between k and m that must be true if $\int_0^2 (kx+m)\mathrm{d}x = 1$
- Find which other condition for the value of k or m that must be satisfied if $\int_0^2 (kx+m)\mathrm{d}x$ is to be interpreted as the area of a region in the first quadrant and $\int_0^2 (kx+m)\mathrm{d}x = 1$ (3/3/¤)

Part II

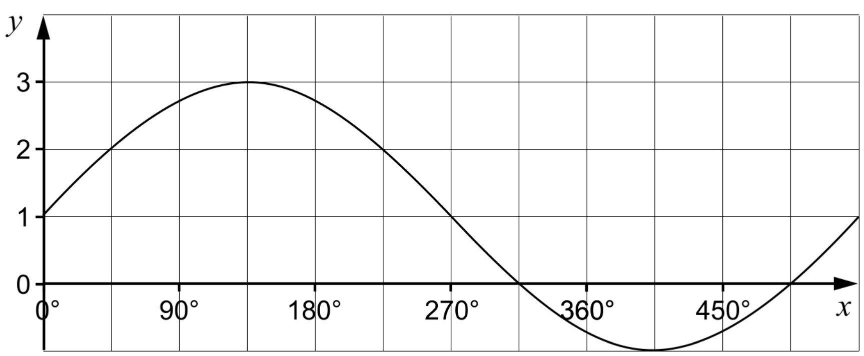
This part consists of 6 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

12. The figure shows the triangle *ABC*. Calculate the length of the side *AC*.



(2/0)

13. A function of the type $y = A + B\sin kx$ can be represented by the graph below.

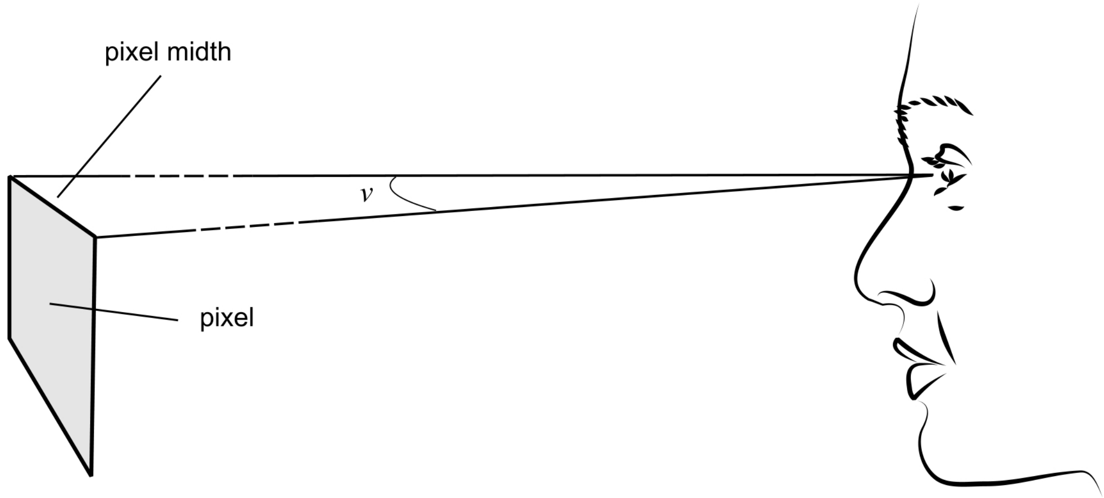


Determine the values of *A*, *B* and *k*. *Only answer is required* (2/1)

14. To make sure that the picture quality of a TV feels as good as possible you should be seated at the distance where you can begin to distinguish a single picture element, that is, one pixel. If you are seated too far from the TV, the details become blurred and if you are seated too close you can see the single pixels.

With normal sight, the resolution v of the human eye is approximately $0.0167°$

A 50-inch TV is 1107 mm wide. The TV has full HD resolution which means that the number of pixels horizontally is 1920



At what distance should a person with normal sight be seated to experience as high a picture quality as possible, that is to begin to distinguish the single pixels? (3/0)

15. A tank that initially is empty is filled with water at a rate of $(8.0 + e^{0.01t})$ litres/min, where t is the time in minutes from the start of the refilling.

a) Calculate the water volume of the tank 60 minutes after the start of the refilling. (0/1)

b) How long does it take from the start of the refilling until the water volume is 2500 litres? (0/2)

16. Peter and Marcus have been given the task of using the derivative to investigate whether the function $f(x) = x^4 - x^5$ has a local maximum, local minimum or saddle point at $x = 0$

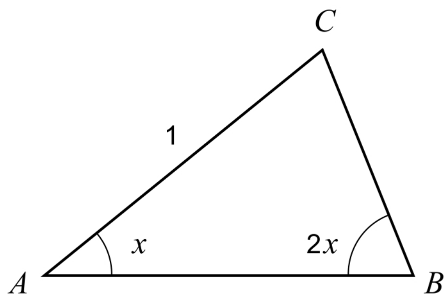
They start by differentiating and notice that $f'(0) = 0$. After that, they are going to investigate whether $x = 0$ is a local maximum, local minimum or saddle point of the function.

”I’m going to study the sign of the derivative $f'(x)$”, says Peter.

”It is easier to use the second derivative. I’m going to solve the problem by calculating $f''(0)$”, Marcus answers.

Evaluate and compare the boys’ methods and solve the problem. (1/1/¤)

17. Side *AC* in the triangle *ABC* has a certain length, 1 length unit (l.u.). The other sides and the angles of the triangle may vary, but the angle *ABC* should always be twice the size of the angle *CAB*.



[l.u.]

What value of angle *A* gives the largest possible value of the area of the triangle? (0/3/¤)