NATIONAL TEST IN MATHEMATICS COURSE D
AUTUMN 2011

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 135 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 16 problems. **Part I** consists of 10 problems and **Part II** consists of 6 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 10 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test

Pass:	13 points.
Pass with distinction:	26 points of which at least 8 ”Pass with distinction”- points.
Pass with special distinction:	26 points of which at least 15 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. a) Express the angle $20°$ in radians. *Only answer is required* (1/0)

b) Express the angle $\frac{\pi}{10}$ in degrees. *Only answer is required* (1/0)

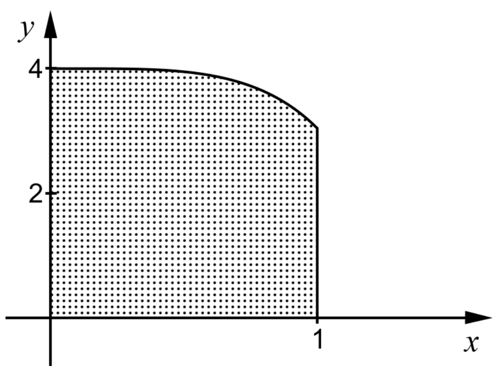
2. Differentiate

a) $f(x) = \cos 4x$ *Only answer is required* (1/0)

b) $g(x) = x \cdot \ln x$ *Only answer is required* (1/0)

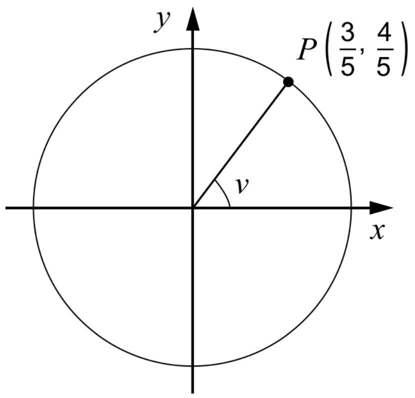
c) $h(x) = (\sin x + 1)^2$ *Only answer is required* (0/1)

3. The figure shows a region bounded by the curve $y = 4 - x^4$, the line $x = 1$ and the positive coordinate axes.



Calculate the area of the region. (2/0)

4. The figure shows a unit circle where an angle v and a point P are marked.



Determine

a) $\tan v$ *Only answer is required* (1/0)

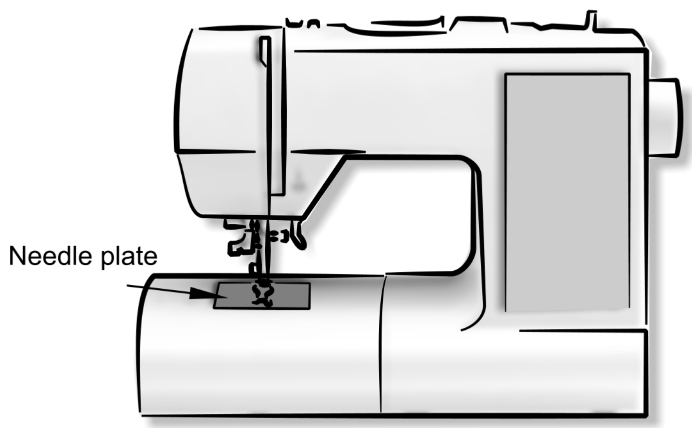
b) $\cos(-v)$ *Only answer is required* (1/0)

c) $\cos(v + 90°)$ *Only answer is required* (0/1)

5. The needle on a sewing machine moves up and down when sewing. The height of the needle point above the needle plate as a function of time can be described by

$h(t) = 1.6\cos(20\pi \cdot t) + 0.2$

where h is the height in cm and t is the time in seconds.



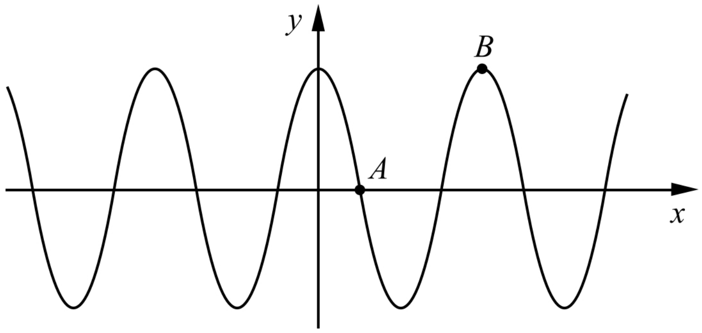
a) Calculate the greatest height of the needle point above the needle plate. (1/0)

b) How many times per second does the needle point sit at its highest position? (0/2)

6. The figure shows the curve $y = 5\cos 3x$, where one zero A and one global maximum B are marked.

a) Determine the x-coordinate of the zero A. (0/1)

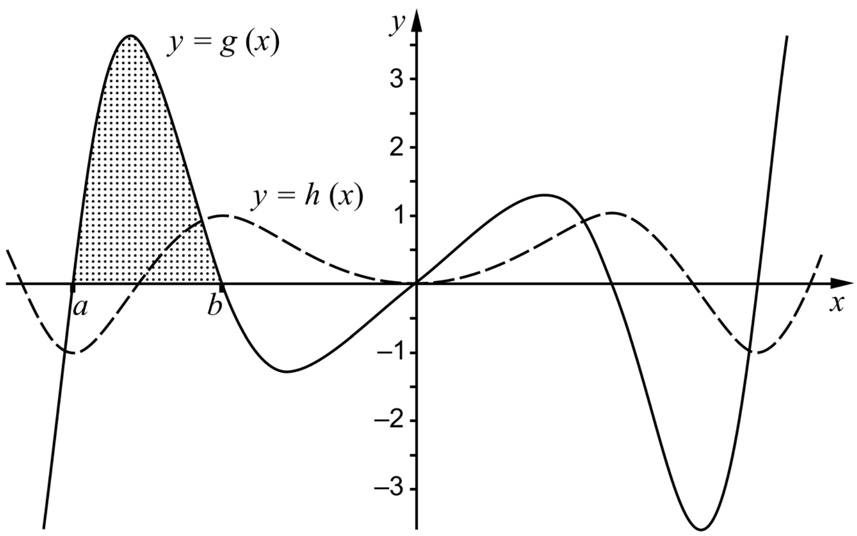
b) Determine the coordinates of the global maximum B. (1/1)



7. Determine the constant k so that the function $f(x) = 2x \cdot e^{kx}$ has a local maximum at $x = 2$ (0/3)

8. Show that $\dfrac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$ for all x where the expressions on both sides are defined. (0/2/¤)

9. The figure shows a function and its derivative. Use the figure to calculate the area of the shaded region. (0/1/¤)



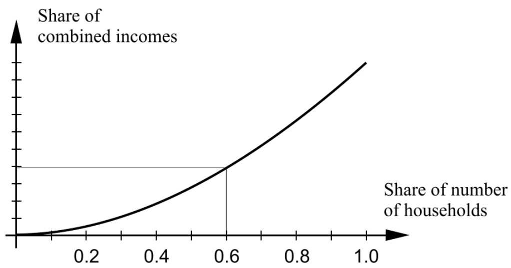
When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

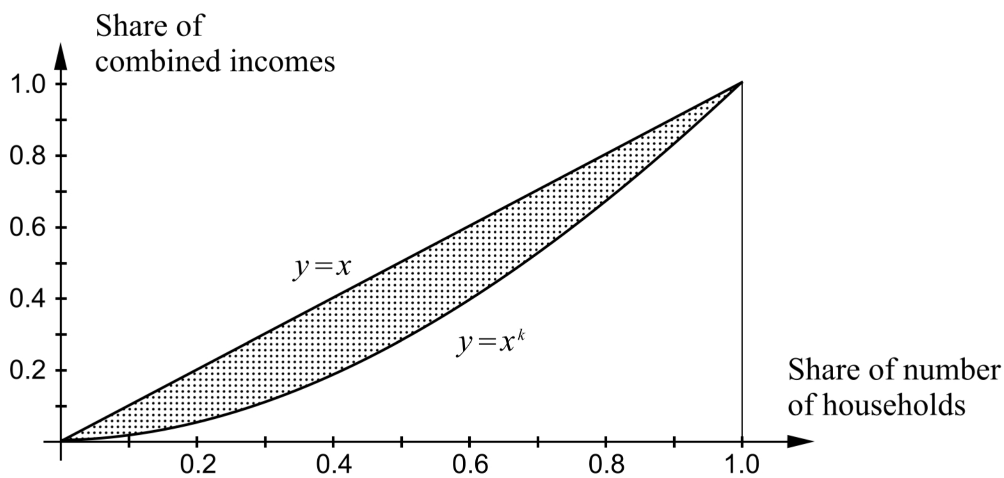
10. This problem deals with a method used to compare different countries with respect to the income distribution of the households. A so-called Lorenz curve is often used to describe the income distribution in a country. See fact box.

Fact box on the Lorenz curve

When the curve is constructed, the households are ranked by income, from the lowest to the highest. The horizontal axis represents the part of the households with the lowest income after ranking. The vertical axis represents the households' combined incomes. The curve shows for example that 60 % of the households with the lowest income have 39 % of the households' combined incomes.



The Lorenz curve does often have the form $y = x^k$ for any $k \geq 1$



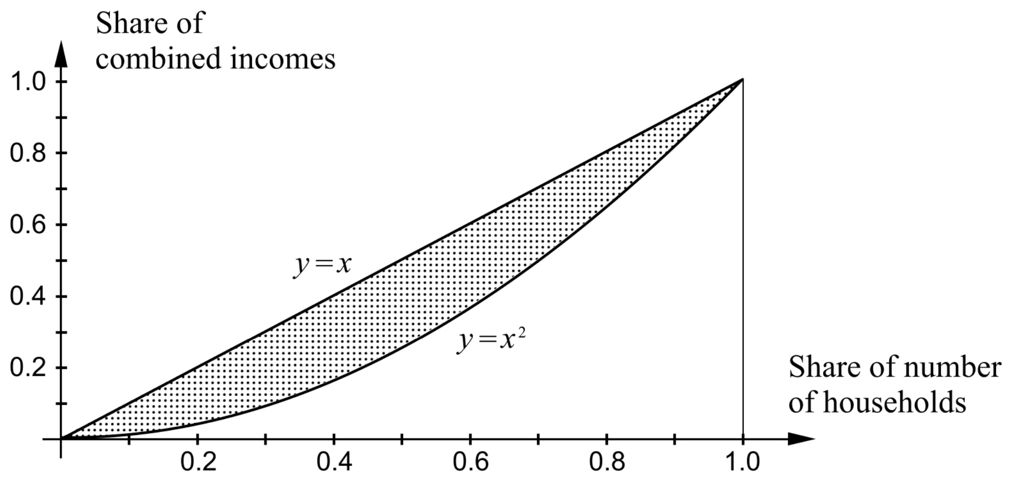
If all households in one country would have the same income, the country's Lorenz curve would coincide with the line $y = x$. The more a country's Lorenz curve differs from the line $y = x$, that is, the larger the area of the shaded region is, the more unequal is the income distribution.

To get a measure of the income distribution of different countries,
the Gini coefficient G is used, where

$$G = \frac{\text{Area of shaded region}}{\text{Area of triangle}}$$

(The area of the triangle is the area of the triangle bounded by $y = x$, $x = 1$ and the x-axis, see figure above.)

- Assume that a country has the Lorenz curve $y = x^2$. First, calculate the area of the shaded region and then G for this country.



- Calculate G for a country with a different Lorenz curve, for example $y = x^3$

The table below shows the Gini coefficient in the year 2000 for some countries.

Italy	0.35
Japan	0.31
Sweden	0.24
Turkey	0.44
USA	0.36

- What information can you get from the table about the income distribution in Italy compared to Sweden?

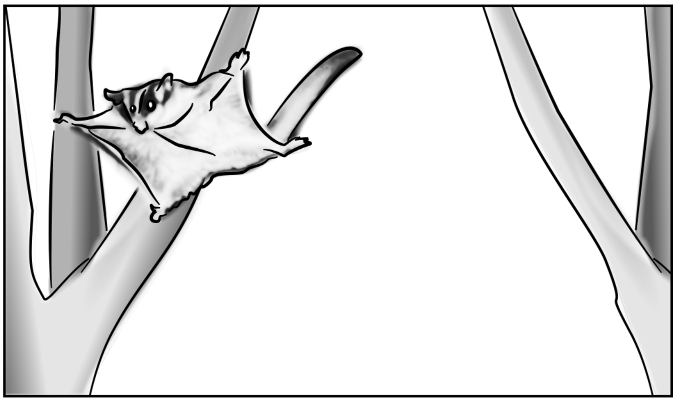
- Write down a general expression for G expressed in k when the Lorenz curve is $y = x^k$

- What values can G assume if $k \geq 1$ (4/3/¤)

Part II

This part consists of 6 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11. The flying squirrel is a small squirrel that lives in trees. By using the flap of furry skin that stretches from the front- to the back legs it can glide long distances. The gliding of a squirrel can be described as a straight line with an inclination of $27°$ to the horizontal plane.



At what height does the squirrel have to start to avoid ending up on the ground during a flight between two trees that are 19 metres apart? (2/0)

12. Calculate the area of the region bounded by the curve $y = \dfrac{10\sqrt{x}}{x+1}$

and the line $y = \dfrac{x}{3}$

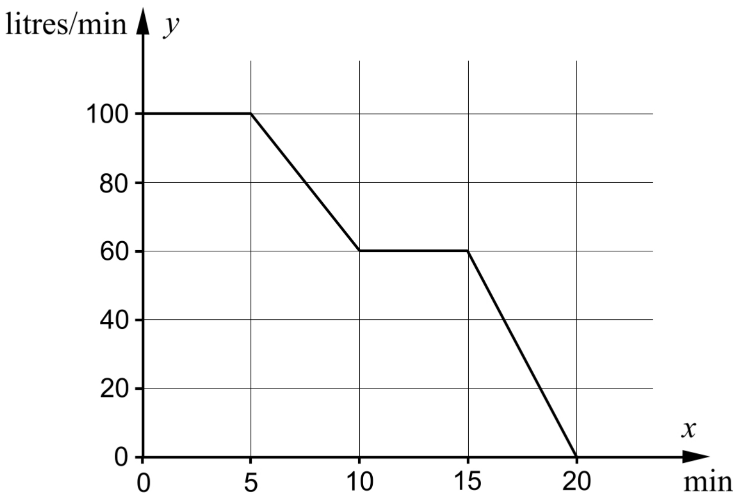
Give answer correct to at least three significant digits. (3/0)

13. In the triangle ABC side $AB = 8.0\text{ cm}$ and side $BC = 12\text{ cm}$.

a) Calculate the third side if angle $B = 50°$. (2/0)

b) Determine what values the length of the third side can have if angle B varies. (0/1)

14. Water is pumped into an initially empty water container for 20 minutes at a velocity of y litres/minute. The velocity varies with time x minutes according to the graph below.



a) How many litres of water have been pumped into the tank during these 20 minutes? (1/0)

b) Evaluate $\frac{1}{20}\int_0^{20} y \, \mathrm{d}x$ and interpret the calculated value. (0/2)

15. It holds for the function f that:

- $f'(0) = 1$
- $f'(3) = -3$
- $f''(x) < 0$ for all x

Show that f has exactly one global maximum. (0/2/¤)

16. In the triangle ABC, $AB = 23.0\text{ cm}$ and $AC = 19.0\text{ cm}$. Angle B is $25°$ larger than angle A. Calculate side BC to three significant digits. (0/3/¤)