# NATIONAL TEST IN MATHEMATICS COURSE D
# AUTUMN 2012

## Directions

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|---|---|
| Test time | 240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.** |
| Resources | **Part I:** ”Formulas for the National Test in Mathematics Course D”. *Please note that calculators are not allowed in this part.*<br><br>**Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D”. |
| Test material | The test material should be handed in together with your solutions.<br><br>Write your name, the name of your education programme/adult education on all sheets of paper you hand in.<br><br>*Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.* |
| The test | The test consists of a total of 19 problems. **Part I** consists of 11 problems and **Part II** consists of 8 problems.<br><br>For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.<br><br>Problem 11 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.<br><br>Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions. |
| Score and mark levels | The maximum score is 44 points.<br><br>The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.<br><br>Lower limit for the mark on the test |

| | |
|---|---|
| Pass: | 13 points. |
| Pass with distinction: | 25 points of which at least 7 ”Pass with distinction”- points. |
| Pass with special distinction: | 25 points of which at least 14 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show. |

# Part I

**This part consists of 11 problems that should be solved without the aid of a calculator.** Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

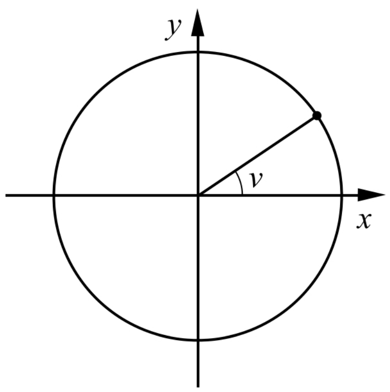
**1.** Differentiate

a) $f(x) = \sin 4x$ *Only answer is required* (1/0)

b) $g(x) = (3x+1)^5$ *Only answer is required* (1/0)

**2.** Evaluate $\int_1^2 (2x^3 - 2x)\,dx$ (2/0)

**3.** The angle $v$ marked in the figure is $v = 40°$




Give two other angles with the same sine value as angle $v$.

*Only answer is required* (2/0)

**4.** Find the antiderivative $F$ of $f(x)=\frac{1}{x}+x-2$ that satisfies the condition $F(1)=\frac{1}{2}$ (2/0)

**5.** Find all the solutions to the equation $\cos 3x=\frac{1}{2}$ (1/2)

**6.** Two of the expressions A-E below equal $\sin 40°$
Which ones? *Only answer is required* (0/1)

A. $\sin 30°+\sin 10°$

B. $2\sin 20°$

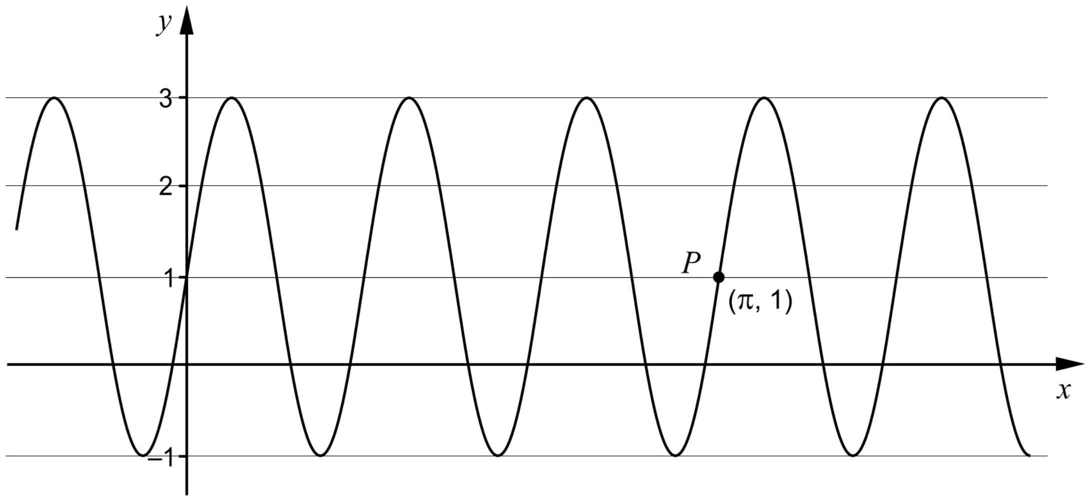
C. $\tan 40°\cdot\cos 40°$

D. $\frac{\sin 80°}{2}$

E. $\frac{\sin 80°}{2\cos 40°}$

**7.** Show that the function $f(x)=2\sin 3x+\cos x-6x$ has a maximum point at $x=0$ (2/1)

**8.** The figure shows a trigonometric curve where the point $P = (\pi, 1)$ is marked.




The equation of the curve can be written in the form $y = A\sin kx + B$

a) What are the values of the constants $A$ and $B$?

*Only answer is required* (2/0)

b) Determine the value of the constant $k$. (0/1)

**9.** The sides of a triangle are 4, 8 and 9 length units respectively.
Investigate whether the triangle is obtuse. (0/2)

**10.** A function is defined by $f(x) = \dfrac{1}{1 + \sin x}$

Investigate the possible values of $f(x)$. (0/2/¤)

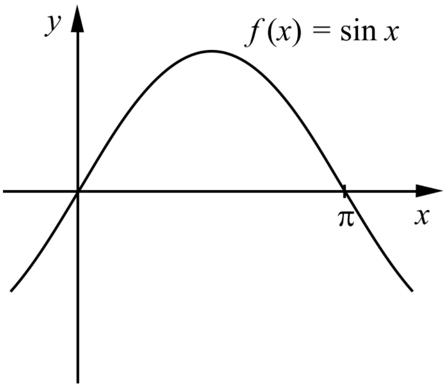
**When assessing your work with this problem, the teacher will take into consideration:**
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

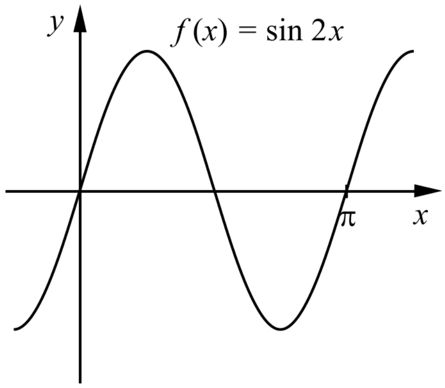
**11.** In this problem you are going to investigate how the value of the definite integral

$$I = \int_0^{\pi} \sin kx \, dx$$

depends on $k$, where $k$ is a positive integer.

- Start by calculating $I$ when $k = 1$ and when $k = 2$




- Calculate $I$ for additional values of $k$ and present the results in a table.

| $k$ | $I$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| | |
| | |

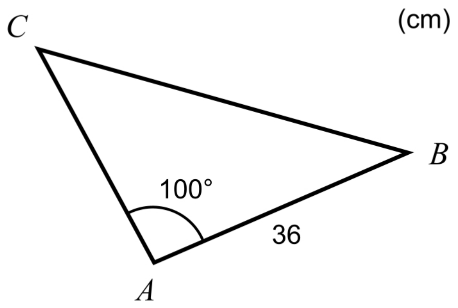
- Formulate a conclusion on how the value of the integral $I$ depends on the value of the constant $k$.

- Show that your conclusion holds for all positive integers $k$. (2/4/¤)

# Part II

**This part consists of 8 problems and you may use a calculator when solving them.**
Please note that you may begin working on Part II without a calculator.

**12.** The area of the triangle *ABC* is $520\ \text{cm}^2$. What is the length of side *AC*?

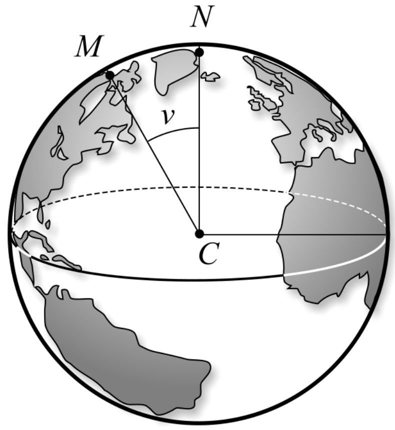



(2/0)

**13.** Find the area of the region bounded by the curve $y = 30x - 3x^2$ and the $x$-axis. (2/0)

**14.** In the figure below, the North Magnetic Pole is marked with *M,* the Geographic North Pole with *N* and the Centre of the Earth with *C*.

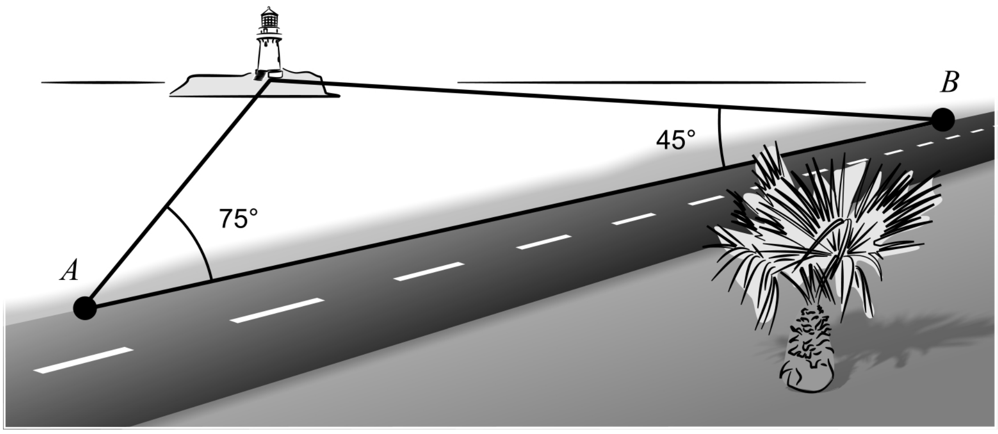
*M* approaches *N* so that the central angle *v* changes with 0.0017 radians per year.




How many years will it take for the central angle to change by $1.0°$?

*Only answer is required* (1/0)

**15.**

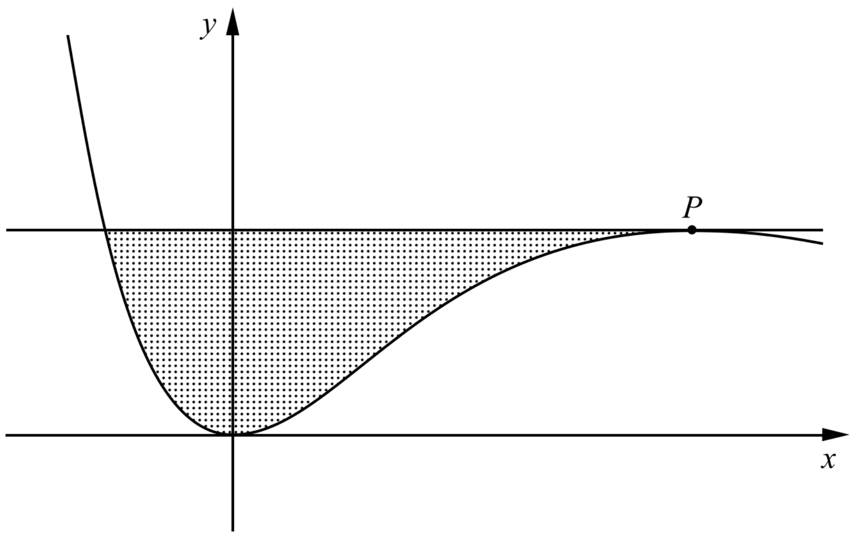



A straight road runs 4.0 km along the shoreline from point *A* to point *B*. When Amir stands at point *A* he sees a lighthouse. His line of sight towards the lighthouse makes an angle of 75 degrees with the road. When standing at point *B*, Amir's line of sight back towards the lighthouse makes an angle of 45 degrees with the road.

Calculate the perpendicular distance from the road to the lighthouse. (2/0)

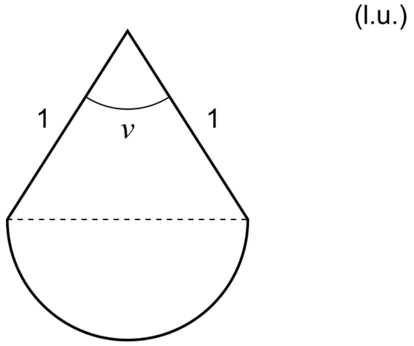
**16.** Show that $\dfrac{\cos x - \cos^3 x}{\sin 2x} = \dfrac{\sin x}{2}$ for all $x$ where the expressions are defined. (0/2/¤)

**17.** The figure shows the curve $y = x^2 e^{-0.5x}$ and the tangent to the curve at the maximum point *P*.



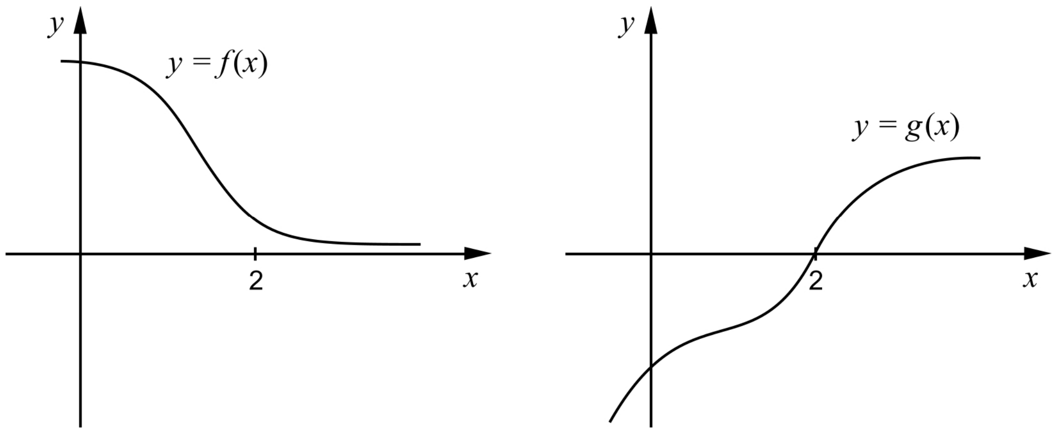

Calculate the area of the shaded region.
Answer to at least three significant digits. (0/3)

**18.** The figure shows a region consisting of an equilateral triangle and a semi-circle. Two of the sides in the triangle have the length 1 length unit, see figure.




Determine the angle $v$ so that the area of the region is as large as possible. Answer to at least three significant digits. (0/2/¤)

**19.** The figures show the curves $y = f(x)$ and $y = g(x)$.




Let $h(x) = f(x) \cdot g(x)$

Investigate whether $h(x)$ is increasing in the interval $0 \le x \le 2$ (0/2/¤)