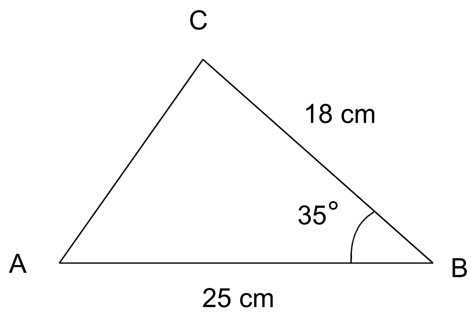
Directions

Test period	December 4 - December 18 1997.
Test time	180 minutes without a break.
Resources	A non-symbol manipulating graphics calculator and a collection of formulas.
Test material	Test material should be handed in together with your solutions. Write your name, gymnasium programme/adult education and date of birth on all the papers you hand in.
Test	The test is made up of 15 problems. Most of the problems are of long-answer type. With these problems, it is not enough to give a short answer, it requires: • that you write down what you do • that you explain your train of thought • that you draw figures when needed • that you show how you use your calculator in numerical and graphical problem solving. For some questions, (where it says ”*Requires only an answer*”) only the answer needs to be given. Try all of the problems. It can be relatively easy, even towards the end of the test, to earn some points for a partial solution or presentation.
The grading levels	The teacher responsible will explain the scores which are required for ”Passed” and ”Passed with Distinction”. On the test one can attain a maximum of 45 points.

1. Find $f'(x)$ if
   a) $f(x) = 5\sin x$ b) $f(x) = 4\cos 5x$ *Requires only an answer* (2p)

2. ABC is a triangle as shown in the figure. Calculate the length of AC.

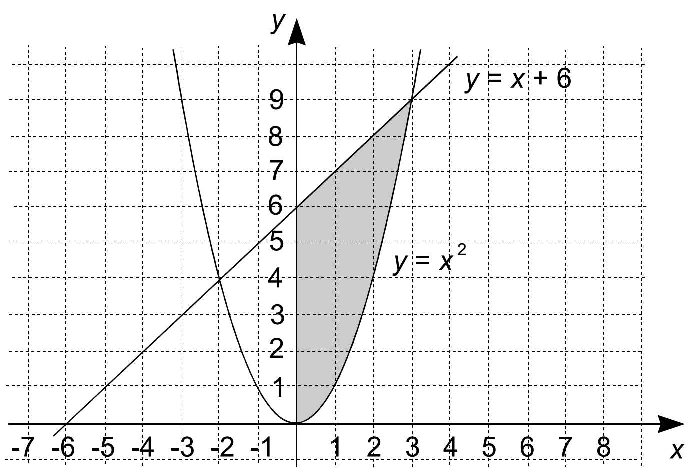


(2p)

3. Use the antiderivative to calculate the integral $\int_1^3 \frac{x^2}{3}dx$ exactly. (2p)

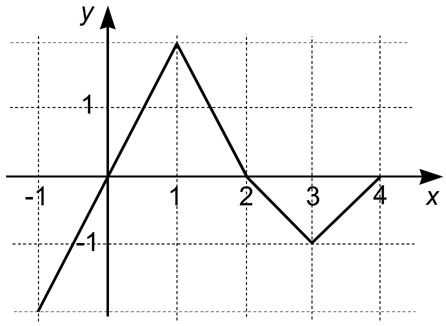
4. In a triangle ABC the angle A is 64.4° and the angle B is 41.4°. The length of side AC is 137 cm. Calculate the length of BC. (3p)

5. Find an expression for an exact calculation of the shaded area. You do not need to calculate the area. (2p)



6. The graph of the function $y = f(x)$ is shown in the figure.
Calculate the integral

$$\int_0^3 f(x)dx$$

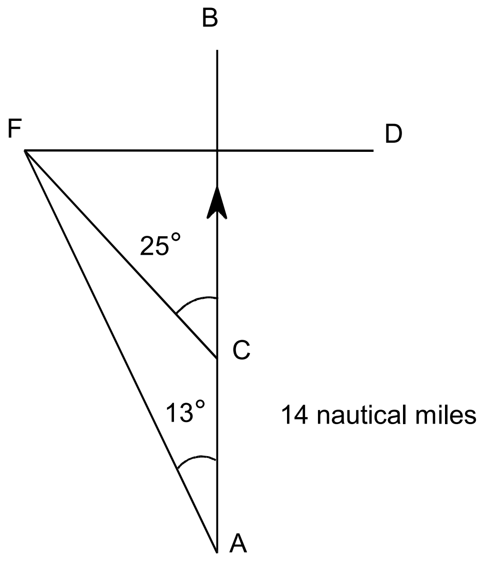


(2p)

7. Solve the equation $1 + \cos x = 2$. (2p)

8. Examine graphically, and show with a simple sketch if there exists any v that satisfies
$2\sin\left(v + 12^\circ\right) = \cos\left(v + 23^\circ\right)$ in the interval $0° < v < 180°$.
If so, state that/those numbers in your answer. (3p)

9. A ship moves from A towards B in accordance with the figure. F is a lighthouse, and the angles 13° and 25° have been measured as the angles between the direction of the ship and the lighthouse in the positions A and C respectively. The distance between A and C is 14 nautical miles. Along the line FD, which is perpendicular to AB, there is a sunken rock 8 nautical miles from F. Is the ship going to hit the rock? (3p)



Note! The figure is not according to scale.

10. A skater fell into a hole in the ice which caused his body temperature to decrease rapidly. Let us assume that the rate of change of temperature is proportional to the body temperature, y °C, described by

$\frac{dy}{dt} = -0.011 \cdot y$ where t is the time in minutes that the skater has been in the water.

a) Show that $y = 37 \cdot e^{-0.011t}$ is a solution to $\frac{dy}{dt} = -0.011 \cdot y$ (2p)

b) How long period of time does it take for the skater´s body temperature to decrease from 37 °C to 31 °C? (2p)

c) What is the rate of change of the body temperature 5 minutes after a person with the temperature 37 °C falls into a hole in the ice? (2p)

11. Show that $y = \cos x \cdot \sin x$ satisfies $2y - y' \cdot \tan 2x = 0$ (3p)

12. The graph of the function $y = f(t)$, $0 \le t \le 9$, is shown in the figure.

Let $g(x) = \int_0^x f(t)dt$

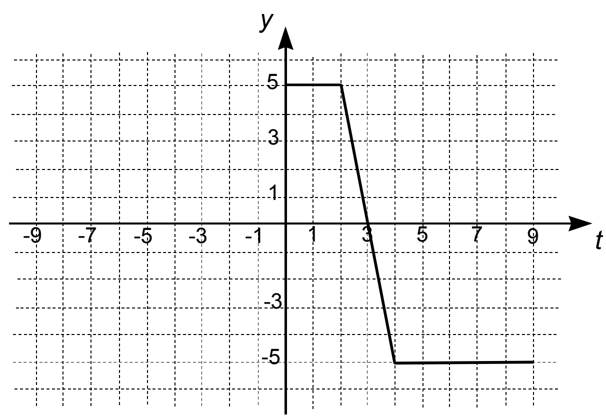
Answer required only to the four problems below.

a) Find $g(2)$

b) Find the highest value of $g(x)$

c) Find the extreme values, if any, of the function $g(x)$ in the interval $0 \le x \le 9$

d) For which number x is $g(x) < 0$?



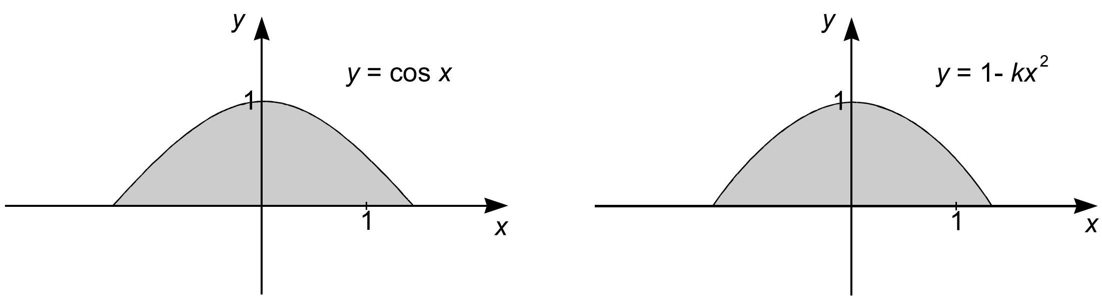
(4p)

13. The flow of traffic to a densely populated area could approximatively be described by the following mathematical model

$$y = 108\sin\frac{\pi x}{12} + 320$$

where y is the number of cars that passes per hour, and x is the number of hours after 7 am.
How many cars pass between 7 am and 5 pm? (3p)

14. The function $y = \cos x$ is approximated with $y = 1 - kx^2$ so that the areas below are of equal size. Find the constant k. (4p)

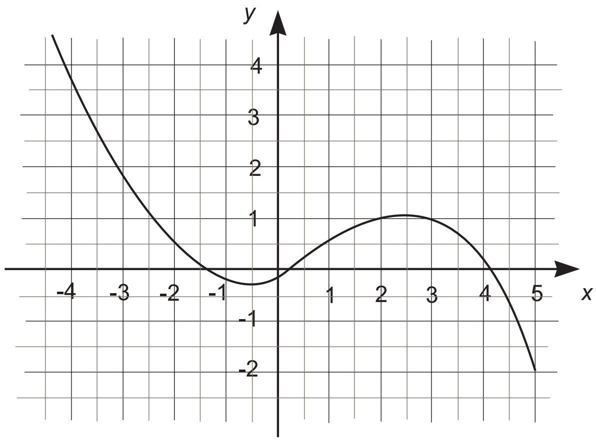


15. The figures below show the graphs of $y = f(x)$ and $y = g(x)$ and their derivatives.
A new function $h(x) = f\left(g(x)\right)$ is formed.
Use the figures to evaluate

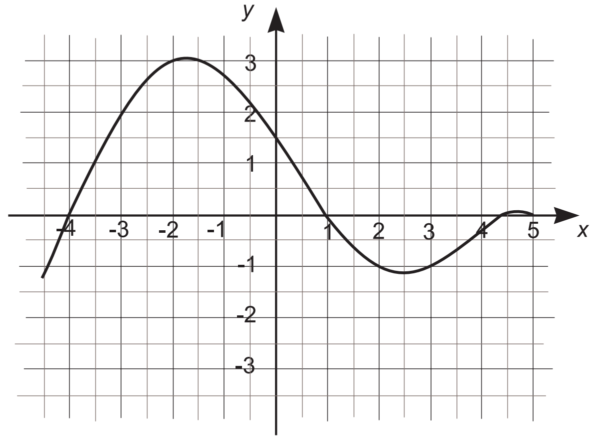
a) $h(-2)$ (2 p)

b) $h'(-2)$ (2 p)

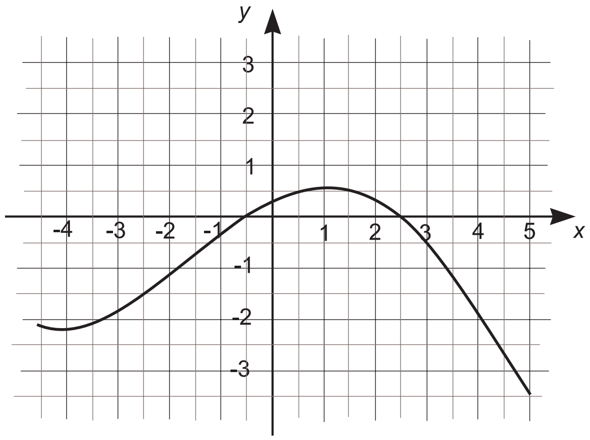
$y = f(x)$



$y = g(x)$



$y = f'(x)$



$y = g'(x)$