NATIONAL TEST IN MATHEMATICS COURSE D SPRING 2001

Directions

Test time	240 minutes without a break.
Resources	Graphic calculators and "Formulas for National Test in Mathematics Courses C, D and E".
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in.
The test	The test consists of 15 problems. To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 15 is a larger problem which may take up to one hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 43 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 "Pass"-points and 1 "Pass with distinction"-point this is written (2/1). Lower limit for the mark on the test Pass: 13 points Pass with distinction: 24 points of which at least 7 "Pass with distinction points"

Name: ______________________________ School: ______________________________

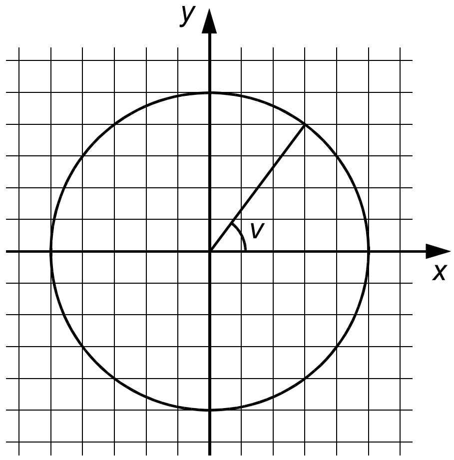
Education programme/adult education: ____

1. Use the primitive functions and calculate $\int_0^2 (x^2 + 3)\mathrm{d}x$ (2/0)

2. Write down all the primitive functions F to $f(x) = 2x + 5$

Only answer is required (2/0)

3. The figure below shows a unit circle.



a) Find $\sin v$ *Only answer is required* (1/0)

b) Find $\sin(180° - v)$ *Only answer is required* (1/0)

4. Let $f(x) = \sin 3x$

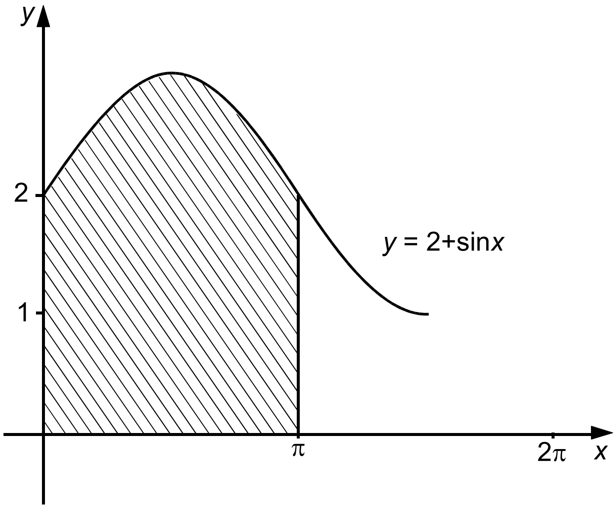
a) Determine $f'(x)$ *Only answer is required* (1/0)

b) Evaluate $f'(0)$ *Only answer is required* (1/0)

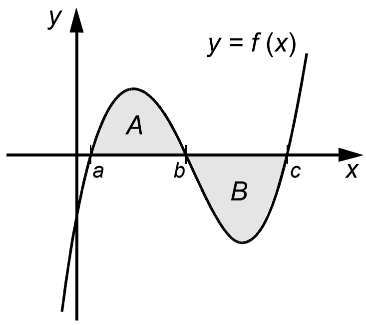
c) Find all the solutions to $f'(x) = 0$ (2/0)

5. In the triangle ABC side AB has length 12.0 cm, angle A equals $42.5°$ and angle C equals $32.3°$.
Calculate the length of side BC. (2/0)

6. Calculate the exact value of the area of the shaded region in the figure below. (2/0)



7. Together with the x-axis, the graph of the function $y = f(x)$ bounds two regions with areas A and B area units as indicated. The graph intersects the x-axis at points a, b and c.



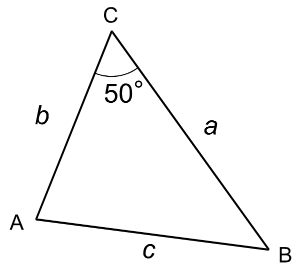
Using integrals, write down an expression for

a) A *Only answer is required* (1/0)

b) $B - A$ *Only answer is required* (0/1)

8. Simplify as much as possible the expression $(\cos x + \sin x)^2 - \sin 2x$ (2/0)

9. In the triangle ABC, angle C is $50°$.



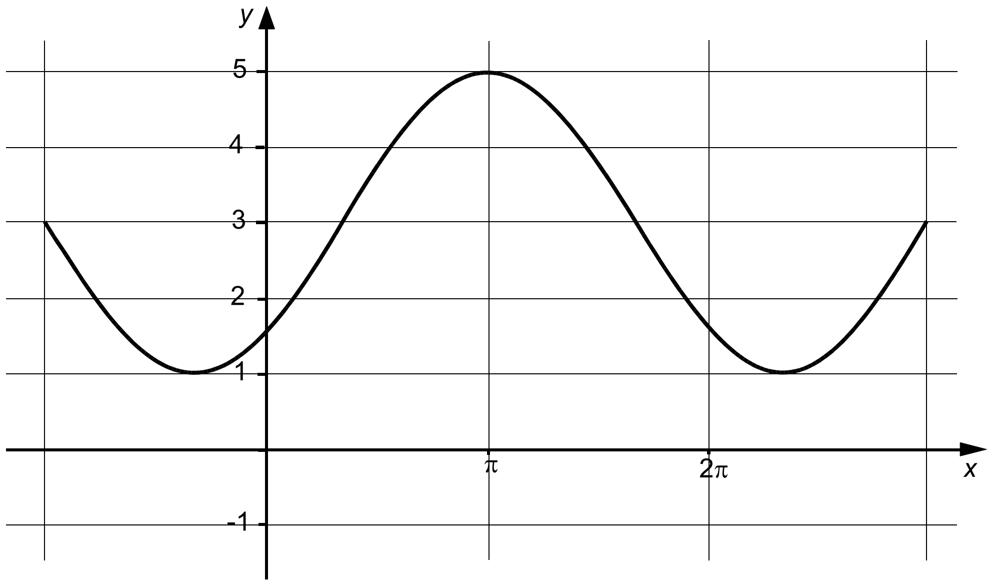
Choose a and b so that the area A of the triangle is given by $A = 12\sin 50°\ \text{cm}^2$. (0/2)

10. Show that $y = x^2 \sin x$ is a solution to the differential equation
$xy' - 2y = x^3 \cos x$ (0/2)

11. The function $y = f(x)$ has a primitive function $F(x) = Ax^2 + Bx$ where A and B are constants.

Determine A and B if $\int_0^1 f(x)\mathrm{d}x = 2$ and $\int_0^2 f(x)\mathrm{d}x = 0$ (0/3)

12. On the sine curve $y = A\sin(Bx + C) + D$ one of the maximum points has coordinates (π, 5). One of the two adjacent minimum points has coordinates ($\frac{7\pi}{3}$, 1), as indicated.



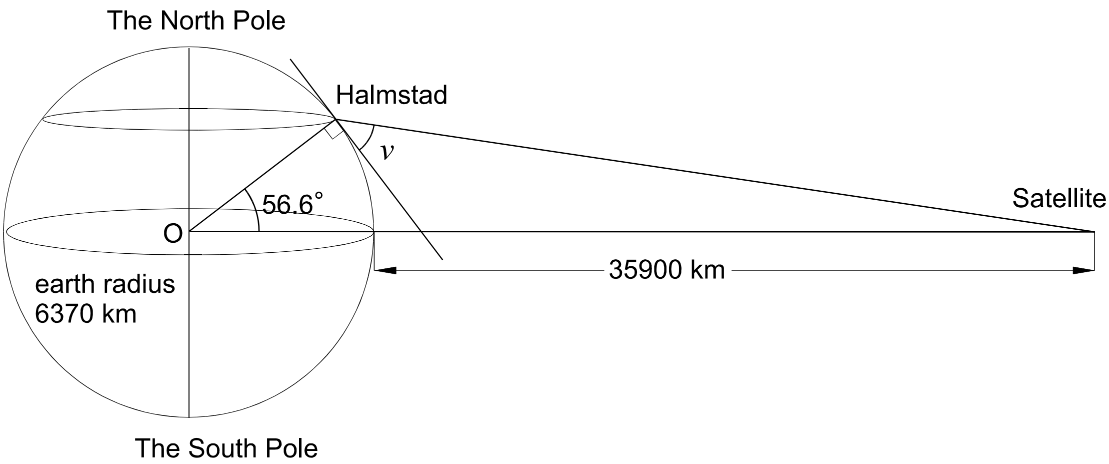
a) Determine A and D. *Only answer is required* (2/0)

b) Determine B and C. (0/3)

13. Stina, who lives in Halmstad, has bought a satellite dish. She plans to mount it on the roof of her house. How should she direct the satellite dish so as to get the best possible reception of TV-signals from a satellite? A TV- satellite remains stationary at a height of 35900 km above the equator (see figure below). Halmstad is situated at a latitude 56,6° north and the earth can be assumed to be a sphere with radius 6370 km.

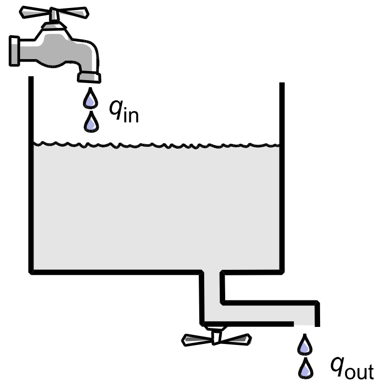
At what angle *v* above the horizon in the south should the satellite dish be pointed so as to best receive signals?

(0/4)

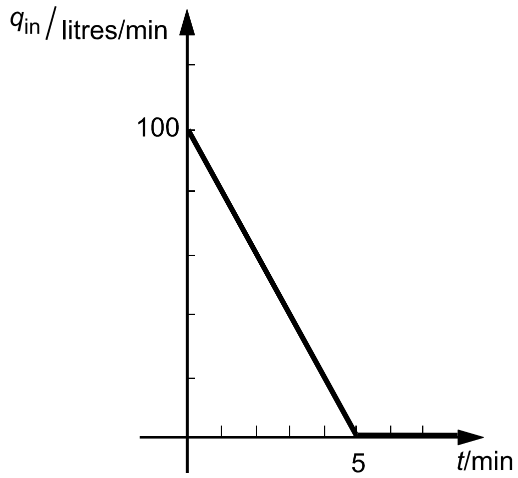


14. In the triangle ABC, the angle A is twice as large as angle B.
What possible lengths can the side BC have, if the side AC is 12 cm? (0/3)

15. A container that initially contains 300 litres of water is filled at a rate q_{in} according to graph 1. The water is leaving the container at a rate q_{out} according to graph 2. The volume of the fluid in the container at a certain time is also dependent on the choice of the rate q_{out} of the outgoing water.

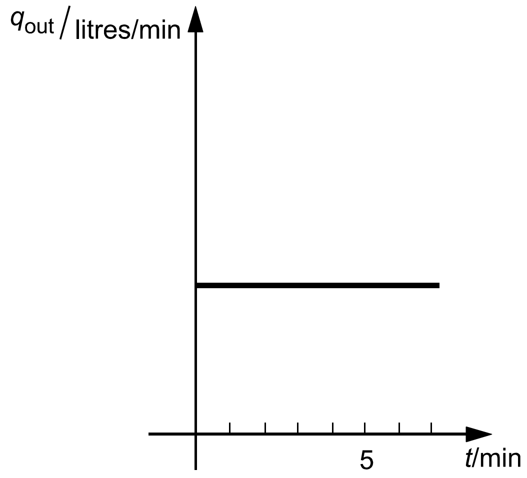


Rate of incoming water



Graph 1

Rate of outgoing water



Graph 2

- Determine the volume of fluid in the container after 2 minutes and 5 minutes respectively, given that the rate of outgoing water q_{out} is 40 litres/min.
- Investigate and describe in as detailed manner possible, how the volume of the liquid depends on time and the choice of q_{out}. (2/4)

When evaluating your work, your teacher will look at:
• what conclusions you have drawn from your investigation
• how close your solution is to a general solution
• how methodical you are in your investigation
• how well you present your work
• whether your calculations are correct