NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2002

Directions

Test time — 240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources — **Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” Please note calculators are not allowed in this part.

Part II: Calculators, and “Formulas for the National Test in Mathematics Courses C, D and E”.

Test material — The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test — The test consists of a total of 15 problems. **Part I** consists of 7 problems and **Part II** consists of 8 problems.

To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 15 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work, is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels — The maximum score is 43 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 “Pass”-points and 1 “Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for Pass with Special Distinction in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	12 points
Pass with distinction:	24 points of which at least 6 “Pass with distinction points”.

Pass with special distinction: The requirements for Pass with distinction must be well satisfied. Your teacher will also consider how well you solve the ¤-problems.

Name: ______________________________ School: ________________________

Education programme/adult education: ____

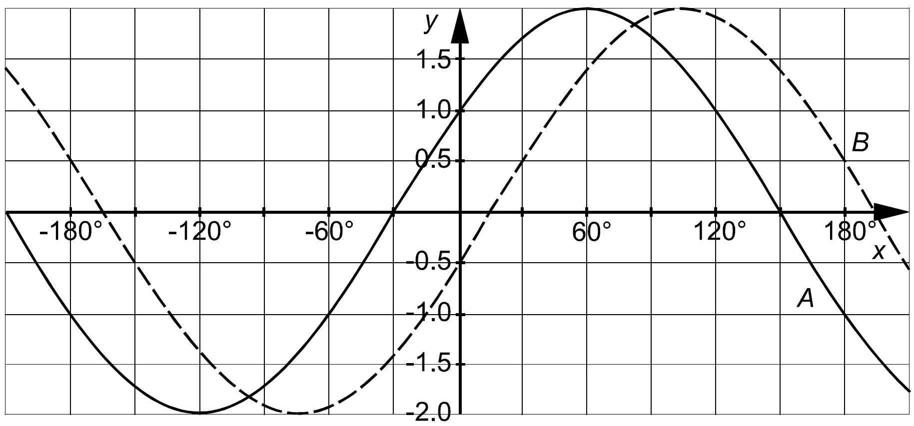
Part I

This part consists of 7 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Evaluate $\int_0^3 (x^2 + 4x)\mathrm{d}x$ (2/0)

2. Calculate $f'\left(\frac{\pi}{3}\right)$ when $f(x) = 2\sin x$ (2/0)

3.



Curve *A* can be described by the equation $y = 2\sin(x + 30°)$
Write down an equation for curve *B*. *Only answer is required* (2/0)

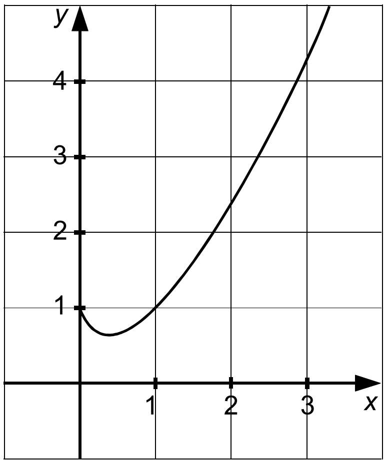
4. Which one/ones of the following equations has/have two solutions in the interval $0 \le x \le \pi$

A: $\cos x = -0.3$
B: $\sin x = 0.8$ *Only answer is required* (1/0)

5. Find $g(x)$ if $g'(x) = \sin 3x + \cos 2x$ and $g(\pi) = 2$ (3/0)

6. Determine the positive number a so that $\int_1^a \frac{1}{x}\,\mathrm{d}x = 2$ (2/0)

7.



The figure above shows the graph to a function $f(x)$ whose derivative is $1 + \ln x$

Use the figure to calculate $\int_1^3 (1 + \ln x)\,\mathrm{d}x$ (0/2)

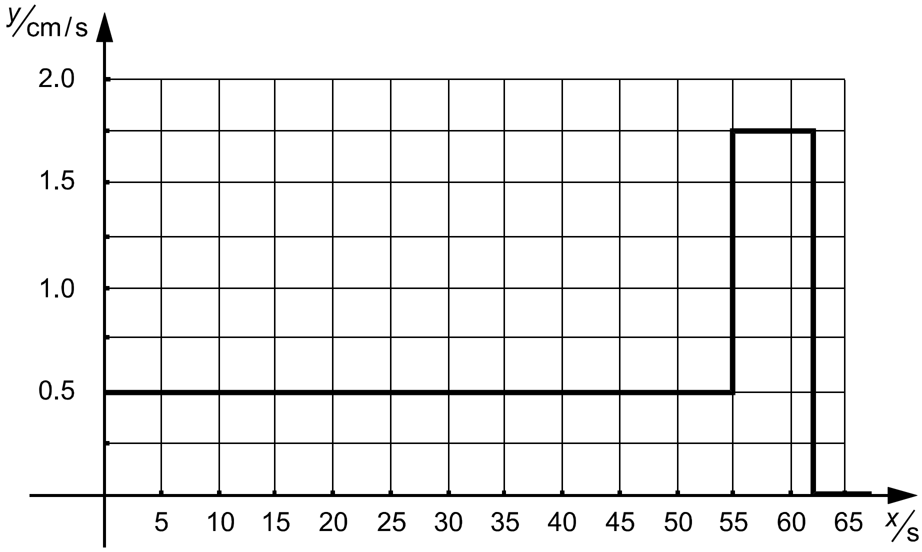
Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without a calculator.

8. Show that $y = 10e^{2x}$ is a solution to the differential equation $y' - 2y = 0$ (2/0)

9. In a triangle the sides are 5.0 cm, 6.0 cm and 7.0 cm.
Calculate the largest angle of the triangle. (2/0)

10. Water pours into an empty container at constant speed.
The figure below shows at what speed , y cm/s, the level of the water rises in the container.

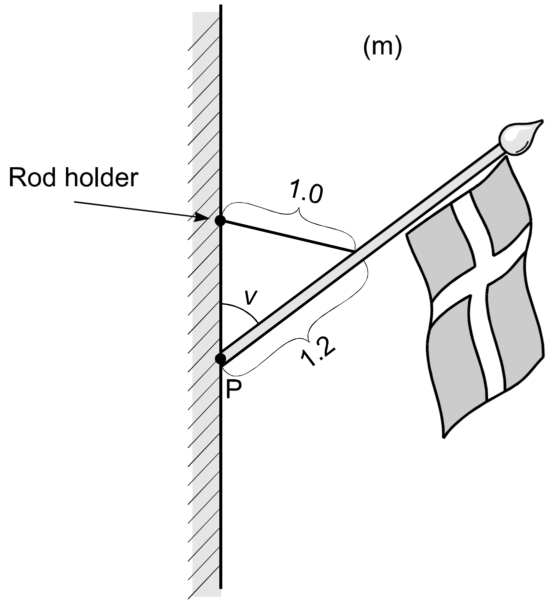


a) How long time does it take before the water stops pouring? (1/0)

b) What is the final level of water? (2/0)

c) Draw a sketch to illustrate what the container might look like. (0/1)

11.



Above the door to a shop there is a flagpole. It is held in position by a rod with length 1.0 m. The shop owner is moving the rod holder so that the flagpole forms an angle $v = 30°$ with the wall. The rod holder is placed straight above the point P. Calculate the distance between P and the new position of the rod holder. (2/1)

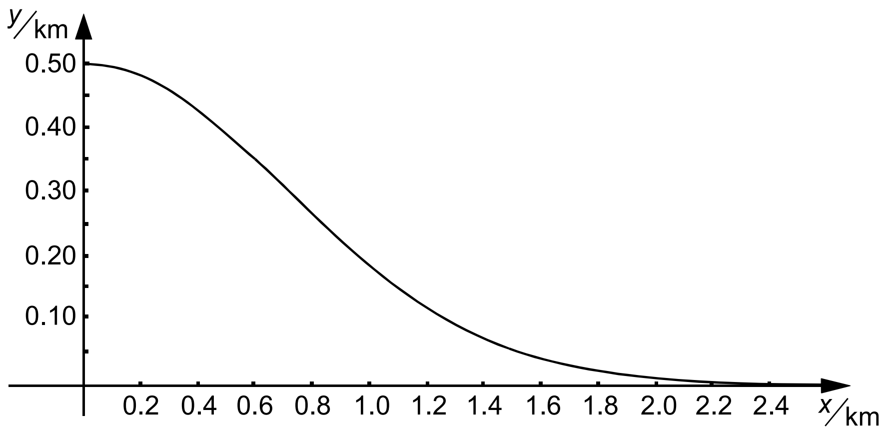
Together with the y-axis, the curves $y = e^{0.2x}$ and $y = x^2$ enclose an area within the first quadrant. Write down the integral for the area and calculate this area to at least three significant numbers. (0/3)

13. a) Show that $1 + \cos 4x = 2\cos^2 2x$ (0/1)

b) Calculate the exact value of the integral

$$\int_0^{\frac{\pi}{8}} 2\cos^2 2x \, dx$$ (0/2)

14. A ski slope has a vertical drop of 500 metres. The profile of the slope can be seen in the figure below.



The height y km is a function of the distance x km.
The relation between y and x is given by
$y = 0.5e^{-x^2}$, $0 \le x \le 2.5$

a) Calculate the gradient of the slope when $x = 0.8$ (0/2)

A general way of describing slopes with similar profiles as the one above is given by the function
$y = 0.5e^{-ax^2}$, $0 \le x \le 2.5$
where a is a positive constant.

b) Write down an equation for the determination of the x-value at the point where slopes with such profiles have the steepest gradient. (0/3/¤)

c) Find a so that the slope has the steepest gradient when $x = 1.0$ (0/1)

When assessing your work with problem 15, your teacher will take into consideration:

- how well you present your work
- if your calculations are correct
- how close to a general solution you are
- how well you use the mathematical language
- how well you justify your results

15. A tone sounds different when played on an organ or violin. This is due to the fact that the sound consists of one fundamental tone and several so called overtones. The overtones can be of different intensity and this is what gives the instrument its tone quality. The relation between the periods of the overtones and the fundamental tone is simple. If we choose a violin-string as an example it can produce a tone that can be described by a sum of terms

$a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + ...$

$a_1 \sin x$ corresponds to the fundamental tone and then follows the 1:st overtone, 2:nd overtone etc.

- Figure 1 shows the graphs to the functions that describe a fundamental tone $(y = a \sin x)$, its third overtone $(y = b \sin 4x)$ and the tone given by these two together $(y = a \sin x + b \sin 4x)$.
  Determine the constants *a* and *b*.

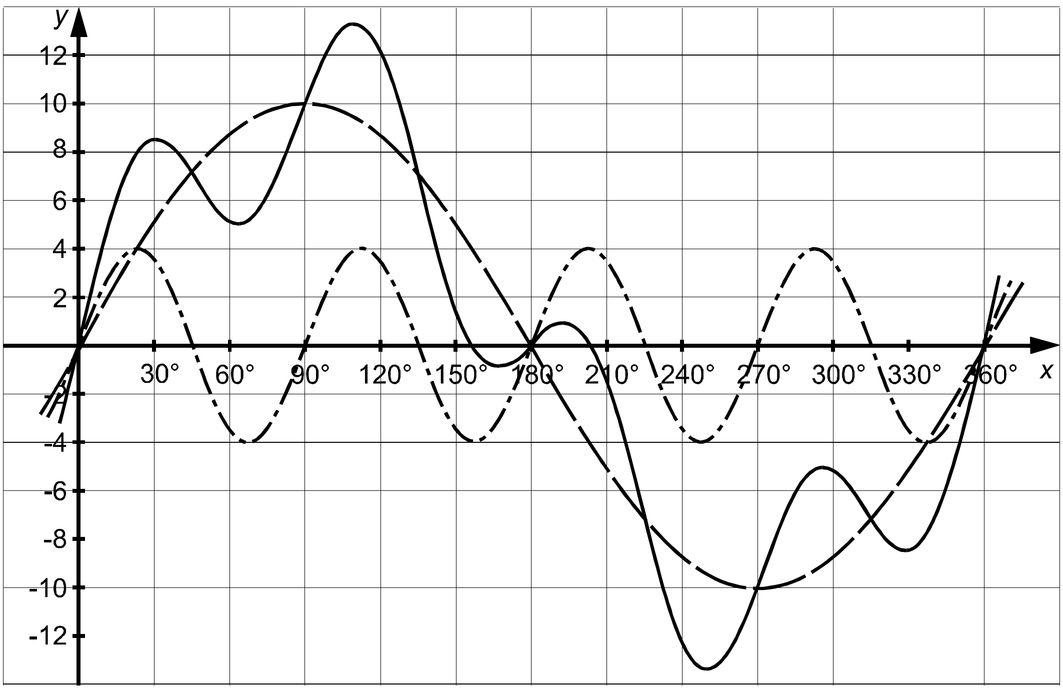


Figure 1

- Figure 2 shows the graph to the function $y = 10\sin x + c\sin 2x$
  The function describes a tone consisting of one fundamental tone and its first overtone.
  Determine the constant c.

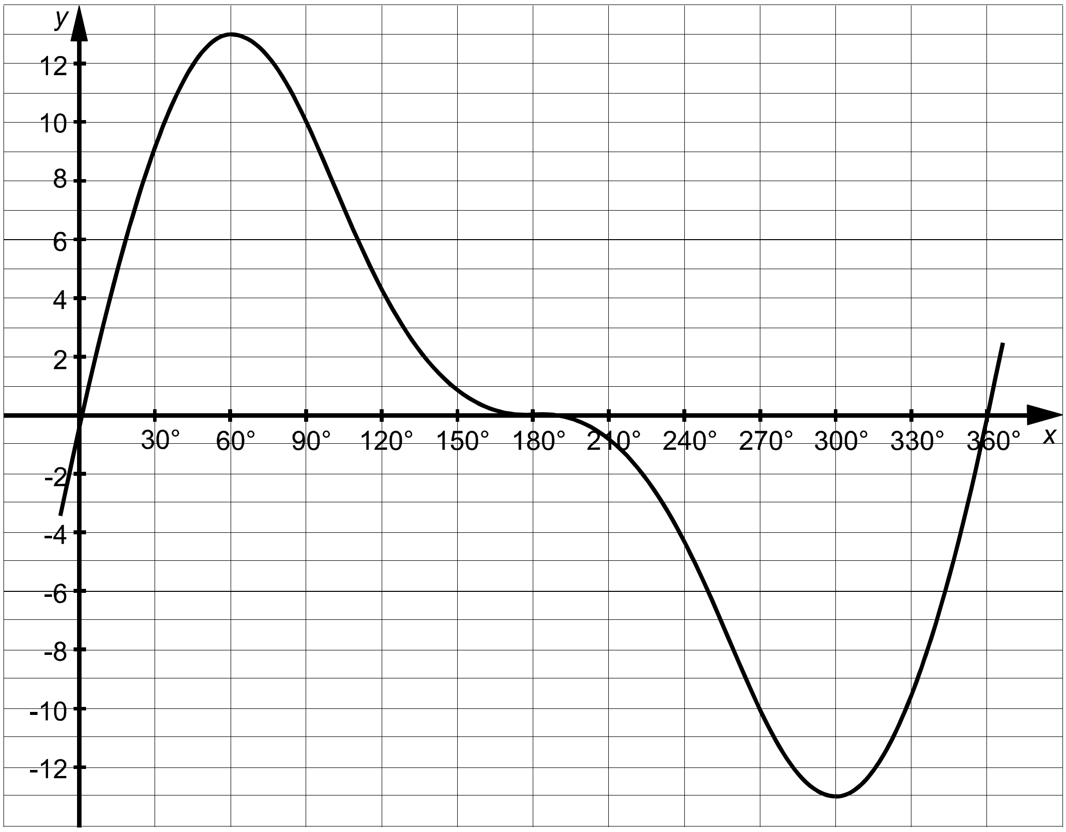


Figure 2

- Figure 3 shows the graphs to the functions that describe a fundamental tone $y = 12\sin x$, and the tone $y = 12\sin x + d\sin kx$ given by the fundamental tone together with an overtone.
  Determine the constants d and k.

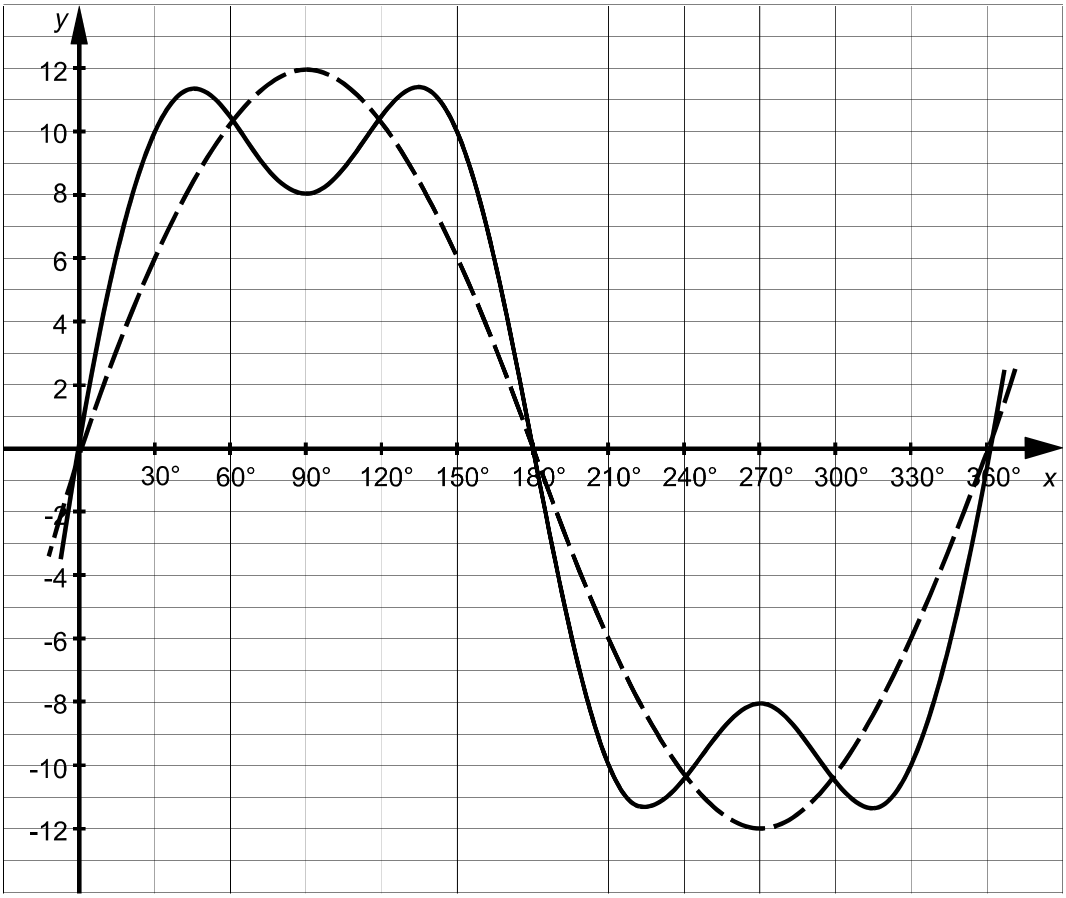


Figure 3

- Assume you have a figure showing the graphs to the functions $y = p\sin x$ and $y = p\sin x + q\sin nx$, where n is an integer greater than two.
  Describe a general method for how to determine the constants p, q and n from these graphs. (2/4/¤)