NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2004

Directions

Test time
240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.

Resources
Part I: ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.*

Part II: Calculators, and ”Formulas for the National Test in Mathematics Courses C, D and E”.

Test material
The test material should be handed in together with your solutions.

Write your name, the name of your education programme / adult education on all sheets of paper you hand in.

Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.

The test
The test consists of a total of 16 problems. **Part I** consists of 8 problems and **Part II** consists of 8 problems.

To some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources.

Problem 16 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem.

Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.

Score and mark levels
The maximum score is 44 points.

The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000.

Lower limit for the mark on the test

Pass:	13 points
Pass with distinction:	26 points of which at least 6 ”Pass with distinction”-points.
Pass with special distinction:	In addition to the requirements for ”Pass with distinction” you have to show *“Pass with special distinction” qualities in at least one* of the ¤-problems. You must also have at least 14 ”Pass with distinction”-points.

Name: ______________________________ School: __________________________________

Education programme/adult education: ____

Part I

This part consists of 8 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

1. The equation $\sin x = \frac{1}{2}$ has two solutions in the interval $0 \le x \le 2\pi$.

One of the solutions is $x = \frac{\pi}{6}$. Find the other solution.

Only answer is required (1/0)

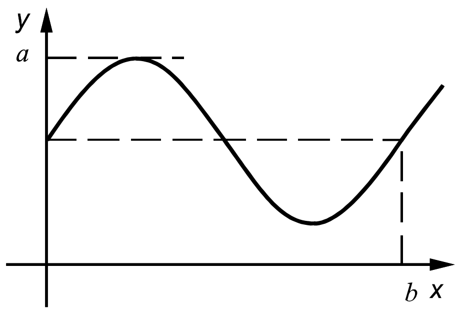
2. Evaluate $\int_1^2 x(1-x)\,dx$ (2/0)

3. Let $f(x) = \ln(2+x) - 2x$

a) Find the derivative of the function *f*. *Only answer is required* (1/0)

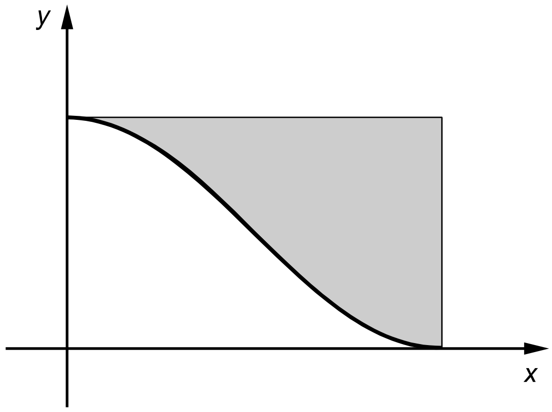
b) Solve the equation $f'(x) = 0$ (1/0)

4. The figure below shows the curve $y = 15 + 10\sin 4x$.
Determine a and b. *Only answer is required* (2/0)

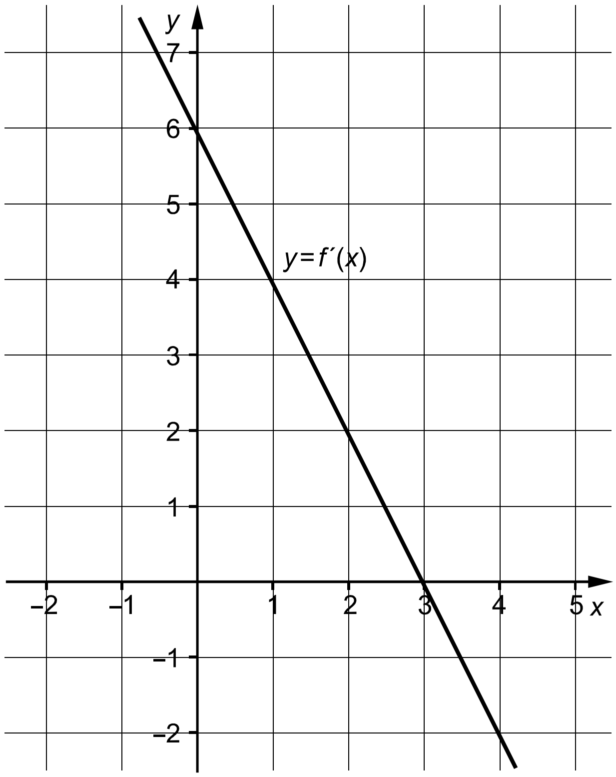


5. The shaded region in the figure below is bounded by the curve $y = 1 + \cos x$ and lines that pass through the points where the curve intersects the coordinate axes. These lines are parallel to the coordinate axes.

Calculate the exact value of the area of the shaded region. (0/3)



6. The figure below shows the derivative $y = f'(x)$ of the function f.

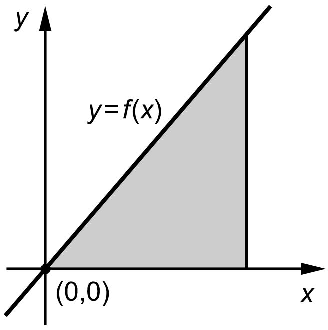


a) The function f has a maximum. Use the graph above to justify this. (1/0)

b) Find the function f when the maximum value of the function is 12. (1/2)

7. The area of the shaded region can be calculated by the integral $\int_0^5 f(x)\,dx$.

Determine the linear function $y = f(x)$ if the value of the integral is 20. (0/2)



8. a) Give a trigonometric equation with solutions $x_1 = 120°$ and $x_2 = 240°$ in the interval $0° \le x \le 360°$ *Only answer is required* (0/1)

b) Give a trigonometric equation that has exactly one solution in the interval $0° \le x < 360°$ *Only answer is required* (0/1)

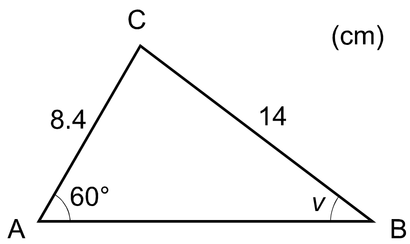
Part II

This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

9. Determine the constant a so that $y = e^{3x}$ is a solution to the differential equation
$y'' - 3y = ae^{3x}$ (2/0)

10. Calculate the angle v. (2/0)

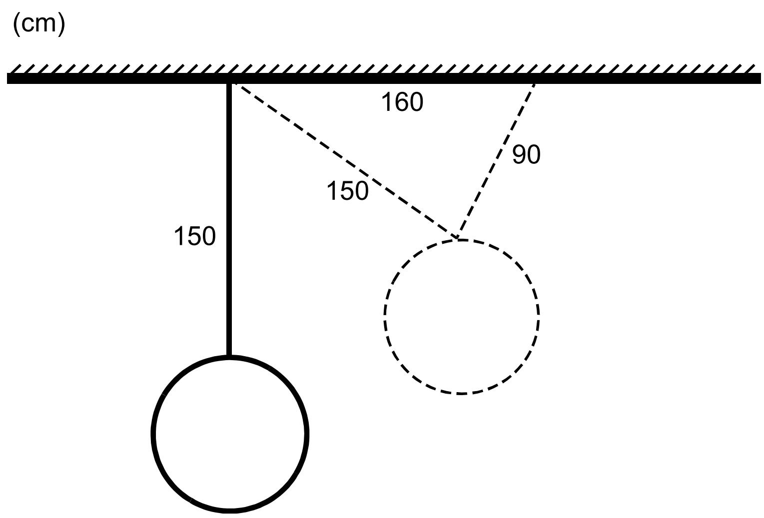
(Calculations based on measurements are not accepted)



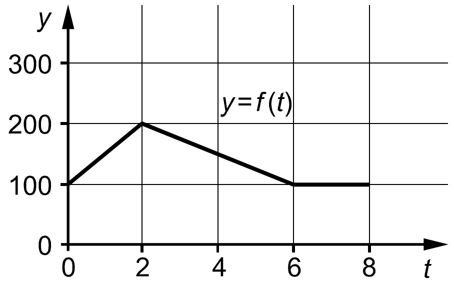
11. Stina and Nisse have a lamp above their coffee table. Sometimes they want to raise the lamp so that it does not block their vision. The lamp hangs from the ceiling on a 150 cm long rope. 160 cm from where the lamp is attached to the ceiling there is a hook, and from there they attach a 90 cm long rope to the lamp.

How much higher will the lamp hang when attached this way? (3/0)

(Calculations based on measurements are not accepted)



12. When studying the working environment in a factory the concentration of harmful dust as a function of time was examined during one working day. The diagram below shows how the concentration $f(t)$ mg/m³ varied with time t h during eight hours.



a) Calculate the value of $\frac{1}{8}\int_0^8 f(t)\,\mathrm{d}t$ (2/0)

b) Interpret what the value of $\frac{1}{8}\int_0^8 f(t)\,\mathrm{d}t$ tells you about the presence of the harmful dust. (1/1)

13. The table below shows some values of the functions f and g and their derivatives.

x	-2	-1	0	1	2
$f(x)$	-2	-4	0	4	2
$g(x)$	1	2	0	-2	-1
$f'(x)$	-7	2	5	2	-7
$g'(x)$	$3{,}5$	-1	$-2{,}5$	-1	$3{,}5$

a) Calculate $h'(-2)$ when $h(x) = f(x)\cdot g(x)$ (2/0)

b) Calculate $s'(-1)$ when $s(x) = f(g(x))$ (0/2)

14. a) Use your calculator to work out the value of k so that

$$\int_{-1}^{1}\sqrt{1-x^2}\,\mathrm{d}x = k\pi$$

Only answer is required (1/0)

b) Investigate the function $y=\sqrt{1-x^2}$ and explain the value of k. (0/1)

15. A formula for $\cos 2x$ can be derived by differentiating the function $f(x) = \sin 2x$ in two different ways.

1) By using the Chain Rule you get $f'(x) = 2\cos 2x$
2) By starting with the connection $\sin 2x = 2\sin x \cos x$ and then applying the Product Rule you get
   $f'(x) = 2\cos x \cos x + 2\sin x(-\sin x) = 2(\cos^2 x - \sin^2 x) =$
   $2(\cos^2 x - (1-\cos^2 x)) = 2(2\cos^2 x - 1)$

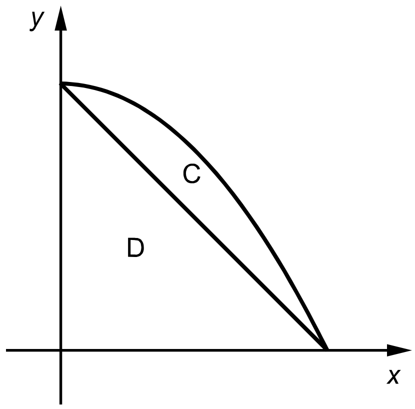
Since both expressions for $f'(x)$ have to be equal it follows that $\cos 2x = 2\cos^2 x - 1$. We have received a formula for $\cos 2x$ that only contains powers of $\cos x$.

Use the same technique to derive a formula for $\cos 3x$, by using the connection $\sin 3x = 3\sin x - 4\sin^3 x$. The formula should only contain powers of $\cos x$. (0/3/¤)

When assessing your work with this problem, the teacher will pay extra attention to:

- how general your solution is
- how well you have presented your work
- how well you have justified your solutions
- how well you have used the mathematical language

16. The figure below shows the graph of a function $y = a^2 - x^2$ where $x \geq 0$ and $a > 0$, and a line that passes through the intersection points between the graph and the coordinate axes. *C* is the area of the region bounded by the graph of the function and the straight line. *D* is the area of the region bounded by the coordinate axes and the straight line.



- Calculate the quotient $\frac{C}{D}$ for $a = 1$ and $a = 2$
- Use the result above to formulate a general assumption about the quotient $\frac{C}{D}$ and investigate if your assumption holds for all values of *a*.
- Investigate the value of the quotient $\frac{C}{D}$ for functions of the type

  $y = a^2 - b^2x^2$

  where $b > 0$, $a > 0$ and $x \geq 0$ (2/4/¤)