NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2005

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C, D and E.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators and ”Formulas for the National Test in Mathematics Courses C, D and E”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 9 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 17 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction”-points. Pass with special distinction: In addition to the requirements for ”Pass with distinction” you have to show most of the“Pass with special distinction” qualities that the ¤-problems give the opportunity to show. You must also have at least 13 ”Pass with distinction”-points.

Name: ______________________________ School: ___________________________________

Education programme/adult education: ____

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

1. Evaluate $\int_1^3 (x^2 - 1)\,\mathrm{d}x$ (2/0)

2. Determine $f'(x)$ if

a) $f(x) = 4\cos 3x$ *Only answer is required* (1/0)

b) $f(x) = (3 - 2x)^6$ *Only answer is required* (1/0)

c) $f(x) = x^2 \cdot \mathrm{e}^{3x}$ *Only answer is required* (0/1)

3. Which two of the functions $F(x)$ below are the antiderivatives to $f(x) = 3x^5 + 1$? *Only answer is required* (1/0)

A $F(x) = \dfrac{3x^4}{4}$

B $F(x) = 15x^4$

C $F(x) = 0.5x^6 + x$

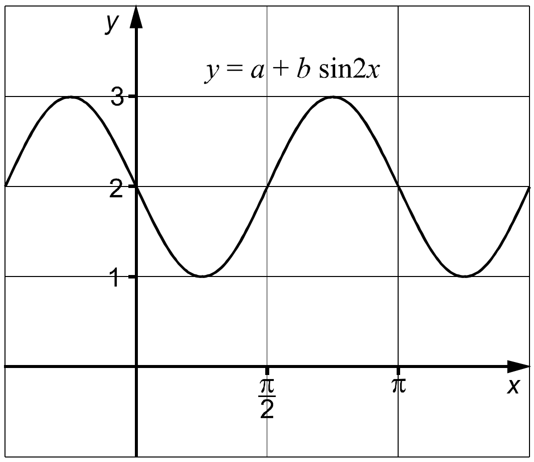
D $F(x) = x^6 + 2x$

E $F(x) = \dfrac{x^6}{3} + x + 1$

F $F(x) = \dfrac{x^6}{2} + x - 14$

4. Arrange the following numbers according to size:
$a = \sin 24°$, $b = \cos 100°$ and $c = \sin 165°$
Justify your answer. (1/1)

5. The figure shows the graph of the function $y = a + b\sin 2x$
Determine the constants a and b. *Only answer is required* (1/1)



6. Which one of the following expressions A – F can be simplified to 1?

Only answer is required (0/1)

A $(\sin x + \cos x)^2$

B $(\sin x - \cos x)^2$

C $(\sin x + \cos x)(\sin x - \cos x)$

D $\cos x(\tan x \cdot \sin x + \cos x)$

E $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

F $2(\sin x + \cos x)$

7.

The number of starlings in Sweden has been investigated since 1979. The results of this investigation can be described mathematically by the differential equation:

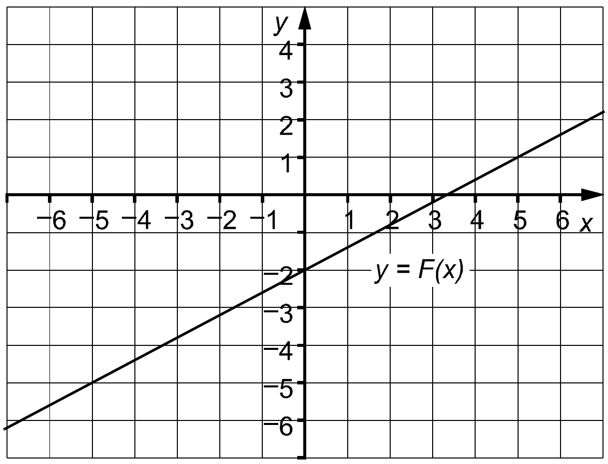
$\frac{dy}{dt} = -0.03 \cdot y$, where y is the number of starlings at the time t years from 1979.

Explain, in your own words, the meaning of the differential equation in this context. (1/1)

8. In the triangle ABC the angle $A = 90°$
Show that $\sin B = \cos C$ (0/1/¤)

9. The function F is the antiderivative to f
The figure below shows $y = F(x)$

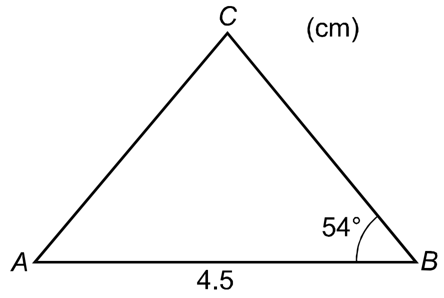
Determine $\int_0^5 f(x)\,dx$ (0/2/¤)



Part II

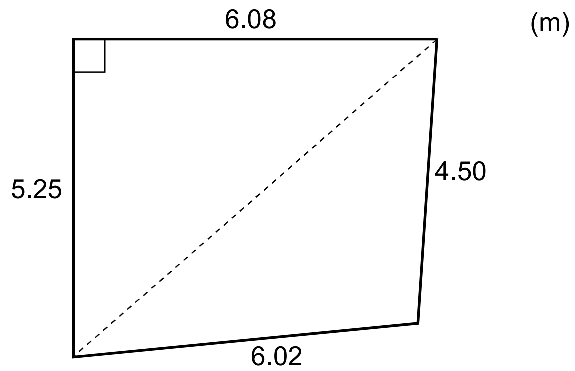
This part consists of 8 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

10. In the triangle ABC the sides AC and BC are of equal length.
Calculate the area of the triangle. (2/0)



11. Use the antiderivative to calculate the area of the region enclosed by the functions $f(x) = x^2 + x + 1$ and $g(x) = 9 - x$ (3/0)

12. Daniel and Linda are looking at a flat. According to the information received the living-room is 31.2 m^2. They want to check if this is correct so they measure the walls and draw a sketch of the living-room. They know that one corner of the room is right-angled. Their sketch looks like this:



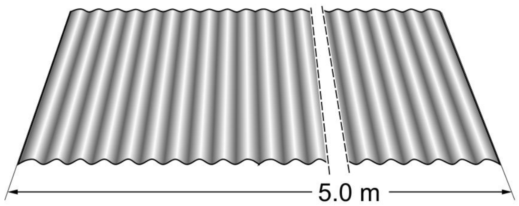
What is the area of the living-room according to Daniel's and Linda's sketch? (2/2)

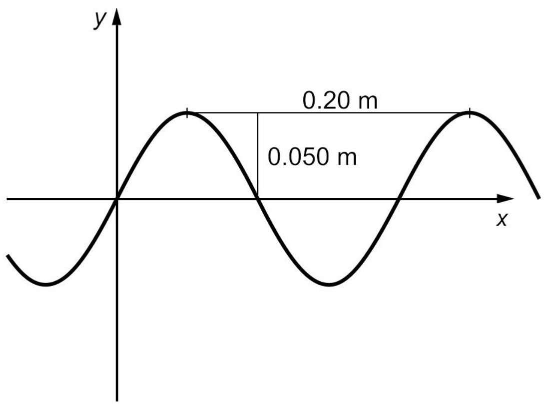
13. Find all the solutions to the equation $\sin 3x = 0.421$ (2/1)

14. Determine *the number* of solutions to the equation $\sin 2x = \dfrac{x^2}{10} - 1$,

where x is measured in radians. (1/1)

15. A sheet of corrugated iron is made by pressing a flat sheet into curving folds. Seen from the side, the corrugated iron in the picture has the shape of a sine curve with period 0.20 m and amplitude 0.050 m.

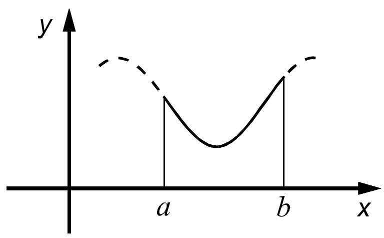




a) Find a formula for the 'iron curve' of the form $f(x) = A\sin kx$ (0/1)

There is a formula for calculating the length of a curve. According to this, the length s of a curve $y = f(x)$ from $x = a$ to $x = b$ can be calculated from

$$s = \int_a^b \sqrt{1 + (f'(x))^2}\,\mathrm{d}x$$

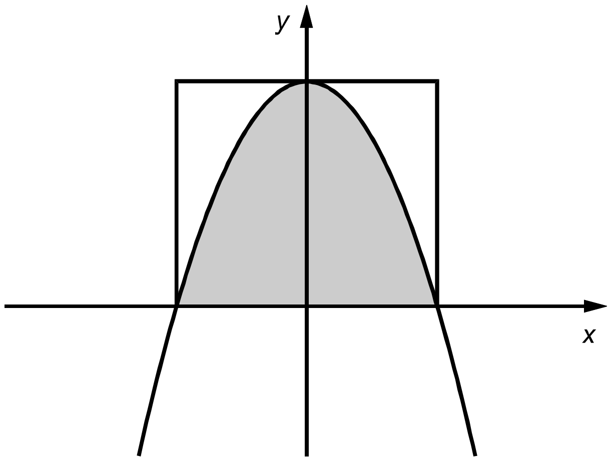


b) How long a *flat* iron sheet should you start with in order to get a piece of corrugated iron with a length of 5.0 m? (0/3/¤)

16. For which values of the constants a and b is it true that the function $f(x) = ax^2 + bx - \sin 3x$ has a local maximum when $x = 0$? (1/2/¤)

When assessing your work with this problem your teacher will take take into consideration:
- How well you carry out your calculations
- How well you justify your conclusions
- How well you present your work
- How well you use the mathematical language

17. The figure shows a parabola and a rectangle in a coordinate system. The shaded region is enclosed by the parabola and the x-axis.
The area of the shaded region will from now on be referred to as the area of the parabola.



Two of the corners of the rectangle coincide with the points where the curve intersects the x-axis. One of the side of the rectangle touches the maximum point of the curve.

In this problem, you are going to investigate the relation between the area of the parabola and the area of the rectangle.
Let the equation of the parabola be $y = b - ax^2$, where a and b are positive numbers.

- You may for example start by letting $b = 9$ and $a = 1$ and draw the graph of the function $y = 9 - x^2$. Then determine the relation between the area of the parabola and the area of the rectangle.

- Choose other examples yourself and try to formulate a conclusion based on your chosen examples.

- Investigate if your conclusion also holds for the general case with the parabola $y = b - ax^2$

If you want to you may go straight to investigating the general case. (3/4/¤)