NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2006

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 60 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Courses C and D.” *Please note that calculators are not allowed in this part.* **Part II:** Calculators and ”Formulas for the National Test in Mathematics Courses C and D”.
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme / adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 9 problems and **Part II** consists of 9 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 12 points Pass with distinction: 25 points of which at least 7 ”Pass with distinction”-points. Pass with special distinction: 25 points of which at least 14 ”Pass with distinction”-points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 9 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

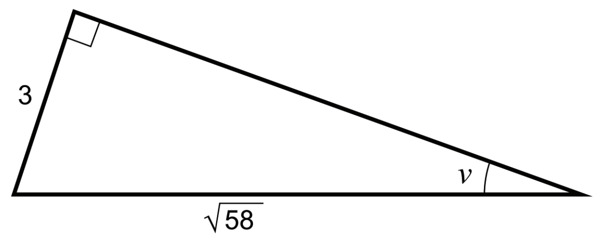
1. Differentiate

a) $f(x) = 3\cos x$ *Only answer is required* (1/0)

b) $g(x) = (3x+1)^7$ *Only answer is required* (1/0)

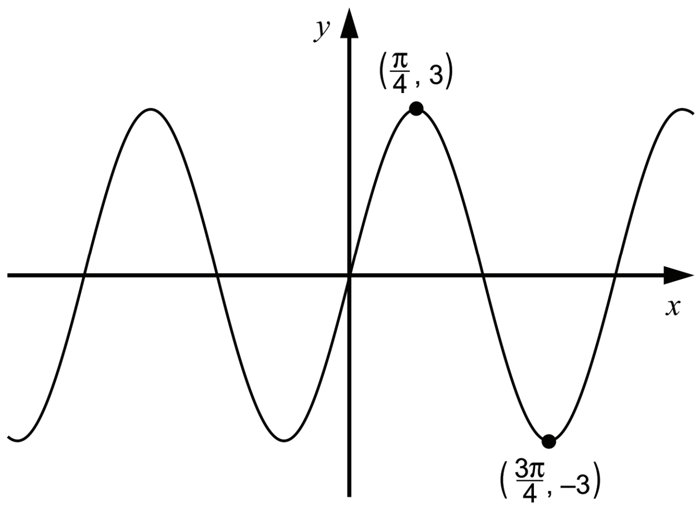
2. Find the antiderivative F of $f(x) = 3x^2$ that satisfies the condition $F(2) = 5$ (2/0)

3. The figure shows a right-angled triangle. Determine tan v. (2/0)



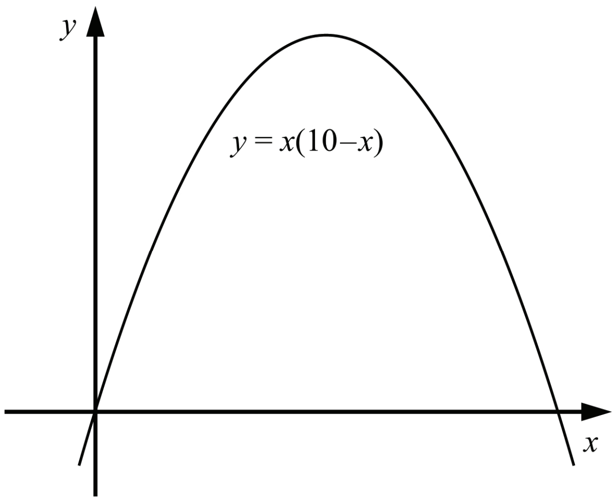
4. The figure shows a sine curve where the coordinates of a maximum point and a minimum point are given. Find the equation of the curve and write it in the form $y = a\sin kx$

Only answer is required (2/0)



5. a) Write down an expression for the area bounded by the curve $y = x(10 - x)$ and the x-axis. (1/0)

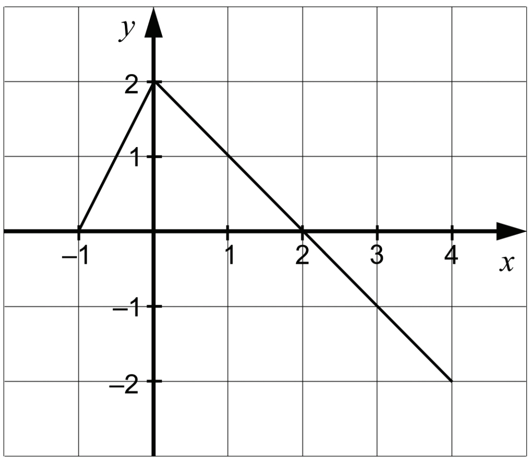
b) Calculate the area. (2/0)



6. Differentiate the function $f(x) = \dfrac{\ln x}{x}$ and determine the zero of the derivative. (0/2)

7. The figure shows the graph of the function $y = f(x)$

Determine $\int_{-1}^{4} f(x)\mathrm{d}x$ *Only answer is required* (0/1)



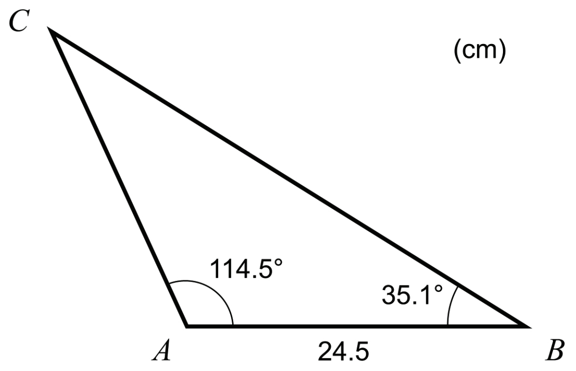
8. For which angles within the interval $0^\circ < v < 360^\circ$ is it true that $\sin v > \sin 111^\circ$? Justify your answer. (0/2)

9. Investigate which of the integers 5, 6, 7, 8 and 9 is a/are reasonable value(s) of the positive constant a in the relation $\int_{\frac{\pi}{2}}^{a} \sin x \, dx \approx 0.9$ (0/3/¤)

Part II

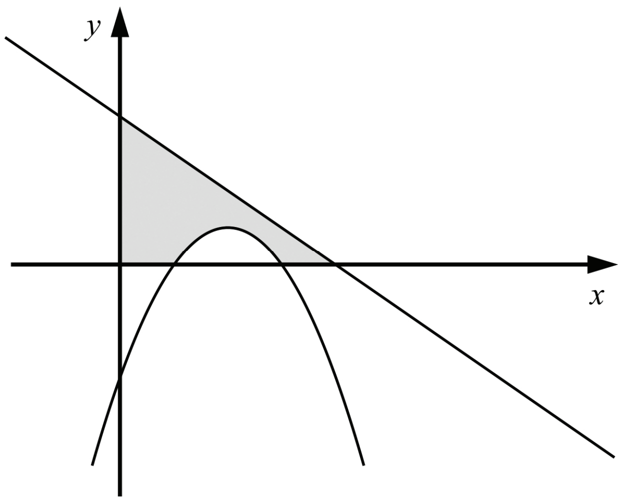
This part consists of 9 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

10. Determine the length of side BC in the triangle ABC. (2/0)



11. Is $y = 6 \cdot e^{0.063x}$ a solution to the differential equation $y' - 0.063y = 0$? (2/0)

12. The figure shows the curve $y = 4x - x^2 - 3$ and the line $y = 4 - x$
Calculate the area of the region bounded by both these graphs and the positive coordinate axes. (3/0)

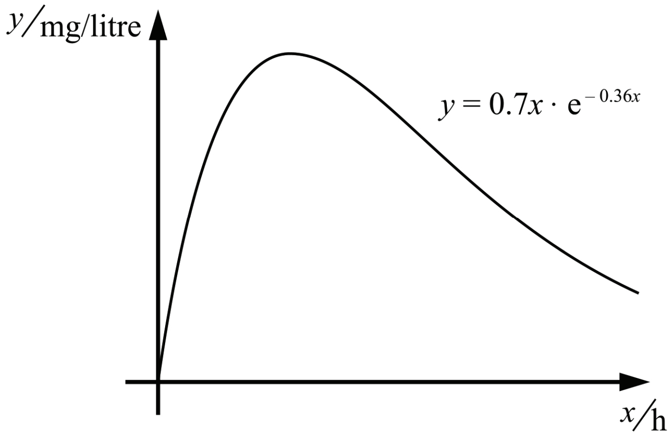


13. Find all the solutions to the equation $\tan 2x = 1.6$ (1/1)

14. A sedative is injected into a muscle from there it goes into the circulatory system. The concentration in the blood , y mg/litre, can be described by the model

$y = 0.7x \cdot e^{-0.36x}$

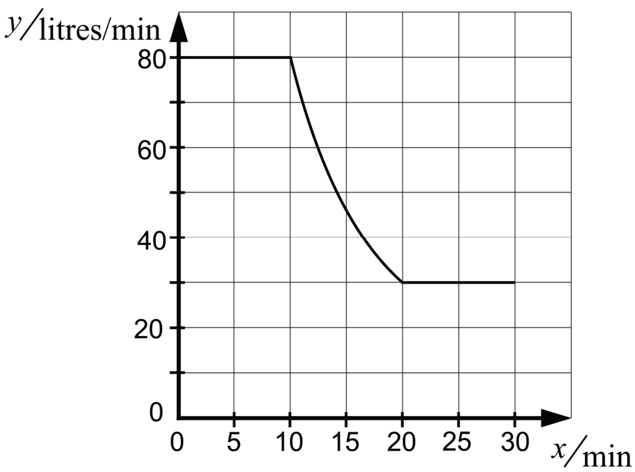
where x is the number of hours after the injection has been given.



The sedative is effective when the concentration in the blood is at least 0.4 mg/litre. Find out how many hours the sedative is effective according to the model.
Answer to one decimal place. (1/1)

15. Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$ (0/2)

16. An empty tank is filled with water. The figure below shows how the re-filling rate y litres/min depends on the time x min. Due to a decreasing water-pressure, the rate decreased during a ten-minute period. During this period, the re-filling rate can be described by the function $y = \dfrac{1000}{x} - 20$



a) Write down an expression for the volume of the water in the tank after 30 min and calculate the value. (0/2)

b) After how much time is the volume of water in the tank 1000 litres? (0/2)

17. The function *f* is defined by $f(x) = x^2 \cdot e^{-x}$

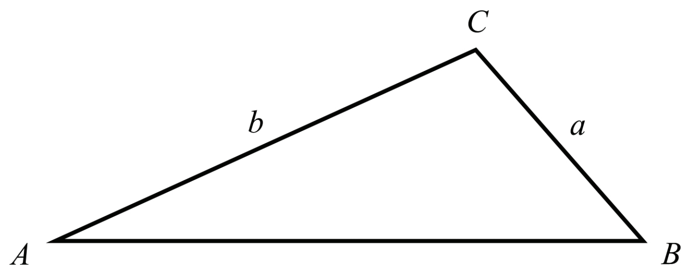
a) Use the method of differentiation to determine the *x*-coordinates of all the stationary points to *f* (0/2)

b) Investigate whether *f* assumes any maximum or minimum value. (0/1/¤)

18.

When assessing your work with this problem, your teacher will take into consideration: • How well you carry out your calculations • How well you justify your conclusions • How well you present your work • How well you use mathematical language

In the triangle *ABC* the angle *B* is always twice as large as the angle *A*



- Determine $\frac{b}{a}$ if the angle A is 25°

- Determine the angle A if $\frac{b}{a} = 1.5$

- Investigate how $\frac{b}{a}$ depends on the angle A. Specifically, write down what values $\frac{b}{a}$ can assume.

(2/4/¤)