NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2008

Directions

Test time	240 minutes for Part I and Part II together. We recommend that you spend no more than 90 minutes on Part I.
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II:** Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 11 problems and **Part II** consists of 7 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 46 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction” in Assessment Criteria 2000. Lower limit for the mark on the test Pass: 13 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction”-points. Pass with special distinction: 26 points of which at least 14 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 11 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without your calculator.

1. Evaluate $\int_0^1 (10x^4 + 3)\ \mathrm{d}x$ (2/0)

2. Differentiate

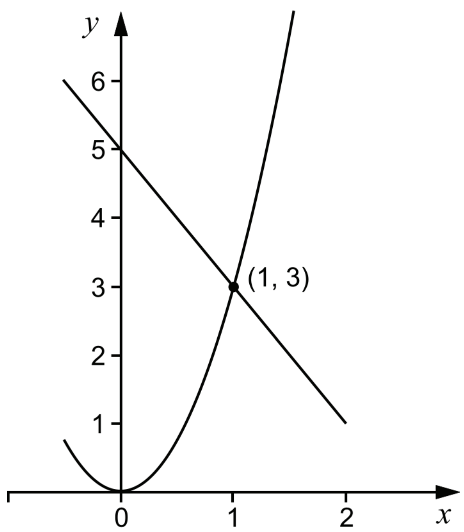
a) $f(x) = 3e^{2x} - 6x$ *Only answer is required* (1/0)

b) $g(x) = \sin 2x + \cos x$ *Only answer is required* (1/0)

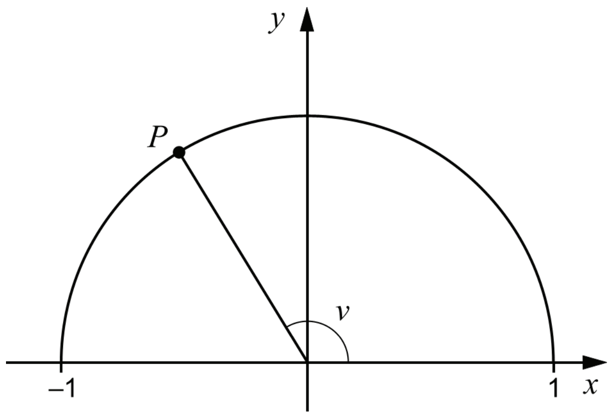
c) $h(x) = e^{\sin x}$ *Only answer is required* (0/1)

3. Find $\sin(-45°)$ *Only answer is required* (1/0)

4. Calculate the area of the region in the first quadrant bounded by the curve $y = 3x^2$, the line $y = 5 - 2x$ and the y-axis. (2/0)



5. The figure shows a part of a unit circle. The point P has coordinates $\left(-\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$

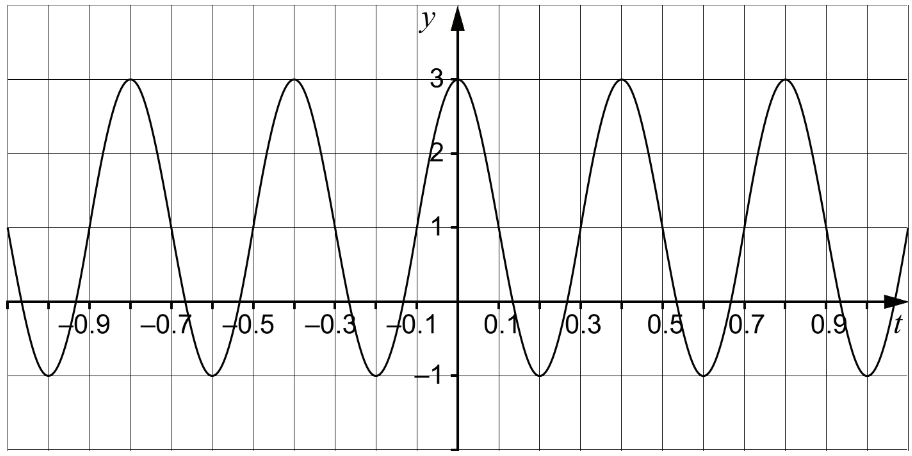


Find the exact values of

a) $\sin v$ *Only answer is required* (1/0)

b) $\sin 2v$ (2/0)

6. A part of the graph of the function $y = f(t)$ is drawn in the figure below.



Which of the alternatives A-E best describes the function?
Only answer is required (0/1)

A. $f(t) = 2\cos 5\pi t + 1$

B. $f(t) = 2\sin 5\pi t + 1$

C. $f(t) = \cos 5\pi t + 2$

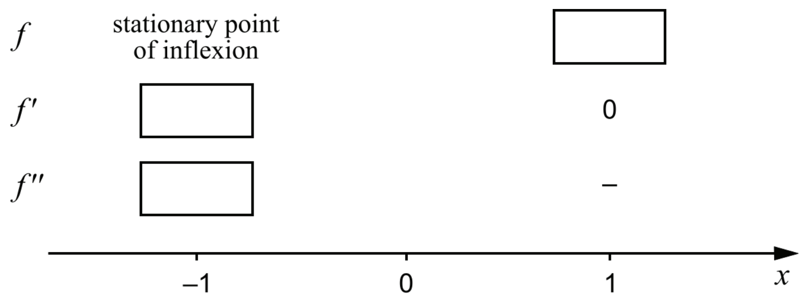
D. $f(t) = \sin 5t + 2$

E. $f(t) = 2\cos 5t + 1$

7. Solve the equation $\cos x = \frac{\sqrt{3}}{2}$ (1/1)

8. The function *f* and its derivatives have properties according to the figure below at the points where $x = -1$ and $x = 1$. Copy the chart onto your paper and fill in the missing symbols in the empty boxes. Choose from min, max, stationary point of inflexion, +, $-$ or 0.

Only answer is required (1/1)



9. Decide for each of the following statements whether it is true for all angles v satisfying the condition $\sin v > \frac{1}{2}$. Do not forget to justify you answer.

a) $\sin 2v > 1$ (1/0)

b) $\cos v < \frac{1}{2}$ (0/1)

c) $\cos^2 v < \frac{3}{4}$ (0/1/¤)

10. Determine the constant k so that the value of $\int_0^1 (3x - k)^2 \, dx$ becomes the smallest possible. (0/2/¤)

11. A formula for $\cos 2x$ can be derived by differentiating the function $f(x) = \sin 2x$ in two different ways.

1) By using the chain rule we get $f'(x) = 2\cos 2x$
2) By starting with the relation $\sin 2x = 2\sin x \cos x$ and differentiating it, we get the following:
   $f'(x) = 2\cos x \cos x + 2\sin x(-\sin x) = 2(\cos^2 x - \sin^2 x) =$
   $2(\cos^2 x - (1-\cos^2 x)) = 2(2\cos^2 x - 1)$

Since both expressions for $f'(x)$ must be equal, it follows that $\cos 2x = 2\cos^2 x - 1$. We have then a formula for $\cos 2x$ that contains only powers of $\cos x$.

Use the same technique to derive a formula for $\cos 3x$ by using the relation $\sin 3x = 3\sin x - 4\sin^3 x$. The formula should only contain powers of $\cos x$.

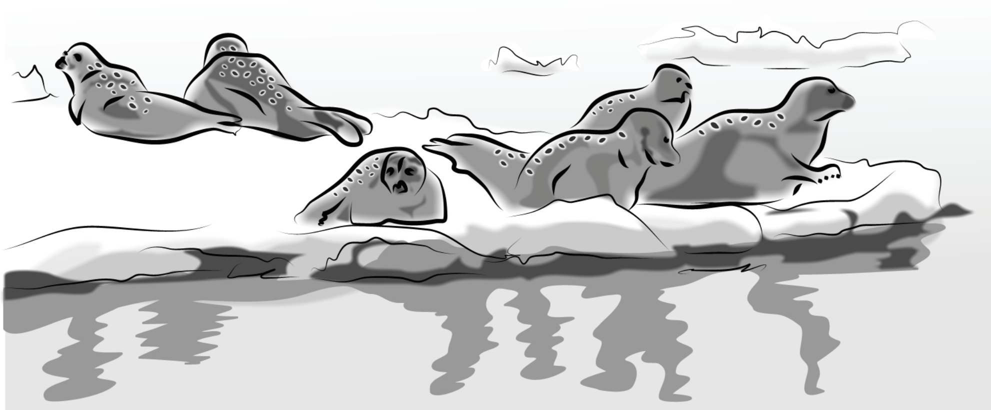
(0/3/¤)

Part II

This part consists of 7 problems and you may use a calculator when solving them. Please note that you may begin working on Part II without your calculator.

12. In the triangle ABC side AB is 22 cm, side BC is 17 cm and angle B $79°$.
Calculate the area of the triangle. (2/0)

13.

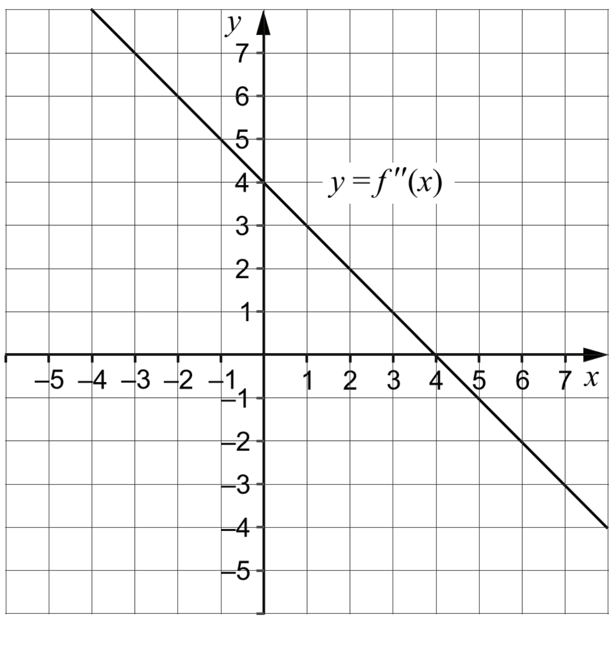


At the beginning of the 20th century there were about 180 000 ringed seals in the Baltic Sea. The population then decreased during almost the whole century. In 1985 there were 5000 ringed seals left. After that the development of the population can be mathematically described by the differential equation

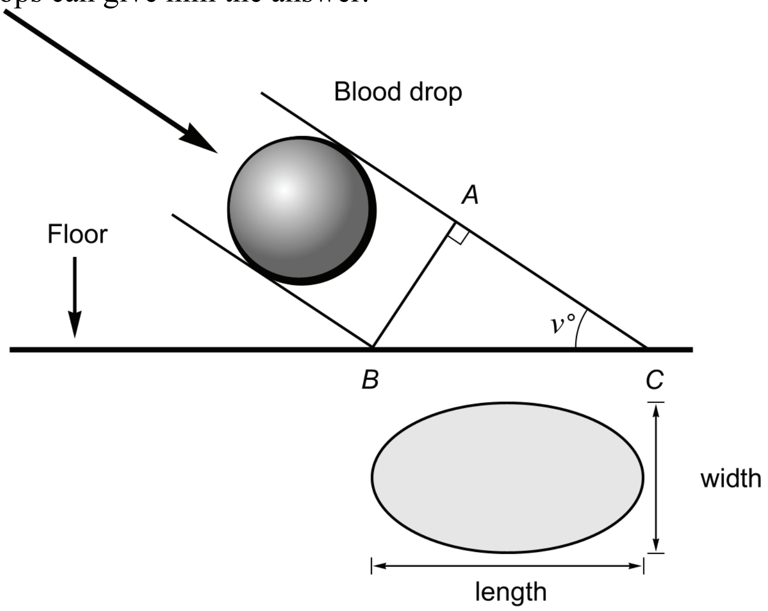
$\frac{dy}{dx} = 0.0216y$, where y is the number of ringed seals at a time x years from 1985.

In your own words, explain the meaning of the differential equation in this context. (1/1)

14. The figure shows the graph of $y = f''(x)$
Determine $f(x)$ if $f'(1) = 2$ and $f(1) = 0$ (1/2)

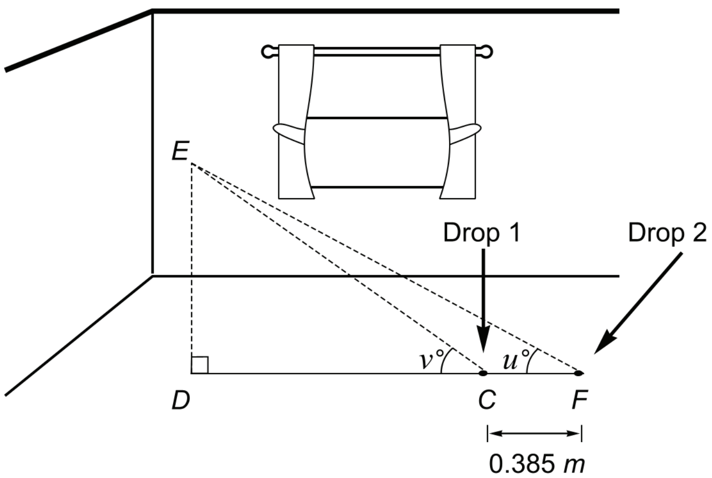


15. Inspector Moore arrives at the scene of a crime. The victim is gone, but he finds evidence in the form of two drops of blood on the floor. To be able to reconstruct the crime he must know where the victim was when the victim was injured. The blood drops can give him the answer.



Moore starts by investigating blood drop 1. He assumes that the drop is spherical and follows a path according to the figure. The width of the blood drop is the same as the diameter *AB* of the drop. He measures *AB* and it is 3.0 mm and the length of blood drop BC which is 4.0 mm.

a) Help Moore to determine angle *v*. (1/0)



Moore continues by investigating blood drop 2. He first measures the width and it is 4.0 mm and then the length which is 6.0 mm. He also measures the distance *CF* between the blood drops and it is 0.385 m.

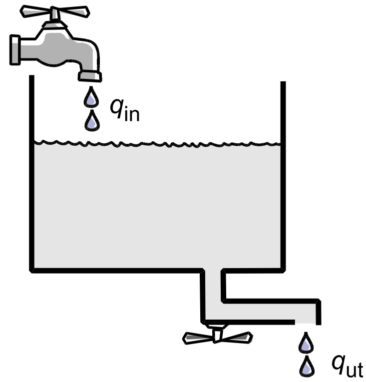
b) Help Moore to decide where the victim was when it was hurt.
You do that by calculating the distance *DC* . (1/2)

16. Calculate the area of the region bounded by the curves $y = e^{x/2}$ and $y = 4 - x$ and the positive coordinate axes.
Give your answer to three significant figures. (1/2)

17. For which x in the interval $0 \le x \le 2\pi$ is $f(x) = \dfrac{1}{\tan x - 1}$ not defined? (1/1/¤)

When assessing your work the teacher will take into consideration:
• what you have concluded from your investigation
• how close to a general solution you are
• how systematic you are in your investigation
• how well you present your work
• if you have done correct calculations

18. A container that contains 300 litres of water to begin with is filled at a rate of q_{in} according to diagram 1. The rate of the outflow of the water is shown in diagram 2. The water volume at a certain time depends on what value that has been chosen for the constant outflow speed q_{ut}



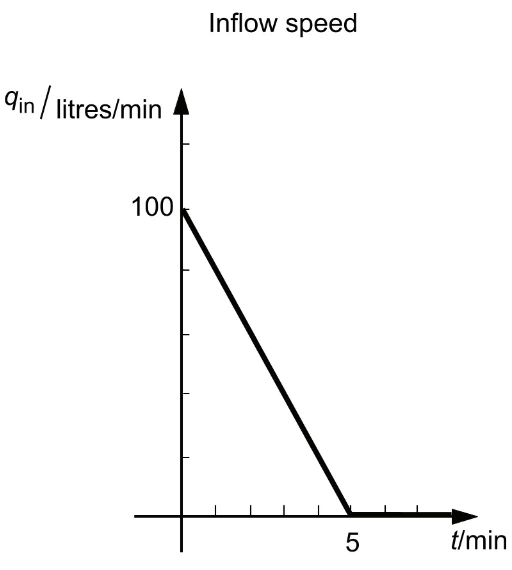


Diagram 1

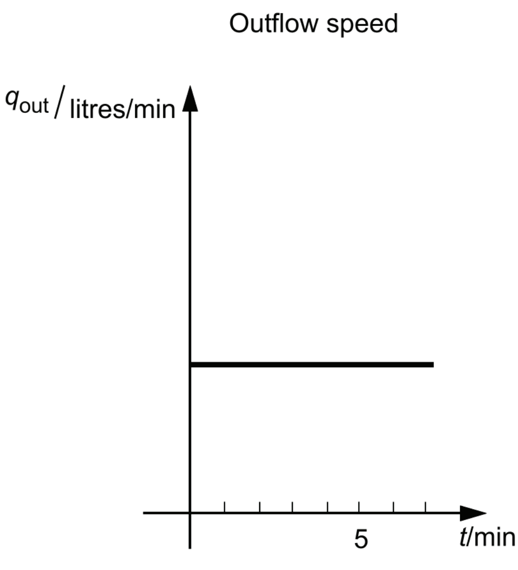


Diagram 2

- Calculate how much water flows in to and out of the container during the first 5 minutes if an outflow speed q_{ut} of 40 litres/min, is chosen.

- Calculate how much water the container *contains* after 2 minutes if an outflow speed of 40 litres/min, is chosen.

- Investigate and describe, as thoroughly as you can, how the water volume of the container depends on time and on the different choices of the constant outflow speed. (3/3/¤)