NATIONAL TEST IN MATHEMATICS COURSE D

SPRING 2009

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 90 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 10 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 18 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 43 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test: Pass: 12 points Pass with distinction: 26 points of which at least 7 ”Pass with distinction”- points. Pass with special distinction: 26 points of which at least 13 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on separate sheets of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

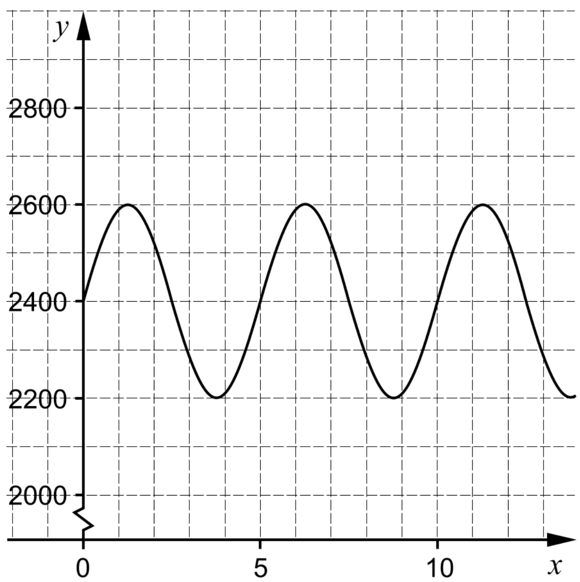
1. Determine an antiderivative F of $f(x) = 4x^3 + 8$ *Only answer is required* (1/0)

2. Differentiate

a) $f(x) = \sin 2x + \cos x$ *Only answer is required* (1/0)

b) $g(x) = \ln(2x+1)$ *Only answer is required* (0/1)

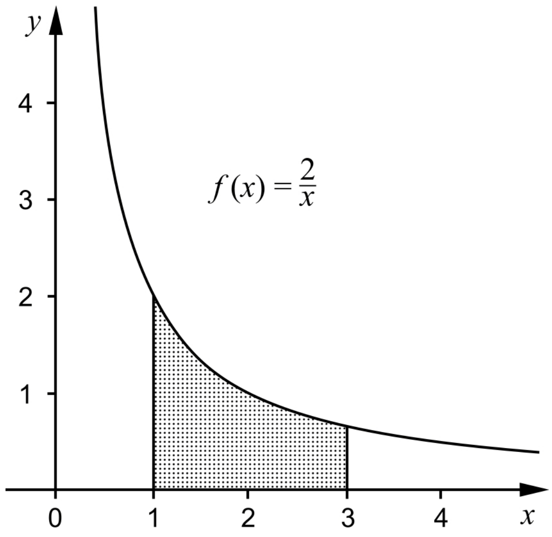
3. During normal breathing, the volume of air in the lungs of a test person varies according to the diagram below. If the volume of air after time x seconds is denoted y ml, the volume as a function of time can be described by $y = A\sin 1.26x + B$



Determine the values of the constants A and B. *Only answer is required* (2/0)

4. The figure shows the graph of the function $f(x)=\dfrac{2}{x}$

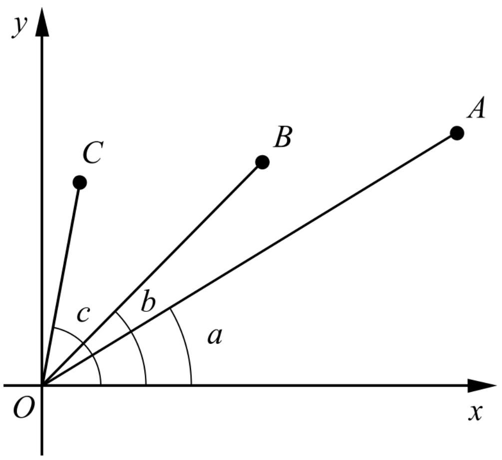
Calculate the area of the shaded region. (2/0)



5. The traffic flow is measured at a certain point on the Öland bridge on the day before Midsummer's Eve. The traffic flow in cars per minute is described by a function $f(t)$, where t is the time in minutes from 12 am.

In words, explain what $\int_{60}^{120} f(t)\,\mathrm{d}t = 3240$ means. (2/0)

6. Lines have been drawn from the points *A*, *B* and *C* in the coordinate system to the origin. The lines form different angles with the *x*-axis.



a = the angle between *AO* and the *x*-axis
b = the angle between *BO* and the *x*-axis
c = the angle between *CO* and the *x*-axis

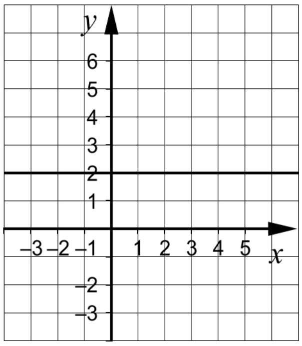
Rank the following values from the *smallest* to the *largest*

a) $\sin a$, $\sin b$ and $\sin c$ *Only answer is required* (1/0)

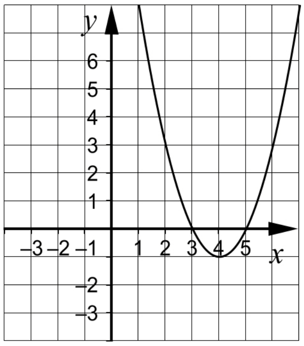
b) $\sin a$, $\cos a$, $\sin c$ and $\cos c$ *Only answer is required* (0/1)

7. Which of the graphs *A*-*F* shows the antiderivative of $f(x) = 2x - 2$?
Don't forget to justify your answer. (0/2)

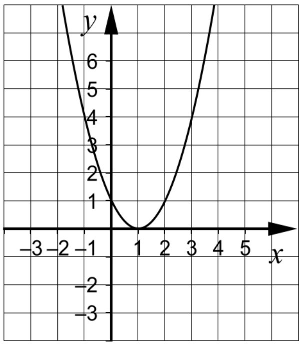
A



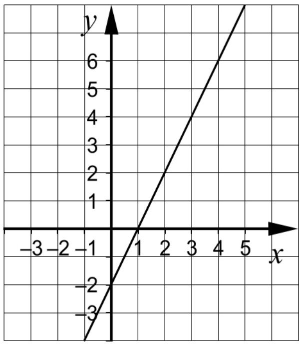
B



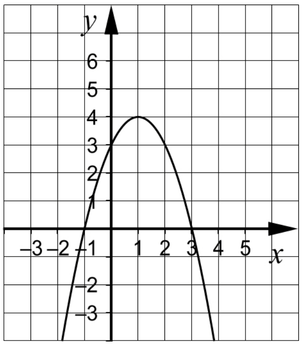
C



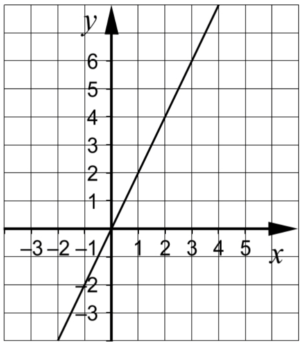
D



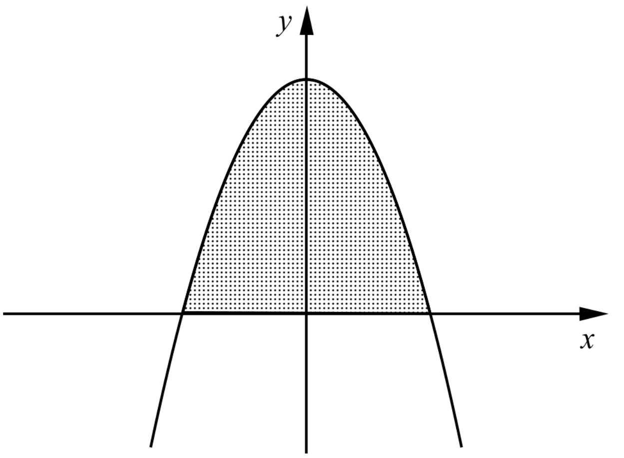
E



F



8. The figure below shows the graph of the function $y = c - x^2$ for a certain value of c where $c > 0$



a) Give an expression for the area of the shaded region and calculate the exact value when $c = 3$ (1/1)

b) Calculate c so the area of the shaded region is 4 area units. (0/2)

9. The function f is defined by $f(x) = 3x + \cos 2x$

Calculate any possible points where the derivative $f'(x)$ is zero and interpret what the derivative tells you about the function f. (1/1/¤)

10. Senad and Marja have tried to solve the equation $2\sin^2 x - \sin x = 0$
This is their solution:

We start by dividing both sides by 2: $\sin^2 x - \dfrac{\sin x}{2} = 0$

We now divide both sides by sin x: $\sin x - \dfrac{1}{2} = 0$

This is the continuation: $\sin x = \dfrac{1}{2}$

$$x = \frac{\pi}{6} + n \cdot 2\pi$$

They have not found all solutions.
Explain why and then find all solutions. (1/1/¤)

Part II

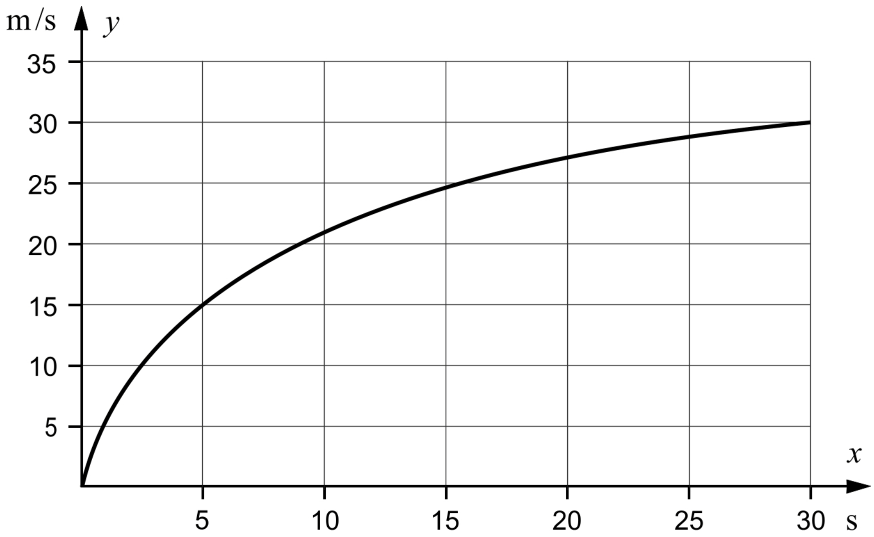
This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11. In triangle *ABC* , angle *A* = 64.4° angle *B* = 41.4°. The side *AC* is 137 cm.

Calculate the length of side *BC*. (2/0)

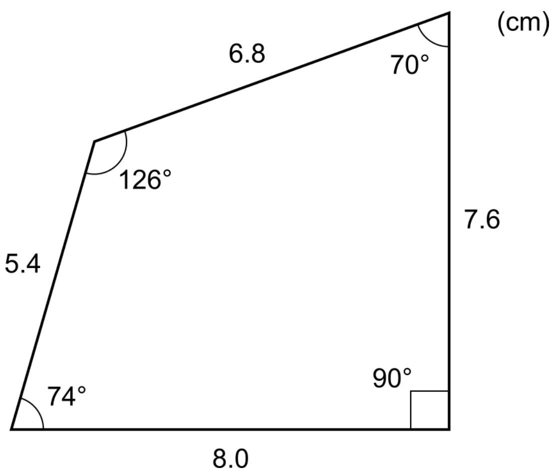
12. The figure shows the velocity graph of a car that accelerates from standing-still to a velocity of 30 m/s during 30 s.

Estimate the distance the car travels during this period of time. (2/0)



13. In the mid-90's, Sven bought a large piece of land. He is planning to sell a part of it at a price of 130 SEK/m^2. He marks the area he plans to sell on a map and then measures the sides and angles (see figure). The scale of the map is 1:500.

How much should he ask for this piece of land? (3/0)

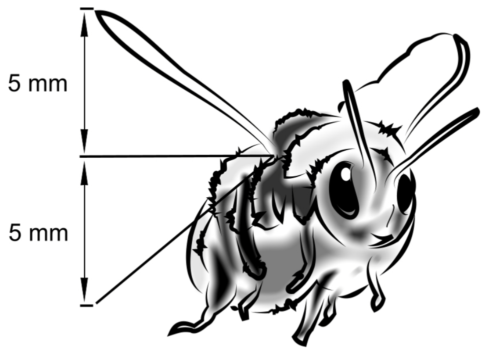


14. The wings of a bumblebee move periodically when it is flying. The vertical movement of the wing-tip can be described by the function

$y = 5 \cdot \sin 390\pi t$

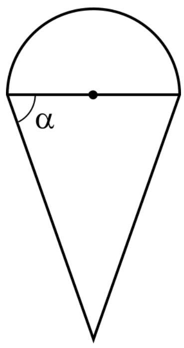
y = the vertical position of the wing-tip, in mm
t = time in seconds

How many times per second does the bumblebee beat its wings?
1 wing-beat corresponds to 1 period.

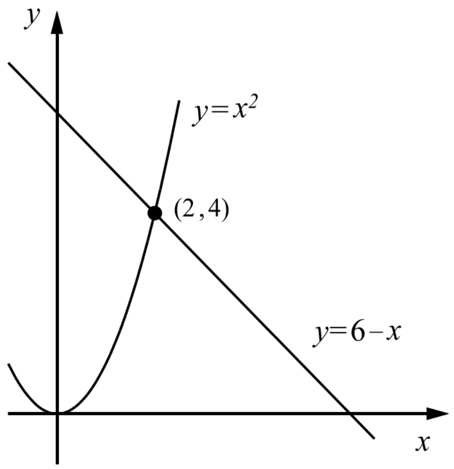


(1/1)

15. The figure shows a semicircle and an isosceles triangle, which both have the same area.
Determine $\tan\alpha$. (0/2/¤)



16. The figure shows a region bounded by the curve $y = x^2$, the line $y = 6 - x$ and the x-axis.



Calculate the circumference of the area. Answer with three significant digits.

You may use the fact that the length L of a curve $y = f(x)$ from $x = a$ to $x = b$ can be calculated with the formula $L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$ (0/2)

17. For what values of x in the interval $0 \le x \le 2\pi$ is $f(x) = \dfrac{1}{\tan x - 1}$ not defined? (1/1/¤)

When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

18. The function f is defined by $f(x) = x^k e^{-x}$, where k is a positive integer, greater than one. Your task is to investigate how the maximum-, minimum- and saddle points of f depend on the value of k

- Start by plotting the graph of f for some values of k (see table below) and determine the x-coordinates of possible maximum-, minimum- and saddle points:

k	x-coordinate for possible maximum	x-coordinate for possible minimum	x-coordinate for possible saddle point
2			
3			
4			
5			

- Formulate a conclusion on the relation between k and the x-coordinates for the maximum-, minimum- and saddle points of f respectively.

- Prove that your conclusions above are valid for all positive integer values of k

  You may use that $f'(x) = x^{k-1} e^{-x} (k - x)$ (2/5/¤)