NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2010

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 120 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 10 problems and **Part II** consists of 8 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 10 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 44 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test:

Pass:	13 points
Pass with distinction:	26 points of which at least 7 ”Pass with distinction”- points.
Pass with special distinction:	26 points of which at least 13 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Calculate $\int_1^2 4x^3 \, \mathrm{d}x$ (2/0)

2. Determine $f'\left(\frac{\pi}{2}\right)$ when $f(x) = x - 3\cos x$ (2/0)

3. Determine the antiderivative F to $f(x) = 4\mathrm{e}^x - x$ that satisfies the condition $F(0) = 1$ (2/0)

4. Differentiate

a) $f(x) = \mathrm{e}^{2x} - \sin x$ *Only answer is required* (1/0)

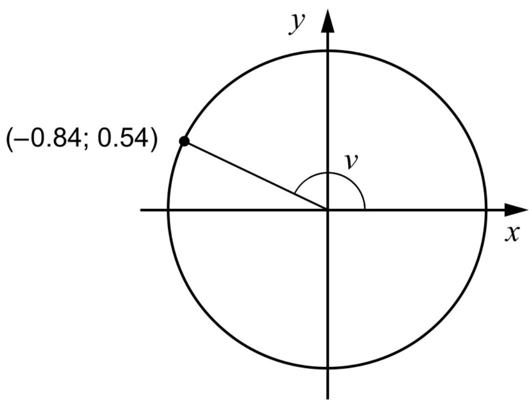
b) $g(x) = x \cdot \mathrm{e}^x$ *Only answer is required* (1/0)

c) $h(x) = \frac{1}{2x+1}$ *Only answer is required* (0/1)

5. The figure below shows a unit circle.

a) Determine $\sin v$ *Only answer is required* (1/0)

b) Determine $\sin(v + 540°)$ *Only answer is required* (0/1)



6. It holds that $\cos 36^{\circ} \approx 0.809$
Use this to determine all solutions to the equation $\cos 3x = 0.809$ (2/1)

7. Simplify $(\sin x + \cos x)^2 - \sin 2x$ as far as possible. (1/1)

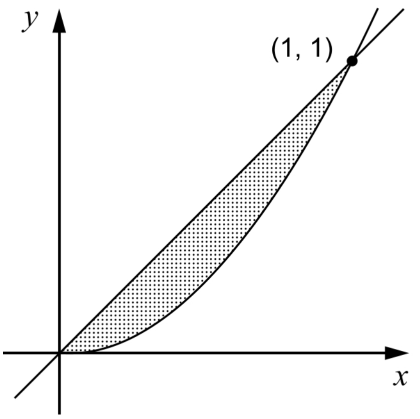
8. A region in the first quadrant is bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = a$ and $x = 3a$. Show that the area of the region does not depend on a. (0/2/¤)

9. The function f is defined by $f(x) = x^2 + \sin x$
Show that if f has an extremum, then it is a minimum. (0/2/¤)

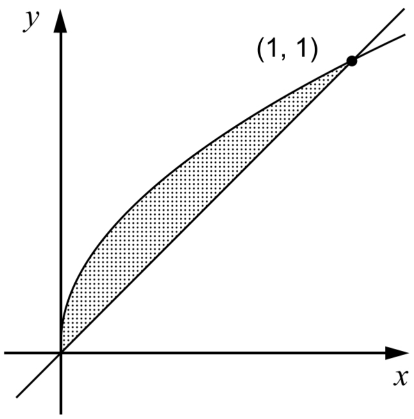
When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

10. In this problem you will compare the area of the region between $y = x$ and $y = x^k$ with the area of the region between $y = x$ and $y = x^{\frac{1}{k}}$ where $k > 1$



The graphs of $y = x$ and $y = x^k$



The graphs of $y = x$ and $y = x^{\frac{1}{k}}$

In the first point, we have chosen $k = 2$

- Calculate the area of the region between $y = x$ and $y = x^2$ and the area of the region between $y = x$ and $y = x^{\frac{1}{2}}$ and compare these areas.

- Compare the area of the region between $y = x$ and $y = x^k$ with the area of the region between $y = x$ and $y = x^{\frac{1}{k}}$ for one/some other values of *k*.

- Formulate a conclusion based on your comparisons.

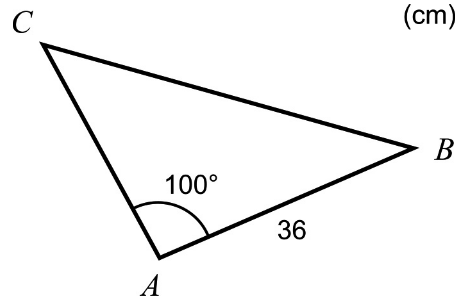
- Show that your conclusion is true regardless of which value of *k* you choose.

(3/4/¤)

Part II

This part consists of 8 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11.

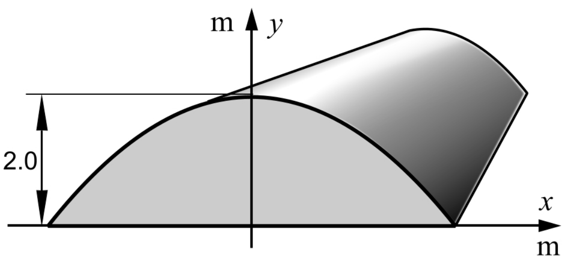


The area of the triangle *ABC* is 520 cm^2. What is the length of side *AC*? (2/0)

12. A 2.0-metre high earth bank will be built next to a motorway as noise protection. The shape of the earth bank can be described by a quadratic equation

$y = 2.0 - 0.125x^2$

Calculate how many m^3 of soil are needed per kilometre of earth bank.



(3/0)

13. The survival of the Peregrine Falcon was seriously threatened from the 1950s until the 1990s. In the mid 1970s the Swedish Society for Nature Conservation (SSNC) launched "Project Peregrine" in order to save the species.

© Photo: Mats Hamrén

When the project had lasted for a considerable time the situation could be described mathematically with the differential equation:

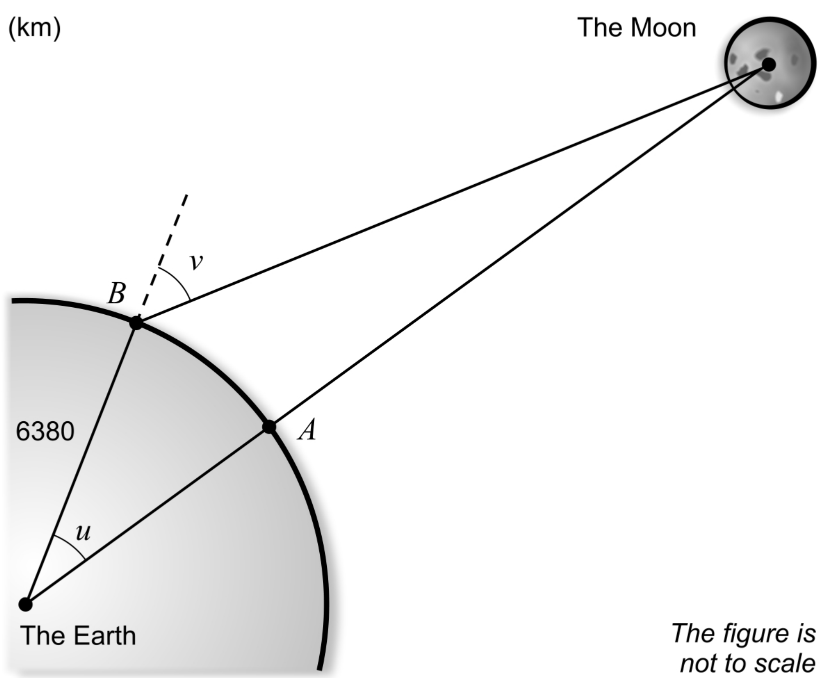
$\frac{\mathrm{d}y}{\mathrm{d}t} = 0.15 \cdot y$, where y is the number of peregrines at time t years from the year 2001.

In your own words, explain the meaning of the differential equation in this context. (1/1)

14. The Greek Ptolemaeus lived in Alexandria around 150 AD. He used the following method to calculate the distance between the Earth and the Moon.

When the Moon was at the zenith (directly above) in A the angle v to the Moon was measured from another spot B far from A . To get simultaneous measurements these were carried out during an eclipse of the Moon.

When the distance between A and B, the radius of the Earth and the angle v were known the distance between the centre of the Earth and the centre of the Moon could be calculated.



The measured values are presented in the table below:

The angle v	4.60°
The distance between A and B (along the earth's surface)	500 km
The radius of the Earth	6380 km

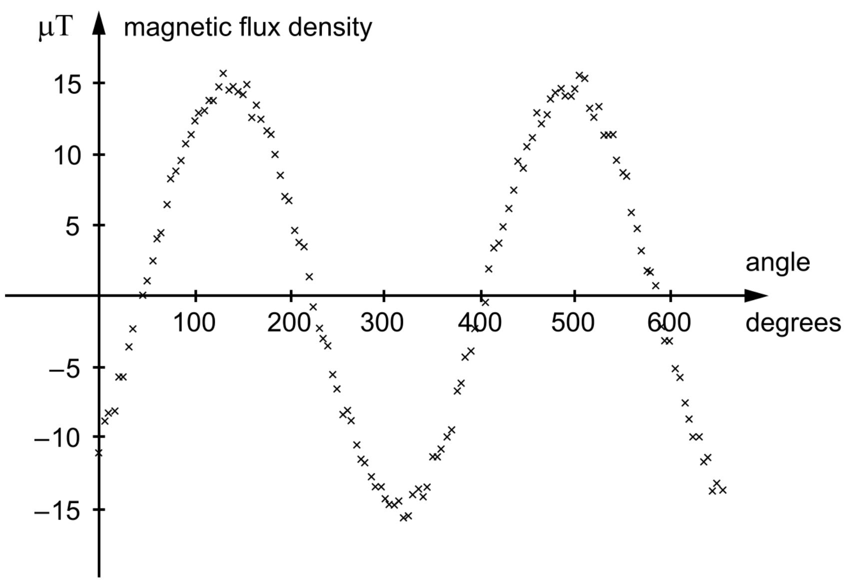
From these values the angle u can be calculated, $u = 4.49°$

Calculate the distance between the centre of the Earth and the centre of the Moon based on the values given in the table above and the calculated value u. (2/0)

15. Calculate the area of the region in the first quadrant bounded by the curve $y = e^{-x}\sqrt{x}$, the line $y = 2 - x$ and the y-axis.
Give your answer with at least three significant figures. (0/2)

16. The graph below shows the results of a measurement of the Earth's magnetic flux density carried out in Uppsala. The flux density is a measure of the strength of the magnetic field and is given in Tesla (T).

The measurement was carried out as follows: First the flux density was measured in a certain direction along the Earth's surface. The measuring instruments were then rotated $5°$ clockwise and a new measurement was carried out. This was repeated until measurements had been carried out for almost two complete revolutions.

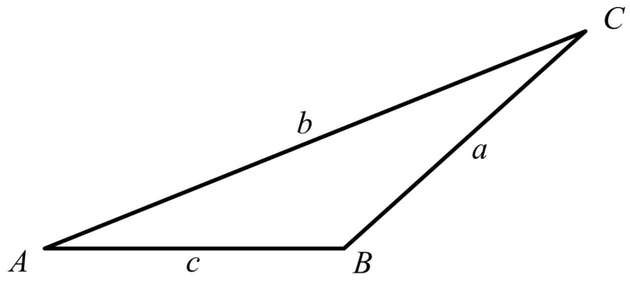


The vertical axis shows the magnetic flux density in μT (microtesla) and the horizontal axis shows the turning of the measuring instruments in relation to the starting position.

The maximum value of the magnetic flux density, $15\,\mu T$, was measured to the north.

Determine a sine function that describes the graph above. (0/2)

17. In the triangle *ABC*, *B* is an obtuse angle.



Show, without using the law of sines, that $b \sin A = a \sin B$ (0/2/¤)

18. Investigate the integral $\int_a^b (x^2 - 1)\,\mathrm{d}x$ when $a < b$

a) Determine *a* and *b* so that the value of the integral is as small as possible. (0/1)

b) Determine what values the integral can assume. (0/1/¤)