NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2011

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 135 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 18 problems. **Part I** consists of 11 problems and **Part II** consists of 7 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 11 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 45 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”.

Lower limit for the mark on the test

Pass:	13 points.
Pass with distinction:	26 points of which at least 7 ”Pass with distinction”- points.
Pass with special distinction:	26 points of which at least 14 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 11 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Evaluate $\int_0^3 (4-x^2)\,dx$ (2/0)

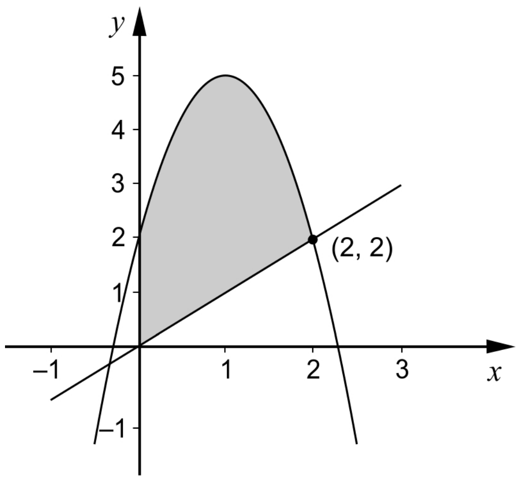
2. Differentiate

a) $f(x)=\sin 3x$ *Only answer is required* (1/0)

b) $g(x)=(1+2x)^{11}$ *Only answer is required* (1/0)

c) $h(x)=x^2\cdot e^{3x}$ *Only answer is required* (0/1)

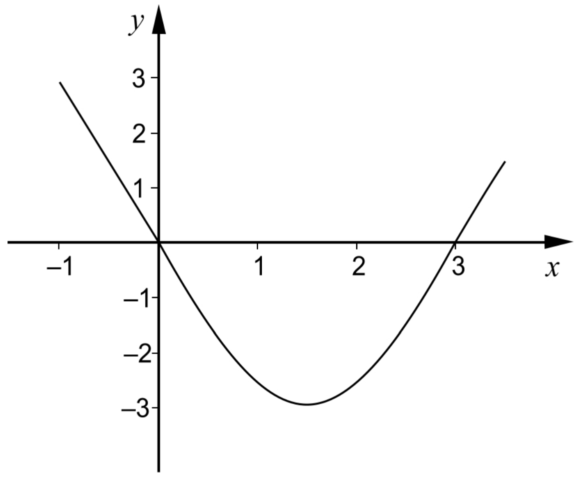
3. The figure shows a region bounded by the y-axis, the curve $y=6x-3x^2+2$ and the line $y=x$. Calculate the area of the region. (2/0)



4. Find the anti-derivative $F(x)$ of $f(x)=\dfrac{2}{x}+3$ that satisfies the condition $F(1)=5$ (2/0)

5. Solve the equation $\cos 2x = 0.9$ if $\cos\ 26° = 0.9$ (2/1)

6. The figure shows the graph of the function *f*.



Arrange the numbers *A*, *B* and *C* in order of magnitude. Start with the *smallest*.

$A = \int_{-1}^{3} f(x)\mathrm{d}x$ $B = \int_{0}^{3} f(x)\mathrm{d}x$ $C = \int_{-1}^{0} f(x)\mathrm{d}x$ *Only answer is required* (1/0)

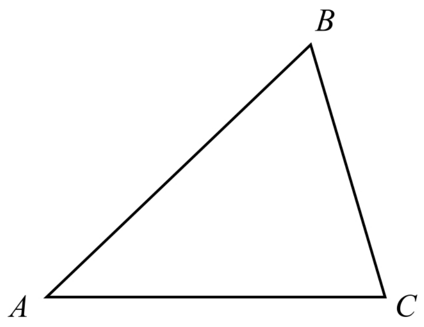
7. It holds for the function *f* that $f(2) = 3$ and $f'(x) = 0.5$ for all *x*.

Evaluate $\int_{2}^{6} f(x)\,\mathrm{d}x$. (0/2)

8. Show that $\dfrac{\sin 2v + \sin v}{2\cos v + 1} = \sin v$

for all *v* where expressions on both sides are defined. (0/1/¤)

9. In the acute-angled triangle *ABC*, $\sin A = 0.6$



a) Determine the value of $\sin(B+C)$ (0/1)

b) Determine the value of $\cos(B+C)$ (0/2/¤)

10. Timo and Peder have been given the task of solving the following problem without a calculator:

> $F(x) = (x+2)(x-2)^3$ is the anti-derivative of $f(x) = 4(x+1)(x-2)^2$
>
> Evaluate $\int_{-2}^{3} (x+1)(x-2)^2 \mathrm{d}x$

Timo says that he will first expand $(x+1)(x-2)^2$ and then evaluate the integral. Peder claims that there is a quicker way to solve the problem.

a) Describe a method that Peder may have considered using. (0/1/¤)

b) Solve the problem with a method of your choice. (0/1)

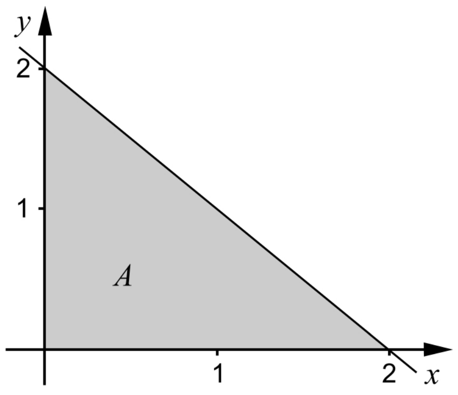
When assessing your work with this problem, the teacher will take into consideration:
- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

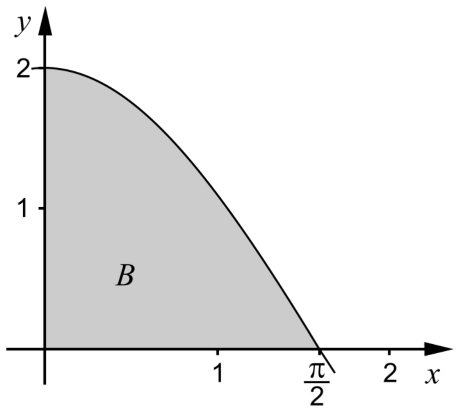
11. In this problem you are going to compare the size of the areas of two regions, *A* and *B*.

Region *A* is bounded by the positive *y*-axis, the positive *x*-axis and the line $y = 2 - kx$

Region *B* is bounded by the positive *y*-axis, the positive *x*-axis and the curve $y = 2\cos kx$

The areas formed when $k = 1$ are shown below.





- Calculate the area of the shaded regions *A* and *B* when $k = 1$, that is when $y = 2 - x$ and $y = 2\cos x$

- Copy the table below and calculate the missing values.

k	Area of *A*	Area of *B*
1		
2		
3		

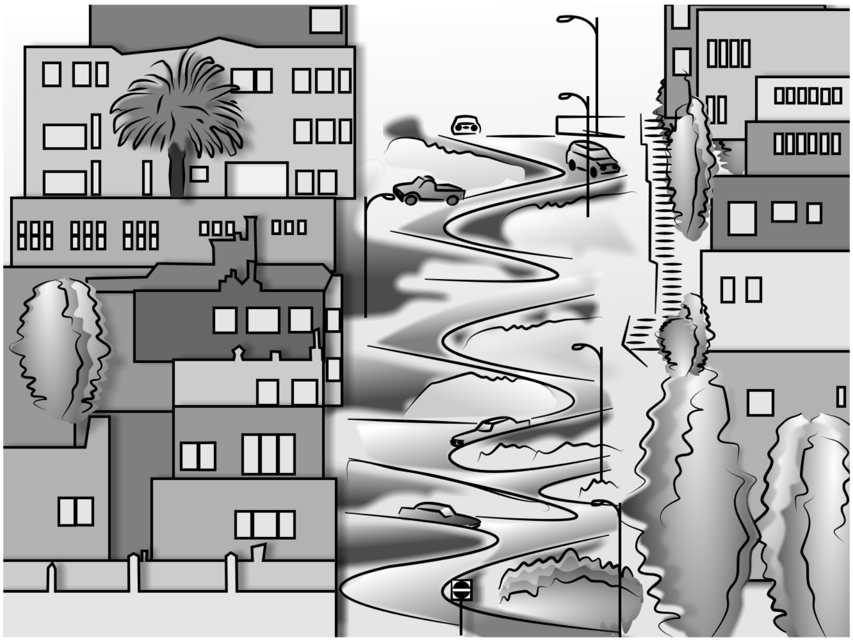
- Compare the areas of the regions *A* and *B* for the same value of *k*. Formulate a conclusion from your comparison.

- Show that your conclusion applies to all $k > 0$ (2/4/¤)

Part II

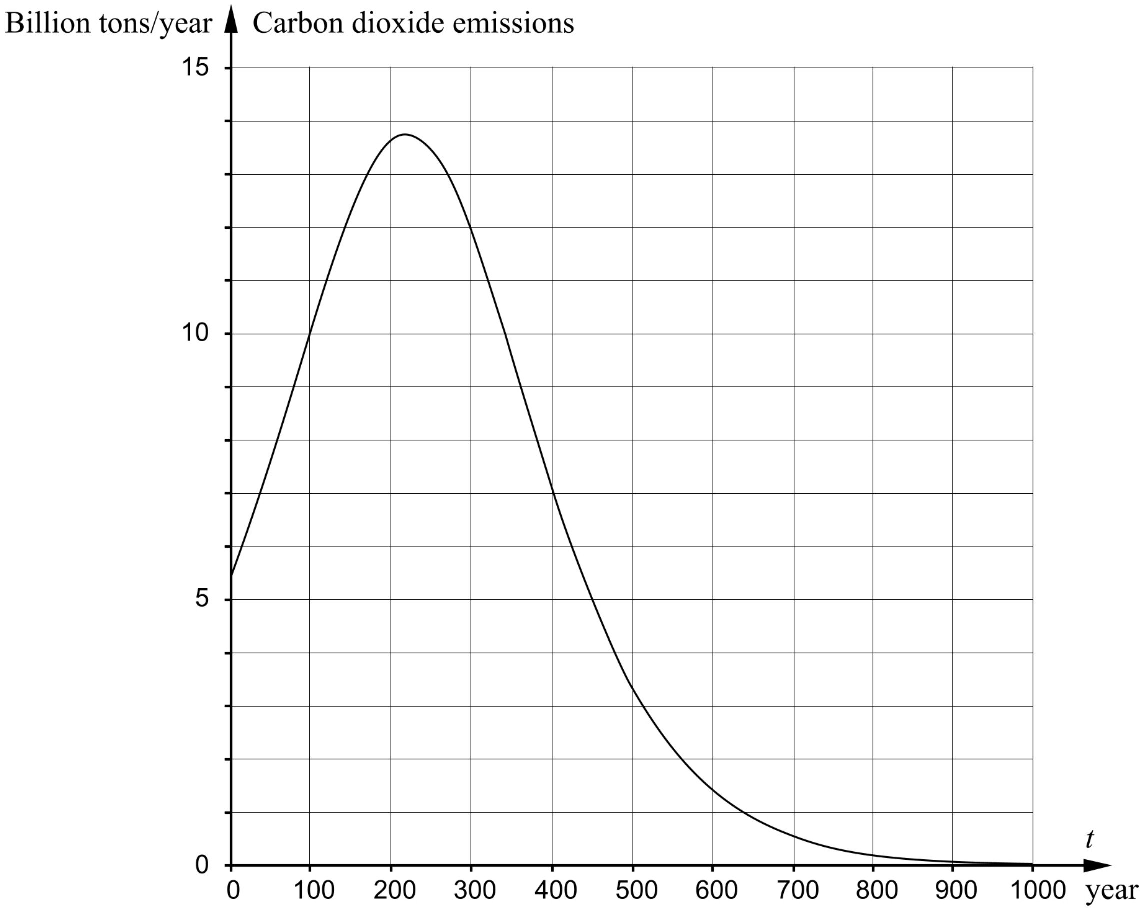
This part consists of 7 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

12. Lombard Street in San Francisco is famous for being located on a hill side with a slope that is so steep that the road has been built in a zigzag.



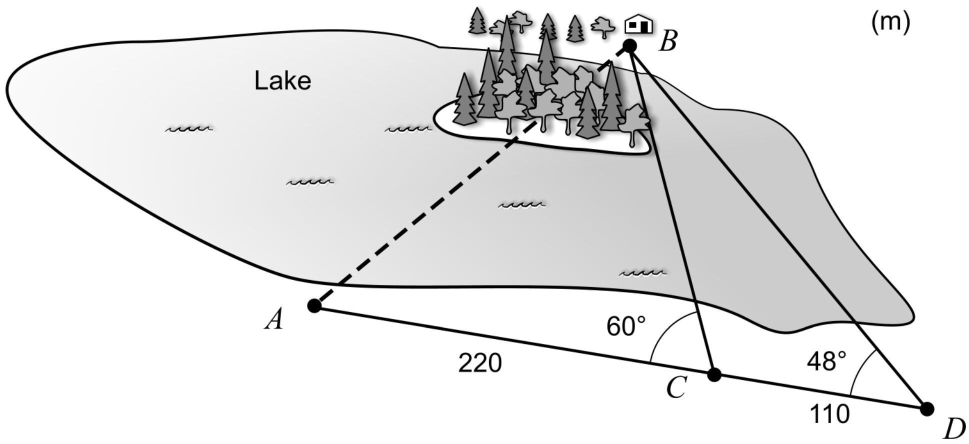
The steepest part of the hill side is 400 metres long and on that distance the difference in altitude is 182 metres.
What is the inclination in degrees between the slope of the hill side and the horizontal plane? (2/0)

13. A prognosis for the yearly carbon dioxide emissions in the world for the coming thousand years is described in the graph below. Time $t = 0$ corresponds to the year 2000.



Use the graph to estimate how much carbon dioxide that will be discharged between the years 2100 and 2400. (2/0)

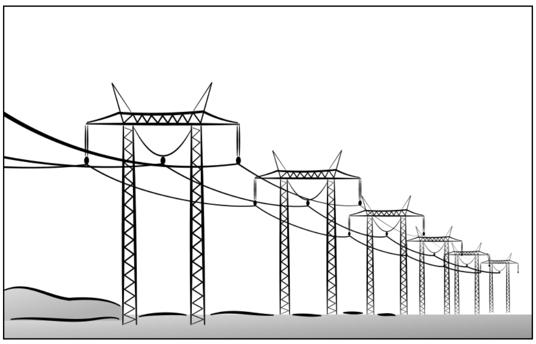
14. The distance between the two points *A* and *B* on opposite sides of a lake is to be determined, see figure.



A land surveyor who is in *A* cannot see *B* that is hidden by an arboreous islet in the lake. She can see *B* from the two points *C* and *D* that together with *A* lie along a straight line. She measures the angle *ACB* to $60°$ and the angle *ADB* to $48°$, the distance *AC* to 220 m and the distance *CD* to 110 m.

Calculate the distance *AB*. (2/1)

15. Aerial lines can handle higher current load when it is windy.



For a certain aerial line, the current load limit is given by the function

$$S(x) = 342 \cdot (1+4x)^{0.25}$$

where x is the wind force in m/s and $S(x)$ is the current load limit in amperes, A.

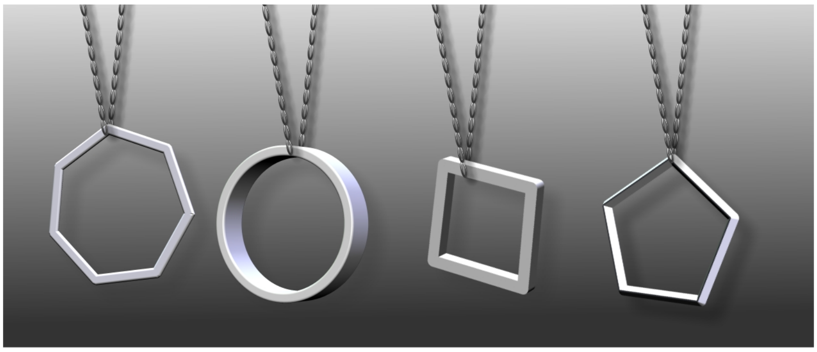
a) Calculate the current load limit when it is windless. (1/0)

b) At what wind force does the current load limit increase at a rate of 50 A/(m/s) ? (0/2)

16. Determine a function of the form $y = A\sin kx + B$ that satisfies the conditions below:

- $A > 0$
- The range is $-4 \leq y \leq 2$
- The local maxima have *x*-coordinates $x = \frac{\pi}{8} + n \cdot \frac{\pi}{2}$ for all integers *n* (1/1)

17.

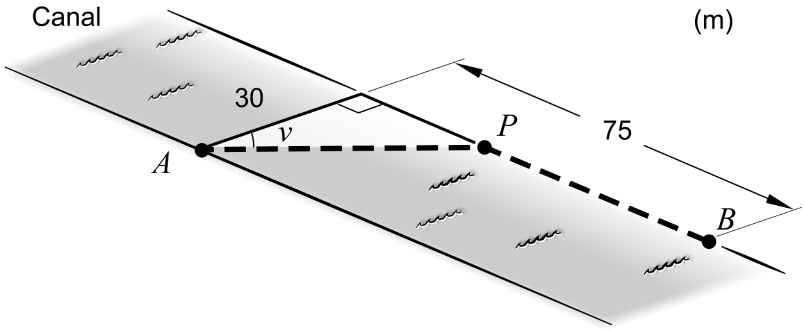


Armand works as a silversmith and his speciality is jewellery in the shape of different geometric figures. He has decided to create a piece of jewellery in the shape of a triangle. At his disposal he has a silver thread of length 9.0 cm that he can bend and cut.

Armand labels the triangle *ABC* and decides that the angle *A* should be 30°, side *AB* 4.2 cm and side *BC* 3.2 cm.

Investigate how this piece of jewellery may be shaped. (1/2/¤)

18. The points *A* and *B* are situated on opposite sides of a 30 m wide canal, see figure.



A cable should run from point *A* to point *B*. The cable should first run through the water to a point *P* and then on land along the edge of the canal to point *B*.
The cost for the cable run is SEK 2500 per metre in water and SEK 1500 per metre on land.
Calculate angle *v* so that the cost for the cable run is as low as possible. (0/3/¤)