NATIONAL TEST IN MATHEMATICS COURSE D
SPRING 2012

Directions

Test time	240 minutes for Part I and Part II together. **We recommend that you spend no more than 135 minutes on Part I.**
Resources	**Part I:** ”Formulas for the National Test in Mathematics Course D.” *Please note that calculators are not allowed in this part.* **Part II**: Graphic calculators or Symbolic calculators and ”Formulas for the National Test in Mathematics Course D.”
Test material	The test material should be handed in together with your solutions. Write your name, the name of your education programme/adult education on all sheets of paper you hand in. *Solutions to Part I should be handed in before you retrieve your calculator. You should therefore present your work on Part I on a separate sheet of paper. Please note that you may start your work on Part II without a calculator.*
The test	The test consists of a total of 17 problems. **Part I** consists of 10 problems and **Part II** consists of 7 problems. For some problems (where it says *Only answer is required*) it is enough to give short answers. For the other problems short answers are not enough. They require that you write down what you do, that you explain your train of thought, that you, when necessary, draw figures. When you solve problems graphically/numerically please indicate how you have used your resources. Problem 10 is a larger problem which may take up to an hour to solve completely. It is important that you try to solve this problem. A description of what your teacher will consider when evaluating your work is attached to the problem. Try all of the problems. It can be relatively easy, even towards the end of the test, to receive some points for partial solutions. A positive evaluation can be given even for unfinished solutions.
Score and mark levels	The maximum score is 43 points. The maximum number of points you can receive for each solution is indicated after each problem. If a problem can give 2 ”Pass”-points and 1 ”Pass with distinction”-point this is written (2/1). Some problems are marked with ¤, which means that they more than other problems offer opportunities to show knowledge that can be related to the criteria for ”Pass with Special Distinction”. Lower limit for the mark on the test

Pass:	12 points.
Pass with distinction:	25 points of which at least 7 ”Pass with distinction”- points.
Pass with special distinction:	25 points of which at least 14 ”Pass with distinction”- points. You also have to show most of the “Pass with special distinction” qualities that the ¤-problems give the opportunity to show.

Part I

This part consists of 10 problems that should be solved without the aid of a calculator. Your solutions to the problems in this part should be presented on a separate sheet of paper that must be handed in before you retrieve your calculator. Please note that you may begin working on Part II without the aid of a calculator.

1. Find $f'(\pi)$ of the function $f(x) = \sin x$ (2/0)

2. Differentiate

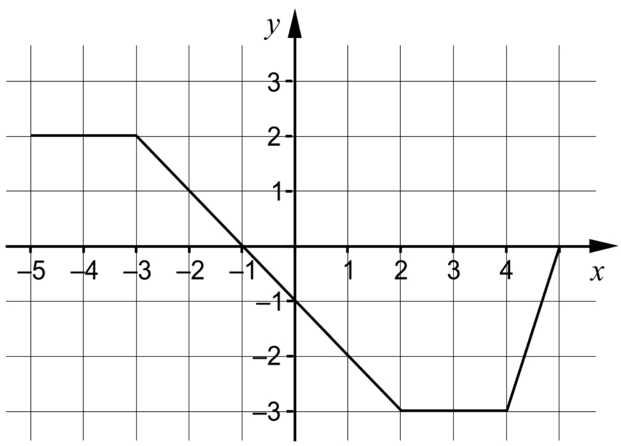
a) $f(x) = 2\cos 3x$ *Only answer is required* (1/0)

b) $g(x) = x^2 \cdot e^x$ *Only answer is required* (1/0)

3. Evaluate $\int_1^e \left(\frac{1}{x} + 2x\right) dx$ and simplify as far as possible. (2/0)

4. Find all solutions to the equation $2\sin x = \sqrt{2}$ (2/0)

5. The figure shows the graph of the function $y = f(x)$ in the interval $-5 \le x \le 5$



a) Evaluate $\int_{-5}^{1} f(x)\, dx$ *Only answer is required* (1/0)

b) Determine a so that $\int_{-5}^{a} f(x)\, dx = 0$ *Only answer is required* (0/1)

6. Given the function $f(x) = 2 + (\sin x + \cos x)^2$

a) Show that $f(x) = 3 + \sin 2x$ (1/1)

b) What is the largest and smallest value of the function f (0/1)

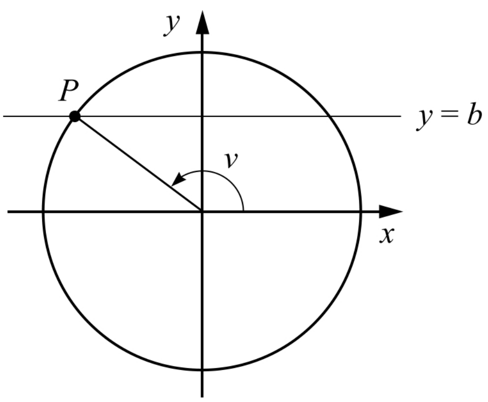
7. The function f has a local extremum at (1, 2) and the second derivative is $f''(x) = 8 - 6x$

a) Decide whether the given extremum is a local maximum or a local minimum. (1/0)

b) Find $f(x)$ (0/2)

8. Show that $\dfrac{2}{1+\cos 2x} = 1 + \tan^2 x$ for all *x* where the expressions are defined. (0/2/¤)

9. The figure shows a unit circle where one angle *v* and a point *P* are marked. The point *P* is in the second quadrant and the line $y = b$ passes through the point *P*.



Find $\tan v$ expressed in *b*. (0/2/¤)

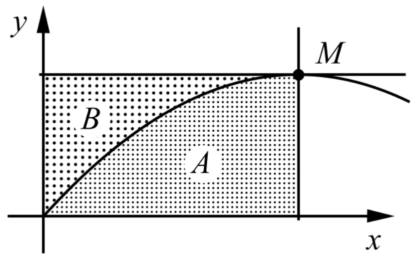
When assessing your work with this problem, the teacher will take into consideration:

- How well you carry out your calculations
- How close to a general solution you are
- How well you justify your conclusions
- How well you present your work
- How well you use mathematical language

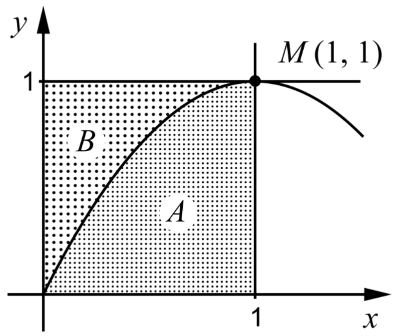
10. In this problem you are going to compare the size of the areas of two regions *A* and *B*.

Region *A* is bounded by the positive *x*-axis, the curve $y = 2kx - x^2$ and a vertical line through the maximum of the curve, *M*.

Region *B* is bounded by the positive *y*-axis, the curve $y = 2kx - x^2$ and the tangent to the curve at the maximum *M*.



- Start with the case $k = 1$. Point *M* now has coordinates $(1, 1)$



Calculate the areas of *A* and *B*.

- Now investigate the cases where $k = 2$ and $k = 3$
Summarize your results in a table.

k	M	Area of A	Area of B
1	$(1, 1)$		
2			
3			

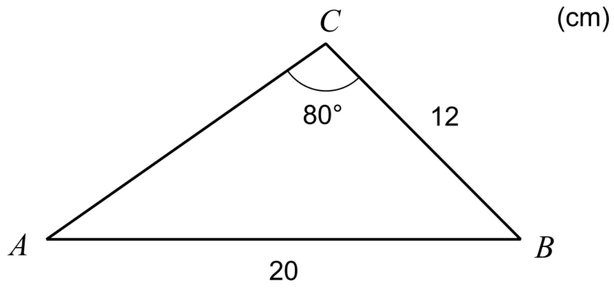
- Compare the areas of *A* and *B* for the same value of *k*.
Formulate a conclusion from your comparison.

- Show that your conclusion holds for all $k > 0$ (2/4/¤)

Part II

This part consists of 7 problems and you may use a calculator when solving them.
Please note that you may begin working on Part II without a calculator.

11. In the triangle ABC, angle C is $80°$ and sides AB and BC are 20 cm respectively 12 cm.

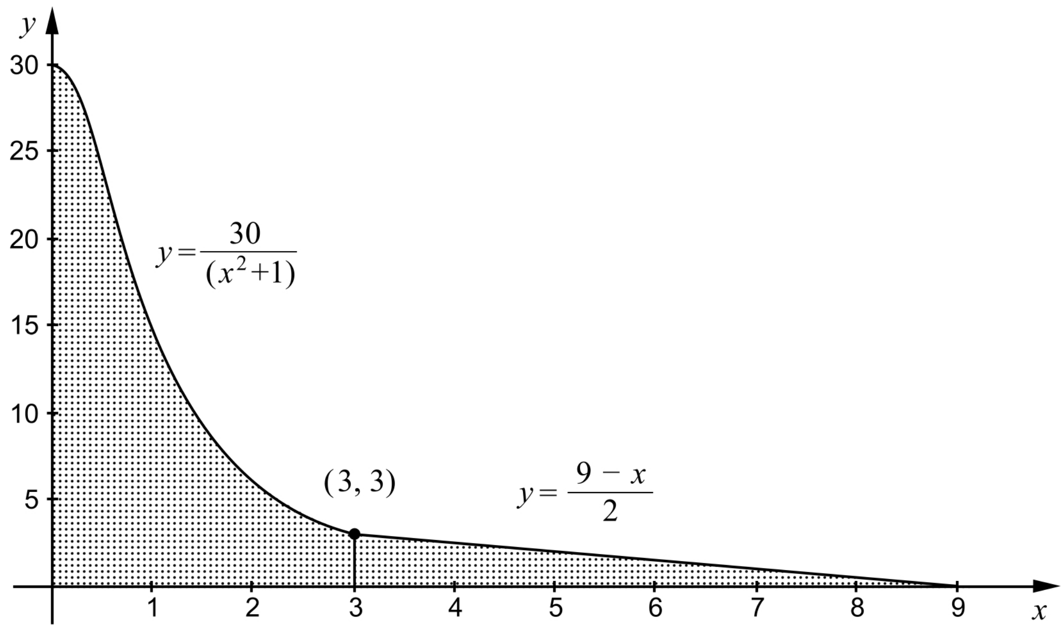


a) Determine angle A. (1/0)

b) Calculate the area of the triangle. (1/0)

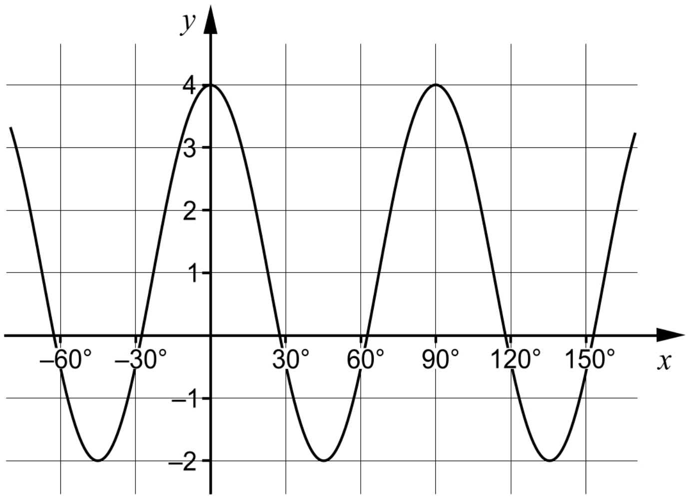
12. The figure shows a region bounded by the curve $y = \dfrac{30}{(x^2+1)}$ in the interval $0 \le x \le 3$,

the line $y = \dfrac{9-x}{2}$ in the interval $3 \le x \le 9$ and the coordinate axes.



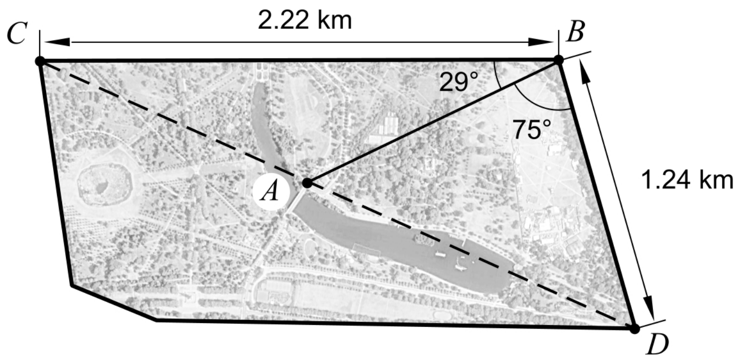
Calculate the area of the region pointed out. Your answer must have at least three significant digits. (3/0)

13. The equation of the curve below can be written in the form $y = A + B\cos kx$.



Determine the constants *A*, *B* and *k*. (1/1)

14. Steve trains carrier pigeons. At a show in Hyde Park, London, Steve will let a pigeon fly from *A* to *B*. Point *A* is situated on the diagonal *CD*, see figure.



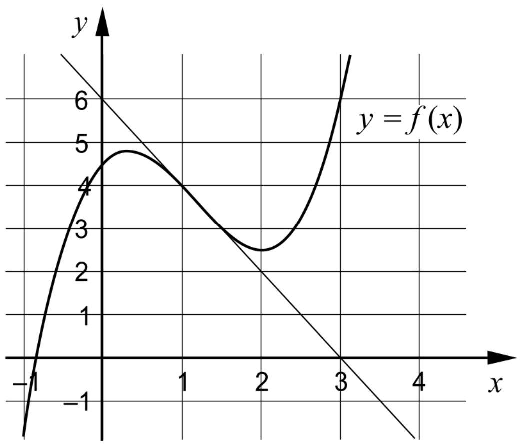
Steve wants to know the distance from *A* to *B* to be able to decide which of his pigeons to choose. Help him calculate the distance *AB*. (2/1)

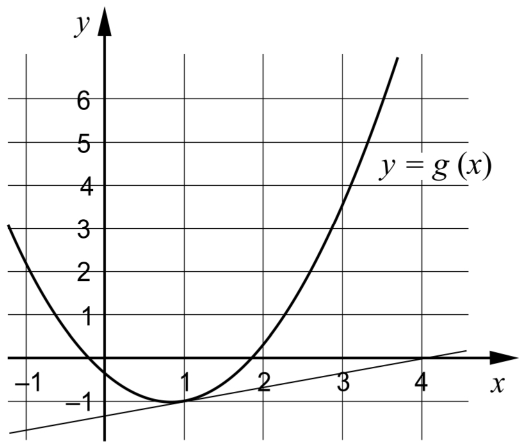
15. An air-filled balloon with a volume of $5000\ \text{cm}^3$ springs a leak. According to a simplified model, the volume decreases at a speed of $(20 - 0{,}01t)\ \text{cm}^3/\text{s}$, where *t* is the time in seconds from the time when the leak arises.

a) What is the volume of air that leaks out during the first 60 seconds? (0/1)

b) How long does it take before the balloon is empty? (0/2)

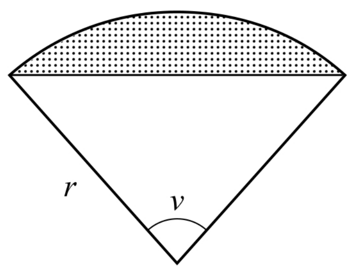
16. The figures show the curves $y = f(x)$ and $y = g(x)$ and also the tangents to these at $x = 1$





Let $h(x) = f(x) \cdot g(x)$ and determine $h'(1)$. (0/2/¤)

17. The figure shows a circular sector where a circular segment is marked.



Calculate the angle v, on the interval $0 < v < \pi$, so that the area of the circular segment is 25 % of the area of the circular sector. Your answer must have at least three significant digits. (0/2/¤)